Manufacturing of Composites
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Lecture - 04
Composites: Properties

Welcome to lecture 4 in the course of Manufacturing to Composites. So, in this lecture we will predominantly focus on how to evaluate the properties of composites. So, till now what we have covered is, we have covered introduction of composites, then we covered matrices function of matrices, then we went into fiber, different forms of fiber, different fiber materials and then we also found out the function of fiber reinforcement in a composite.

So now, what we do is, we will get into the properties and understand little bit of composites calculations. So, in this lecture, we will try to cover how to calculate the volume fraction, then you will try to cover the elastic behavior and longitudinal loading.

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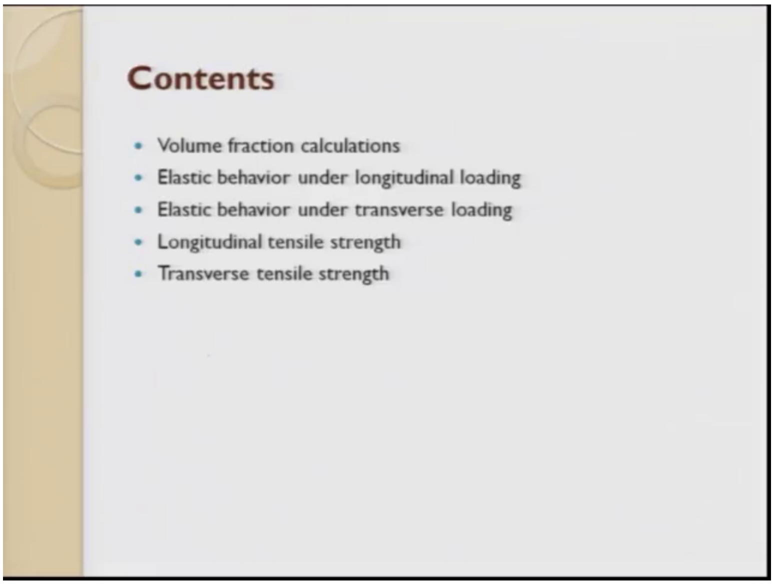

Then we will see the elastic behavior under transverse loading, then we will try to continue with longitudinal tensile strength and transverse tensile strength.

So, basically if you see in elastic region and in plastic region, we are trying to find out properties in for a given composite. So, the property of a composite, are function of the properties of constituent phase, and different relative prepositions size shape distribution and orientation of the dispersed phase, tries to dictate the property of a composite.

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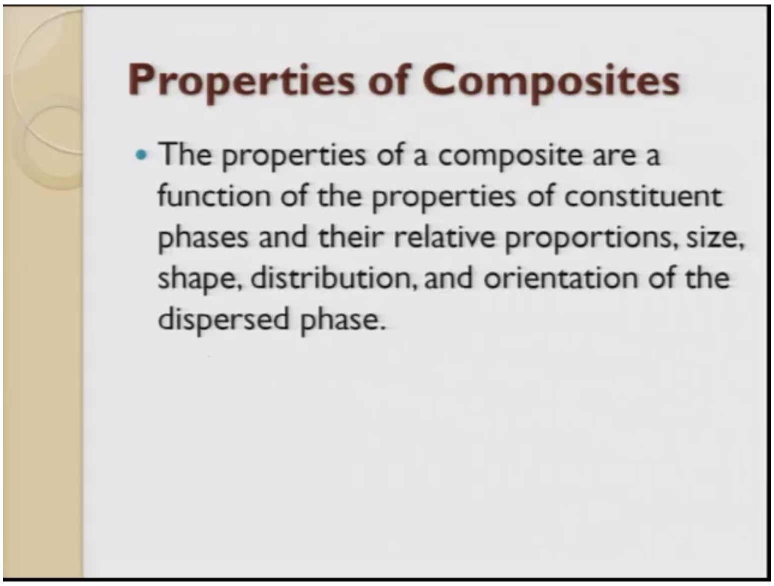

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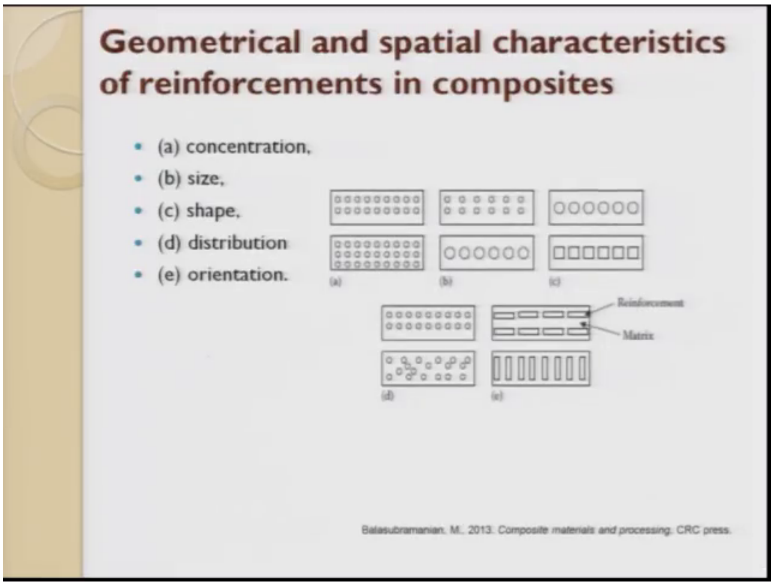

So, if you would look into the geometrical and special characteristics of reinforcement in a composite, the concentration size shape distribution orientation plays very important role in deciding the property of a composite. The figure a shows two different concentrations; basically concentration here means, the amount of reinforcement present in a composite to a crude extent or to a large extent, it is called as a volume fraction, and you can see as a volume fraction increases in the bottom figure of a, you can see the properties change.

The next is size. So, you can see a smaller circle and a larger circle. Smaller circle means it has larger surface area, lesser volume. So, it has more interface. So, this interface tries to enhance their wettability property, or the adhesion between the polymer and the reinforcement. So, smaller the size larger the surface area, larger the surface area means. So, you will have more surface area to come in contact with the polymer, more wetting more interface, so stronger composite. So, if you move to the next one, is the shape. You can see there are two different shapes, we have put here this shapes influence the characteristics of the composite.

Next one is the distribution, you have uniform distribution and non uniform distribution of reinforcement. Non uniform reinforcement, this is very common as far as metal matrix composite per or particulate composite, ceramic matrix composite and all. It is a very common feature where in which the agglomeration happens in the reinforcing zone. So, all these reinforcement get called or gets attached or agglomerated at one portion, and they do not have uniform distribution of strength across the composite. The last one is the orientation. So, you can see the orientation of fiber in two different directions. When you have orientation in different directions, it will try to give you different properties.

Suppose, let us assume whichever is in the horizontal line is zero degrees, and vertical lines which are there, which are referring to 90 degrees, then you will have different properties. There was zero orientation if the load is taken along the direction. they will have higher strength as compared to that of 90 degrees. If you want to stack alternatively these two, then it will give a hybrid property.

So, concentration size shape distribution and orientation. All these factors add and play an important role, as far as the property of the composite is concerned. Now let us try to understand the volume fraction and weight fraction. The volume fraction and weight fraction are generally talked about in composites, because they play an important role in deciding the amount of reinforcement added.

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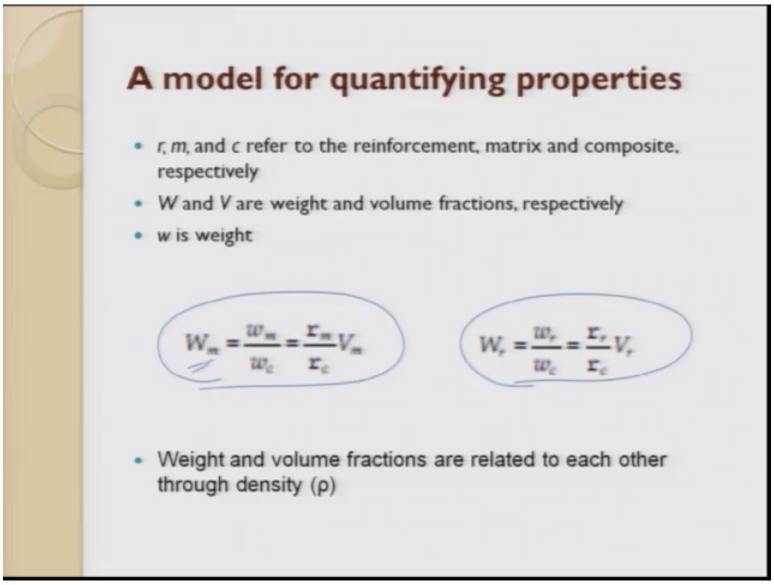

Generally when a manufacturing engineer talks, he always talks about weight fraction, a theoretician or if you want to do a first principle theoretical calculation, we always talk in terms of volume fraction. These two are interrelated by density. Density is nothing but mass by volume. So, now, let us look into the model, which we have developed for quantifying the properties. So, here a suffix r m c refers to reinforcement matrix, and composite respectively. This is going to be common for all through the presentation of today's lecture, where W and V are the weights and volume fractions. So, these two can be interrelated by the term using density small w is called as the weight.

So, if you look at this w whichever is there; that is w suffix m, means the weight of the matrix is equal to w, which is weight of the matrix divided by weight of the composite, which is equal to r suffix m divided by r suffix c. So, this r is the amount of reinforcement in the matrix, and the reinforcement in the composite. This is multiplied volume of the matrix.

So, this tries to give the weight fraction of the composite next of the matrix. And if you want to find out the weight fraction of the reinforcement, what we do is. We divide weight of the reinforcement divided by weight of the composite, which is equal to the reinforcement divided by the reinforcement of the composite multiplied by volume fraction. So, I have given here weight fraction and volume fraction, are related to each other through density.

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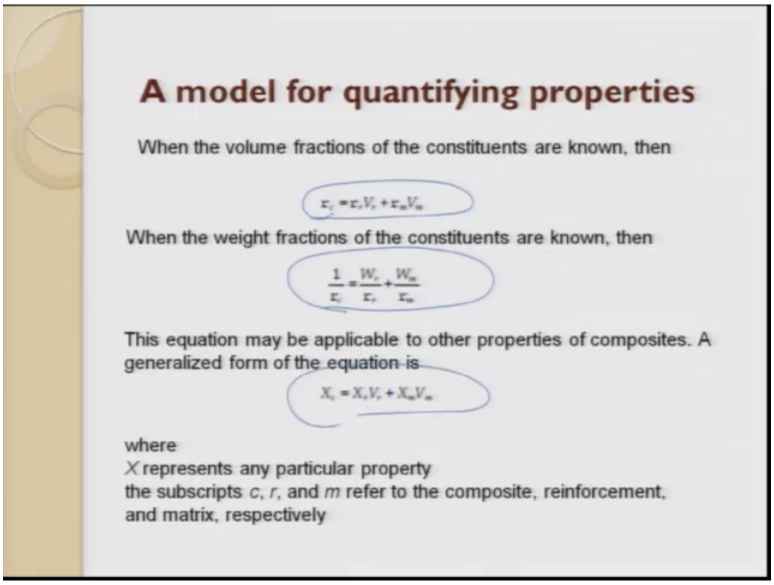

When the volume fraction of the constituents are known, then the same equation can be related as r c equal to r suffix of f; that means, to say fiber multiplied volume of the fiber plus r of suffix m matrix and volume of matrix. So, this calculation came from the previous one, and when we wanted to do with respect to weight constituent. So, we just do 1 by r c is equal to weight of the reinforcement divided by r suffix f plus weights of the matrix divided by r suffix m. This equation may be applicable to other properties of composite, also a very generalized equation is given here. So, where X represents any particular property and the suffix c r and m refers to composite reinforcement matrix respectively.

So, the next topic of discussion will be a model for quantifying the properties.

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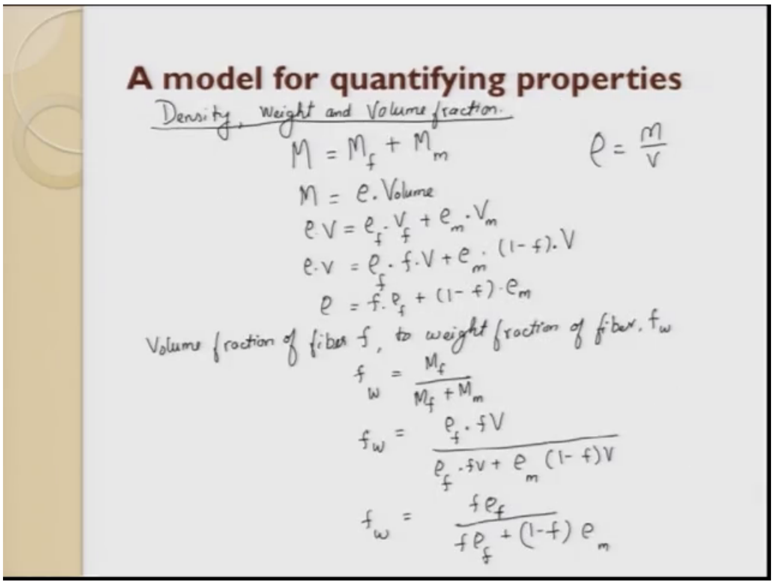

So, as we discussed earlier there is a relationship between density weight; that means, to say weight fraction and volume fraction going to our school days. We remember density is nothing but mass by volume. We just have to use this in this equation to find out a relationship between density weight fraction and volume fraction. So, let us start M which is the mass, total mass it is nothing but mass of the fiber plus mass of the matrix, as I written earlier mass is equal to rho into volume, just it is volume.

So, this rho into volume can be written as rho of fiber into volume fraction of fiber plus rho of matrix into volume fraction of matrix. So, this further can be written as rho of f into f. I will tell you what is f into V plus rho of matrix 1 minus f into V. So, rho can be written as f into rho of f plus 1 minus f into rho m. So, here to convert the volume fraction: the volume fraction of fiber f to weight fraction weight fraction of fiber; that is nothing but f w. We just need to establish a ratio between the mass of the fiber to the total mass. So, this can be expressed as f of w equal to m mass of the fiber divided by mass of the fiber plus, we write it as mass of matrix.

So, this can be within as rho of f into f V divided by rho f into f V plus rho m 1 minus f into V. So, this can be further written as f rho of f divided by this is suffix f divided by f rho of f plus 1 minus f into rho of m.

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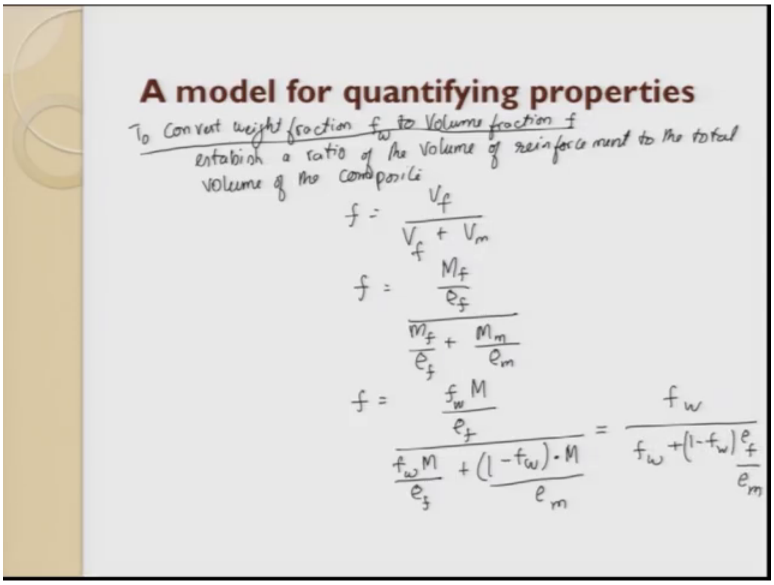

So, to convert now to convert weight fraction; what is weight fraction? Weight fraction is s f w to volume fraction to volume fraction f. What we need to establish a ratio of what do we need to do. We need to establish ratio of the volume, volume of reinforcement to the total volume of the composite. This we have to do. So, to convert the weight fraction f w to volume fraction f, we have to just established this. So, how do we do it, f equal to volume fraction; this is volume fraction plus volume fraction V of m. So, f can be written as mass by density. This is written by mass by density of fiber plus mass of a matrix by density of a matrix.

So, this can be further simplified, and it can be written as f w M divided by rho of f that whole divided by f w, then m, then divided by rho of f plus 1 minus f w into m divided by rho m. So, this can be written as f w divided by f w plus 1 minus f w into rho of f by rho of m. So, what did we do? We were trying to find out a relationship between density weight fraction and volume fraction. These two terminologies are very important and we know that, there is a relationship between density weight and volume fraction, and we have derived those two derivations in this slide.

So, we have written as mass first, mass we have written. Then we have written it for mass, is can be represented by this. Then we assumed that density multiply by the volume or the total composite is nothing but fiber plus matrix. So, we have written it in this form, and then we are moved into volume fraction of fiber f to weight fraction of f w. How do we get it?

Then we have gone to weight fraction to volume fraction that also we have done. So, by this derivation, we will be easily able to find out a relationship between the two relationships. Now, let us keep moving further, the stress strain behavior this graph. I have already told the stress strain behavior of fiber matrix and composite. This is a stress verses strain graph, you can see the strain, this is for a glass fiber composite is drawn.

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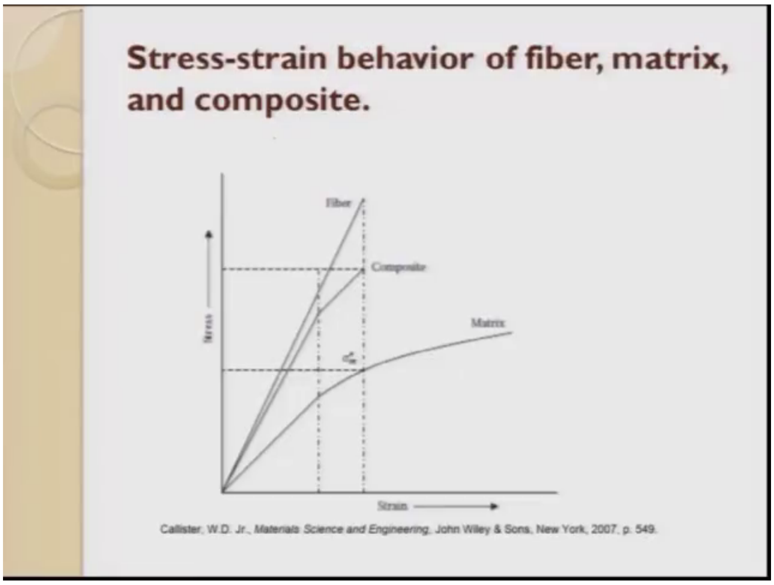

So, you see there is not much of plastic elongation. So, e has a fiber has very good e and a composite, whatever it is in between the property of a matrix, and then of a fiber. So, sigma m is the matrix failure, which happens the sigma m is a point sigma m star is the stress at the matrix, which is taken here

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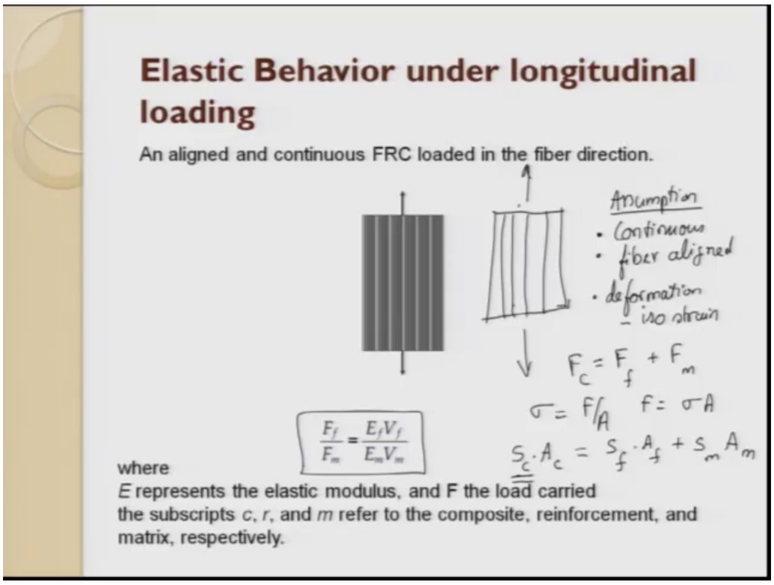

So, now let us try to understand the elastic behavior under longitudinal loading. So, here what we are assuming is, all the fibers are aligned in one direction, and the loading happens in the same direction, the loading happens in the same direction, and we also assume that all the fibers take the same load. So, here the final derivation what we have done is, the f of the fiber, the f is the load which carries by the fiber by matrix can be represented by next.

So, let us try to drive. So, what are the assumptions? I have made assumptions is, I have assumed that it is a continuous fiber, and next all the fibers are aligned in one direction, and I assume that the deformation of both matrix and fiber is same. So; that means, to say it is iso strain condition. So, the f c is the load, which is carried by the composite is nothing but is the load carried by the fiber by the load carried by the matrix.

So, this f can be returned in terms of stress. So, stress is nothing but which is f by a force by unit area. So, hence f equal to sigma by a. So, now, we know that s c into a c is nothing but s of f into a of f plus s of m, and it is a of m. What is this s? S is nothing but the failure stress strength. The failure strength is s c here. So, s c is multiplied with a c s f with a f and s m with a m.

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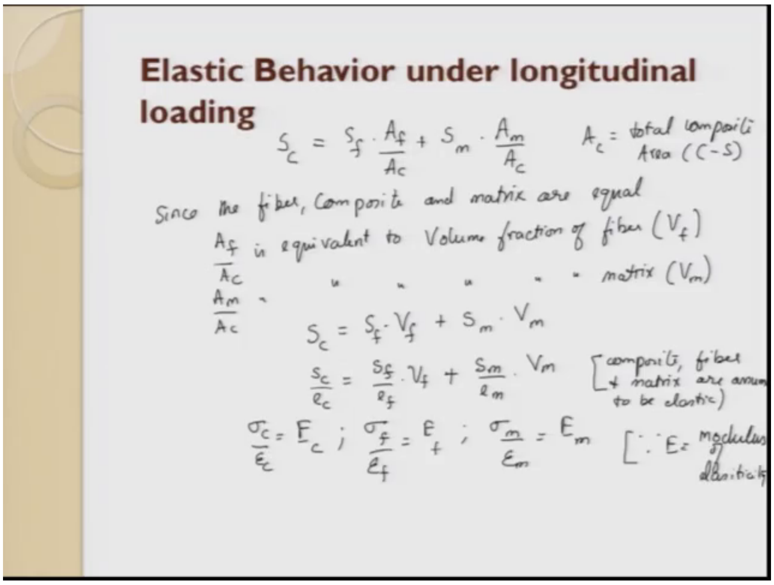

So, to further proceed we can obtain this equation S of f into A of f by A c plus S of m into a of m divided A c in this equation; the total cross section area of the composite. We are dividing this A c is the total., This A c is nothing but the total composite total composite area; that means, to say I am talking about the c s cross section ok.

Since the composite fiber and the matrix length are all equal. Since the fiber composite and the matrix are equal. So, we consider A of f by A c is equivalent to the volume fraction of the fiber, which is nothing but V of f and A of m by Ac is equal to the volume fraction of matrix which is nothing but V m. So, s c can be written as S of f into V of f plus S of m into V of m. So, it is assumed that the strains in the composite fiber and matrix are equal, I said about iso strain condition. So, S of c divided by E c is equal to S of f divided by E of f into V f plus S of m by E m into V m.

So, when the deformation of the composite fiber and the matrix is assumed to be elastic. So, then sigma c by E c is nothing but E c sigma f by E f is equal to Young’s modulus of f, then sigma m by e m is nothing but E f m. Here we assume; what is the assumptions. We assume that composite fiber and matrix are assumed to be the elastic, assumed to be elastic, what is E? E is the Young’s modulus. This is Young’s modulus and writes it down E is Young’s modulus, modulus of elasticity.

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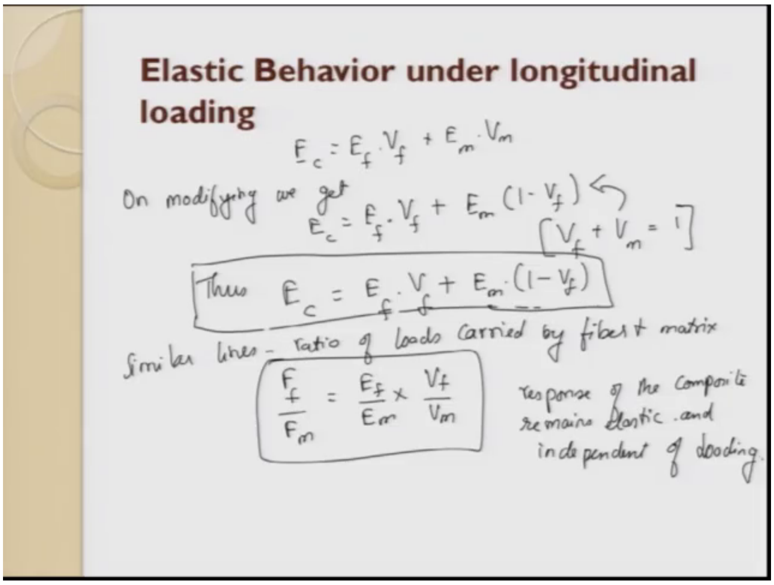

So, which can be represented as E c is equal to E f f into V f f plus e of m into V of m. So, by modifying this equation what we finally, get is modifying this equation modifying on modifying we get the E c equal to E of f into E of f plus E of m 1 minus V of m for the composite, consisting of only fiber matrix. These are all assumptions we make.

So, we assume that V of m plus V of m equal to 1. So, because of that only we are able to get this thing. Thus E c is equal to E of f into V of f plus E m into 1 minus V f. So, the composite modulus is equal to volume fraction of the fiber multiplied by Young's modulus of the fiber, Young's modulus of matrix into 1 minus V f. This is a very important derivations. You can expect some simple trivial problems to be solved in this, using this formula on similar lines. On similar linens we can also do a ratio of loads carried by fiber and matrix. So, the ratio of loads carried by fiber and matrix is given as F of f divided by F of m which is nothing but E of f divided by e of m into V of m by V of m. So, the response of the composite remains elastic. This proportion will be independent of the applied load.

So, here you should remember this point response of the composite, whatever we have taken for derivation reminds elastic. This is independent of the remains elastic and independent of loading. Independent of loading higher the modulus, the volume fraction of the fiber is high; the higher will be the load carrying capacity. So, in this derivation, we have till now what we have done is? Two derivations we have done, one we have done. How do you find out a relationship between density weight and volume fraction. How do you convert volume fraction into weight fraction, how do you convert weight fraction into volume fraction, then we have seen a stress strain behavior of a fiber matrix and composite. So, you can see fiber. So, now, it is very clear as and when you increase the volume fraction of the fiber, the strength of the composite also increases.

So, we have derived this, we have derived elastic behavior under longitudinal loading. We have taken assumptions, it is continuous fiber is aligned and the deformation is iso strain; that means, to say uniforms strain. So, we have derived it and then we have proved it finally, that the F of f, the ratios of the load carried by the fiber and matrix we have found out. We have said F of f divided by F of m equal to Young’s modulus of the fiber Young’s modulus of the matrix, which is multiplied by the volume fraction of the fiber and volume fraction of the matrix

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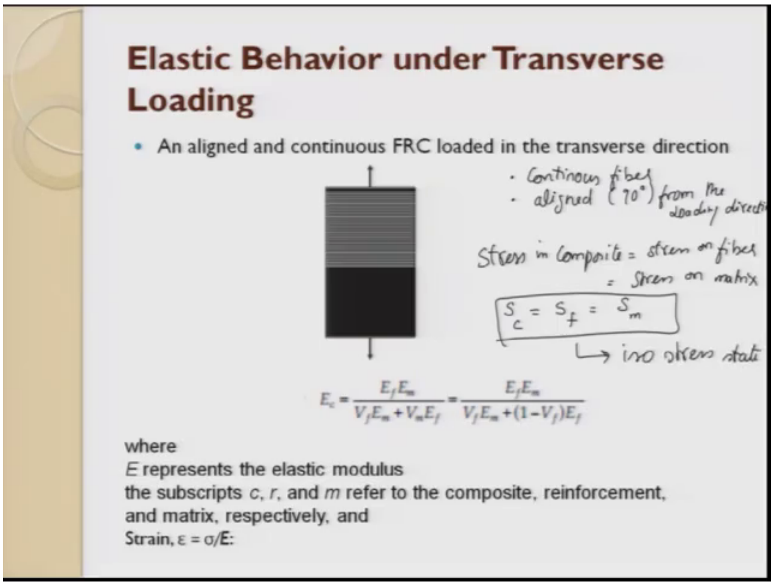

So, now let us see the elastic behavior under transfer conditions. So, here again we consider it is a continuous fiber. So, again here the assumption is. It is a continuous fiber and it is all the fibers are aligned at 90 degrees from the loading direction. So, here 90 degrees and the other thing is, the stress in composites is equal to; that means, to say uniform stress is applied stress on fiber, and this is equal to stress on matrix. So; that means, to say S of c equal to S of f which is equal to S of m. So, it is termed as, what is this terminology called, as here it is iso stress in a previous thing, we saw it was iso strain state, here it is iso stress state.

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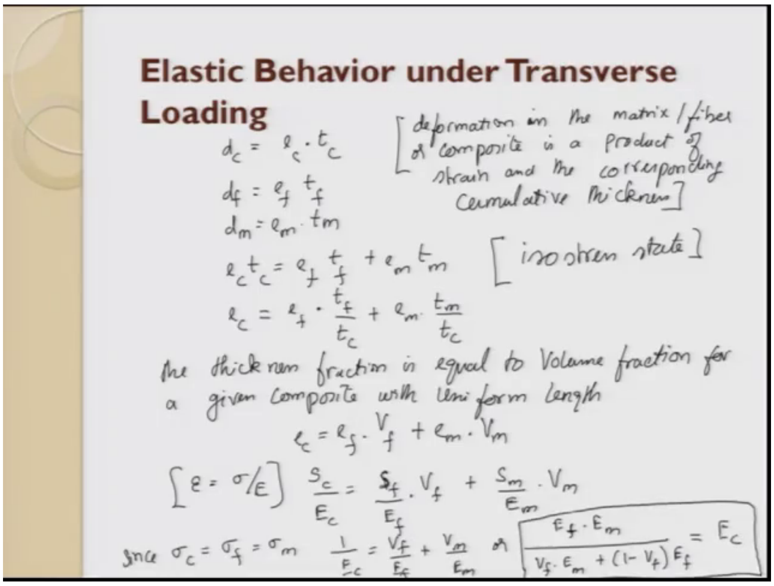

So, the deformation in the constituent or the composite can be written as the d c is equal to E of c into t of c. So, here I will write it in words. So, that you can remember it deformation in the matrix slash fiber, or composite is product of strain and the corresponding cumulative thickness, cumulative thickness. So, d c is nothing but e c into t c. So, same way you can write d f is nothing but e f into t f, and then d m you can represent it is as does d m into t m. So, finally, what we write. We write e c t c is equal to e f t f plus e m t m. why did we do this e c t c and all, because all these things came into existence, because of iso stress state. So, clear because of this iso stress state it came into existence.

Now, what we do, if we try to take that t c a side. So, this can be rewritten as e c equal to e of f into t of f divided by t c plus e of m into t of m divided by t c. So, the thickness ratio, whatever we find out the thickness fraction is equal to volume fraction for a given composite with uniform length. So, this is nothing e c is equal to e of f into v of f plus e m into v m. So, when the constituent are assume to deform elastically, the strain can be written as sigma by e. So, this is nothing but this can help and writing s of c divided by e of c is nothing but s of f divided by e of f into v of f plus s of m divided by e of m into v of m. So, this can be written as; since sigma c sigma f equal to sigma m, we can write it as 1 by e c is equal to v f by e f plus v m by e m, or we write it as e f into e m divided by v f into e m plus 1 minus volume fraction of e f into m.

So, what is this? This is equal to e c. So, this is the relationship which is established for elastic behaviour under transverse loading. So, what did we see? We first saw the elastic behaviour under longitudinal loading, then we saw elastic behaviour with respect to transverse loading.

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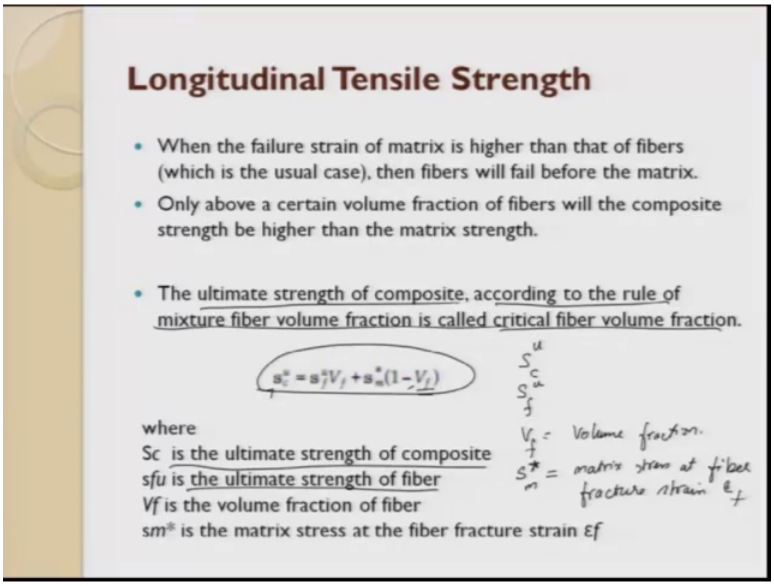

So, the next topic of discussion is going to be longitudinal tensile strength. So, generally what happens in a composite material, the failure mode is very complex. So, first what happens, if the fiber try the matrix and tries to play a role of uniformly distributed, the load onto a fiber, then the fiber starts taking a load, then moment the fiber start taking load after some point of time the fiber fractures, then it gets distributed. So, it is not like a unique failure. What happens in metals is a phenomenon here.

So, generally here there are several failure modes. So, modes that operates in a specific composite depends on both the volume fraction as well as the matrix property, and also the interface property. When the failure strain of the matrix is higher than that of the fiber, then the fiber will fail before the matrix; once the fiber fracture then what happens to the load. The load is born by the fiber, is now transferred to the matrix; however, the matrix cannot take heavy loads for a long time. So, it also immediately fails. So, now, it is very clear that we are supposed to find out the volume fraction that used to say a critical volume fraction, which is required for the composite.

So, a certain volume fraction of the fiber will be the composite strength. So; that means, to say a critical volume fraction level we have to find out so that we can have the strength properties taken by the fiber. So, the volume fraction is called as a critical volume fraction, when you consider a composite which is made out of a fiber, which is brittle and a matrix which is ductile. Then we are, it is very important to find out the volume fraction of the matrix.

So, this is a very critical equation. So, we will try to see, how do we get to this equation. So, this equation is nothing but the ultimate strength of the composite, which is according to the rule of mixture what did we study. We 1 minus v of, we studied in the previous derivation. This is called as the rule of mixture. So, the ultimate strength of the composite according to the rule of mixture, the fiber volume fraction is called as critical fiber volume fraction.

So, here this is very important, where and which you see the first term, which is nothing but s c u which is nothing but the ultimate strength of the composite s f u, which is the ultimate strength of the fiber v f is the volume fraction, v f volume fraction, which is given here volume fraction and s star m. s star m is nothing but the matrix stress at fiber fracture at fiber fracture strain, which is nothing but e of f. So, if you want to really derive this equation.

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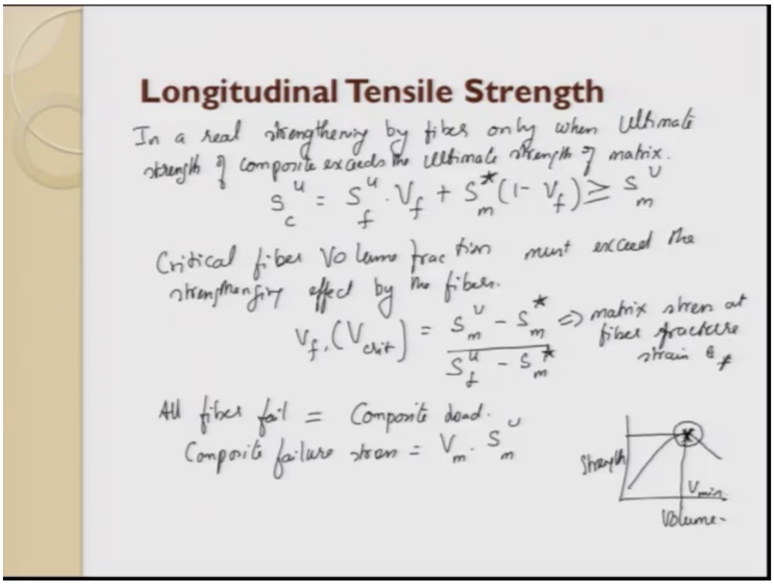

So, we can in a real sense in real strengthening by fiber only when ultimate strength of composite exceeds the ultimate strength of matrix. So, c s u is equal to c f u into v f plus s m star 1 minus v f, which should be greater than this.

So, in this equation the critical fiber volume fraction must exceed the strength. The strength I mean effect of fiber. So, it may effect by the fibers. So, if we rearrange all these terminologies what we get is, v of f; that is v of f which is otherwise called as v critical, which is s m u minus s m star divided by s f u minus s m star. So, what is s m s m is nothing but s m is nothing but the matrix stress at fiber fracture, fracture strain which is e of f. So, the composite will not fail at a critical stress, which is derived here, which is derived here. The composite will not fail at the stress indicated in this equation.

So, after the failure of all fibers, we go head after the failure of all fibers. So, what happens all fibers fail, all fibers fail then what happened the composite; that means, to say the matrix takes the load the composite will fail at a stress. So, what is a composite failure stress? Composite failure stress is nothing but v of m into s of m ultimate. So, there is a decrease in strength of the composite on increasing the fiber volume content then no. So, what happens is, if you see the strength verses volume fraction, there is something like a graph like this. So, you are supposed to find out this point, where you get a best benefit out of it, adding the number of, adding the volume fraction to the composite. So, that you get a very good strength.

So, what are we trying to do is- we are trying to find out this v min. Why is this v min adding more than this reinforcement does not play any major role?

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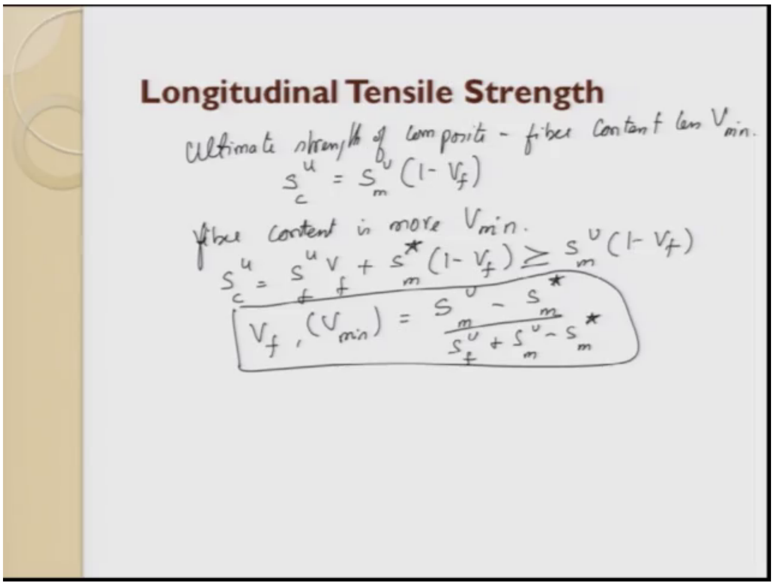

So, the ultimate strength, the ultimate strength of composites is composite when v min, when the v min when I think fiber we can say when. So, I will rewrite it ultimate strength of composite, and the fiber content, fiber content if it is less than v min what we get is. We get c ultimate is nothing but s m u 1 minus v volume fraction of f. So, when v, the fiber content is more than v min then what happens, then s c u is equal to s f u v f plus s s 1 minus v of f s m u 1 minus v of f.

So, we can say v min; that is v min is equal to s m u minus s m star by s u plus s m u minus s m star. So, if you look at it. So, this derivation is very important, which tries to talk about what should be the critical v; so to get proper output. So, if you go, if you plot it in a curve which is shown here, you can very clearly say, this is the v min. What did we do if only this calculation and you see here v critical, this is v critical, this is v min we are found out the both.

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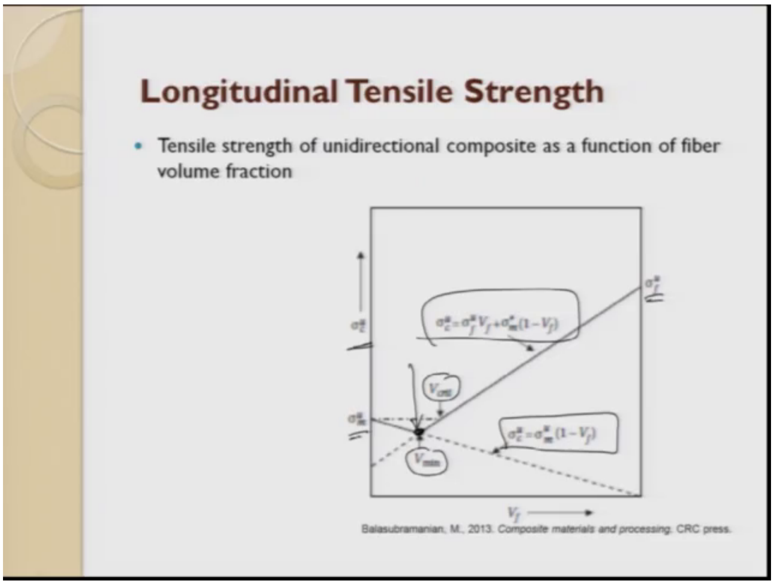

So, this is for sigma, sigma c for a composite 1 minus v c and this is for sigma u. So, this is for sigma with respect to ultimate in a composite ultimate with respect to matrix, this is sigma which is ultimate with respect to fiber.

So, this graph very clearly tells us there is a critical volume fraction for the fiber, which you have to take, and there is a minimum which have to take; such that you try to balance the load taken by the composite, and by the matrix and the fiber. So, when we proceed further, when we have to look for the transverse tensile strength.

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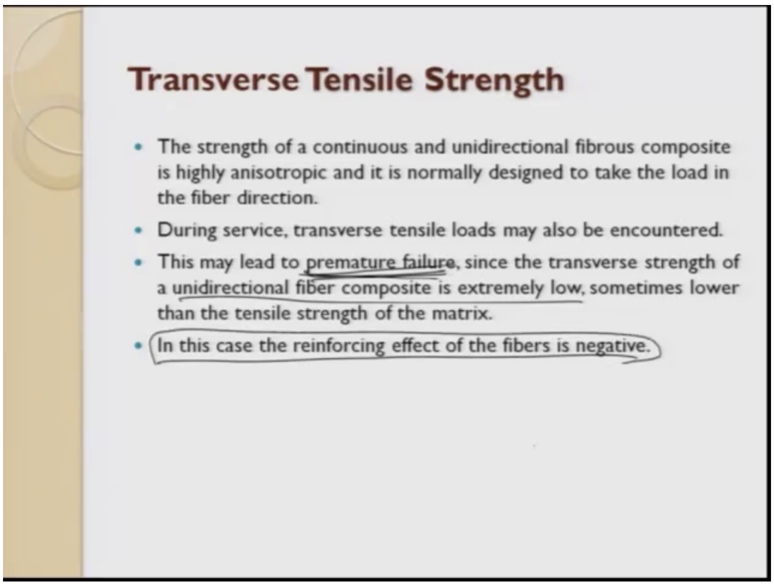

So, in the transverse tensile strength, the strength of the continuous and the disc and the unidirectional fiber composite is highly anisotropic, and it is normally designed to take the load in the fiber direction. So, during service conditionally, the transverse tensile load may also be encountered. So, if you want to figure out this. So, generally what happens in the transfer tensile load, there is always a premature failure.

Since the transfer strength of the unidirectional fiber is extremely low. There is always a premature failure. So, sometimes there is a composite as a premature failure, it is because that the loads are not properly balanced in by the loads taken by the composite is not properly balanced. So, in case of reinforcement affect, the fiber is sometimes has a negative effect, if you load it in the transverse direction.

So, with this we would like to come to an end for lecture 4. At the end of lecture 4 we will try to post some m c q questions. You have to solve those m c q questions and posted, but the answer.

Thank you very much.