Lecture – 34
Solidification: Eutectic(cont.)

Students we are going to complete our discussion today on this liquid to solid transformation. So, in the last lecture I gave you some idea about eutectic solidification. Today we are going to look at eutectic solidification in little detailed manner, and show you how we can actually use theoretical formalisms, to understand the eutectic growth. As I told you eutectic can have different morphologies, but we have been concentrating on lamellar growth. Lamellar type eutectic is the one which is easy to understand, and easy to deal with mathematically also. So, now I am going to discuss how eutectic grows, using little bit of thermodynamics, and also kinetics I will show you how things happens.

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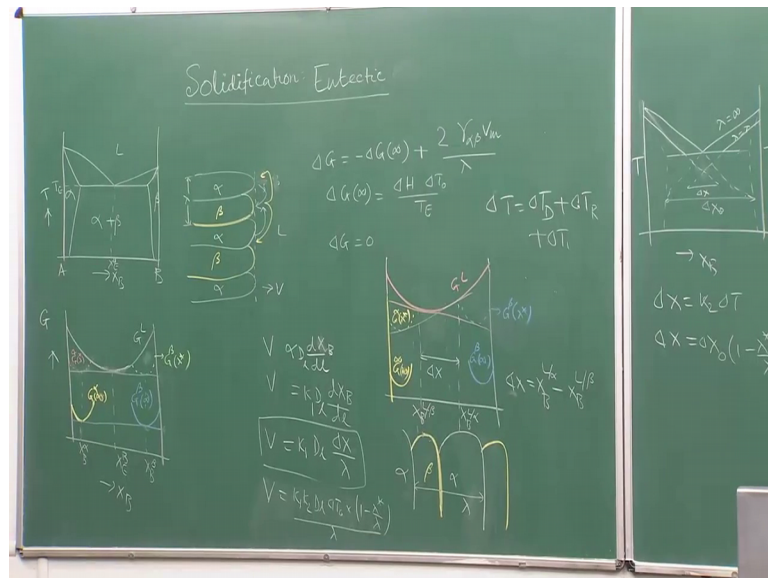
So, as I told you, let us first draw a phase diagram which shows the eutectic growth. As usual it is a phase diagram with two terminal solid solutions liquid transform to two phases alpha and beta, at the eutectic temperature, and we are going to discuss only the case for which the composition of the alloy is exactly same as eutectic; that is equal to X E ok.

Now, the eutectic actually grows a lamellar manner. So, I can actually draw this eutectic in a lamellar likes. So, this is suppose alpha, and this is beta, and slowly I can built up the eutectic lamely. So, therefore, there is a planner interface which is not exactly planar, and a eutectic is growing with a velocity V into the liquid, this growth front also you see this is a lamellar. So, therefore, inter lamellar spacing, but the way we define, also define the book is, like this is the lambda that a thickness of the two lamely; one is alpha other one is a beta. And this is the most important characteristics of micro structural characteristics, which is used to determine the important mathematical equations.

So, we; obviously, this growth is basically possible, because of the under cooling of the liquid. As we know the liquid is under cooling then only any phase will grow. So, here as you have seen properly earlier lecture, the alpha growth requires a rejection of B, because alpha is a rich as you see in the phase diagram. So, this B which is rejected by alpha must diffuse array in front of from the, in front of the alpha to the liquid, or basically the B, which is rejected by alpha will be required for the growth of beta, because beta is reached in B.

So, similarly beta growth requires rejection of A, why because beta is be reached. So, a for that force extra A has to be reached in to liquid, and this A can then be transported to the alpha. So, by using this color arrows, I clearly show you that the growth of alpha requires a rejection of B. So, therefore, B will be transported from the front, or a tip of the alpha to the beta region, and similarly growth of beta requires rejection of A, or basically the A has to be written in the liquid, and that A can be transported to the alpha, and then used for the growth of alpha. So, therefore, this is a diffusional process.

Now, as you know the free energy difference, which is the available energy to the system for is to, growth, has two parts; one is the free energy, corresponding to the alpha, corresponding to the lambda equal to infinity. This is when you have very large lambda, the interfacial energy between alpha and the beta, is basically almost 0. So, that is the interfacial energy corresponding to delta G within bracket infinity, plus. When you have finite value of lambda, then this interfacial energy term has to be taken into consideration.

And this is the way this energy is evaluated. This is equal to 2 by lambda multiplied by lambda interfacial energy between alpha beta multiplied with a molar volume of the

eutectic, where lambda is given in the (Refer Time: 05:29). So, now, we know that delta G infinity is nothing, but delta H into delta T divided by T e. So, it is better to write it is delta T 0, because that is the under cooling, required for the growth. So, delta H is the latent heat evolved, and T is the eutectic temperature.

Now, we can plug in this equation. So, therefore, when delta G is equal to 0; that is where the balance of the free energy, because of the heat evolution, and the interfacial energies of the alpha beta will give us the maximum, basically give us the minimum value of lambda. So; that means, I can write lambda star is equal to very simple that is this; the delta H. Let us put it properly twice gamma alpha beta V m lambda star is equal to delta H T 0 by T e. So, therefore, lambda star is equal to twice gamma alpha beta V m T e delta H delta T 0, as you see that lambda star which is the critical value of lambda, for which this delta G becomes 0, is inversely proportional to the under cooling and directly proportional to the interfacial energy

So, as we see if I have to draw the free energy versus composition diagram, to show you this aspect. Let me just draw it and show you. Suppose this is G, so liquid, let us suppose liquidus is a free energy curve like this, this is G l, and the free energy of the alpha free energy will be alpha. Let us write down alpha in a colour way free energy of alpha is, when lambda is infinity is this. So, free energy of beta, when lambda is infinitely like this and if I draw a common tangent; that is what gives me f t equilibrium compositions of the alpha and the beta, when lambda is very large

But as we know in the real system, lambda has a finite value its, cannot infinity. So, therefore, because the lambda will increase free energy by this amount; that means, that is correspond to the shifting of this free energy curves up. So, let us do that, I will do using a colour chalk. So, this is what it is. Let us suppose, or let us draw it this way. So, this is dotted as you see here. So, G alpha, when lambda equal to lambda some value or lambda star ok

Similarly, this is a, no this is beta G, beta G, beta lambda equal to lambda star. So, now, if I draw a common tangent between all three of them, I will, let us to put it together, as you see here, this becomes X e, because this is X b line and this is, it is a little bit change of the composition of alpha beta. So, that is what actually equilibrium will look like for lambda, is as a finite value correct. Now that is explains how, why that term to alpha beta

gamma alpha beta V m by lambda needs to be, how this needs to be added in to the free energy composition diagram.

Now, question let us look at how the actually growth happens, what the mechanism of growth. As I have already discussed, the growth is basically, because of the diffusion of the atoms from the tip of the alpha to the tip of the beta and vice versa. So, therefore, for the growth, to happen that must be sufficient, you know compositional gradient develop between the tip of alpha to the tip of beta for the B to be transported.

Similarly, for the A to be transported the compositional gradient, must develop in a between A tip of beta and alpha. So, if I basically do a little bit of calculation, as you see the V is basically proportional to D C by D l, or rather let us write DC B by D l, where the C. This is you can write down even better way X B. This is the X B corresponding to the mole fraction of B and D l is the distance. So, therefore, this tells you the compositional gradient.

So, therefore, we can write own V is basically a constant K, this is basically has to be proportional to D diffusivity, where D XB D l right. This must be written like this, correct. So, now, question is this, in a system like this, as you clearly see the compositions a head composition, a head of the alpha and a head of the beta is basically same. This is equal to X B E right, X B E as you see here; that means, the interfacial compositions or rather, let me write down this way. This is the composition interfacial composition of the tip of alpha is equal to X B E. similarly interfacial composition in the tip of beta is X B E, because this is the common tangent; though therefore, the solid compositions is at X B alpha ok.

Similarly, solid composition is X B beta. This is in equilibrium with the composition of the liquid given by X B E. So, therefore, in front of the both that alpha and beta compositions of the liquid is same. There is no compositional gradient available, but for the growth to happen, there must be a composition gradient created.

So, how we can actually do that, let us little bit reformulate this diagram, and so that we can understand better way, but we keep it this one constant same. So, basically velocity of the interface is equal to a constant K 1 multiplied with diffusivity D in the liquid and multiplied by the compositional gradient. We need to find out what is the value of this compositional gradient, then we can actually find out the value of V. To know that we

have to basically modify these constructions, how we can do that. Let us do it properly here.

So, let me do it in a little bit different manner. What I do is? I draw a diagram like that. So, that is your beta, then alpha, then is beta again. So, now, if I plot it, if I plot this things properly, if I plot this things properly little bit. So, let us suppose this is my G l. Now as you see clearly, this is my; that is you clearly see this is my, the. It is defined lamely alpha beta alpha. So, to do that, let me see how we have done in my book to do that. Yes let this, it has to be changed little bit, because otherwise it will not look nice. So, I will make it little bit bigger.

So, that it looks better. So, now, question is this, how we are going to draw this, such kind of things. You can draw such a kind of things by using again free energy of the alpha and beta. So, free energy of the alpha will, for the infinity will look like this. This is alpha, when it becomes infinity as you have drawn. So, this is your, again beta infinity. So, we are shifting this things, because of the infinite values of alpha like this. So, this is my G alpha lambda star, as we have done this is your G beta lambdas lambda star.

So, if I draw the common tangents here, this is one and then this is another one, as you clearly see the compositions equilibrium compositions are A given by the tip of the alpha and beta are given by these dotted lines. So, this is equal to X B liquid beta is equal to X B liquid alpha, why, because what I am doing? I am doing a common tangent between the G alpha, when X lambda is equal to lambda star with G liquid; that is what has to be done. This is what exactly I have done in this case actually, because they are lying on the same, you know common tangent between all of them.

So, there is no driving force available, but here it is not the case. Here actually the common tangents cannot be drawn, only thing we can draw is, tangent between common tangent between G l and the alpha. Again G beta and the alpha and by drawing that I can find on the equilibrium composition at the tip of beta equilibrium composition, at tip of alpha. So, therefore, this compositional difference is, what is basically existing between alpha and beta tip in the liquid, and that is what is driving the transport of the beta.

So, we can write down this equal to K 1 D l delta X by, and this is equal to your lambda as we have drawn here; that is this is what it is. We can always assume that the diffusional distance is basically equal to lambda. Remember this is a basically A naught

A exactly correct equation expression why, because compositional values will depend upon exact value of lambda. Not only that, it will depends upon where actually we are lying, we are lying here, you are lying there or there, wherever it depends on the exact location of the interface, we are not doing that, because we are doing order of matured calculation. Therefore, we are not doing that, so that is what it is.

So, V is equal to a constant K 1 multiplied by D l into X, delta X is the compositional difference. So, delta X is nothing, but you can write down X P alpha liquid alpha interface minus X B liquid beta interface; that is the compositional difference, which is driving the deficient to happen across these sideways. So, now, as you know from the phase diagram.

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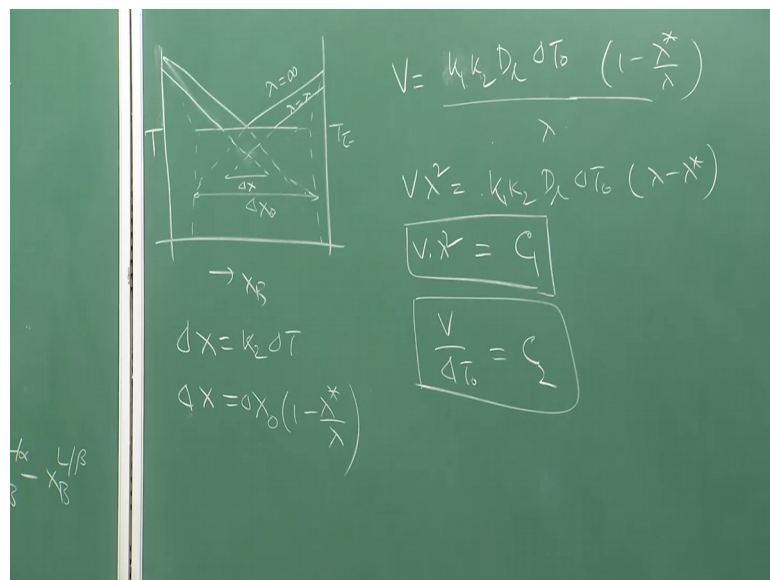
You can actually do that, if I draw the, if I have drawn only the phase diagram, yes, or let me draw only the important part, as you know the phase diagram allows you to convert this delta X to delta T 0 delta T.

So, let me just draw the important lines, this is my T e. I am drawing only this line. Remember this equal to lambda to be equal to infinity. So, if I extend this lines; that is tells you the compositional difference from the alpha and beta interfaces, this is the compositional difference between these two; that is actually can be written as X is 0. So, A lambda is very high, this is the compositional difference would exist between this tips, but when lambda is equal to lambda.

So, what will happen, these will go down actually. So, temperature will simply go down, or let me see how we can draw it properly; that is very easy to draw; obviously, everything will go down; why, because when the lambda is finite. Lambda is basically finite, then what will happen, lambda is. If lambda is finite and then there is a problem of everything, temperature will go down, as this is the effect of interface, that is the interface is becoming energy is higher, where delta T is when decreases, say lambda equal to lambda.

So, we can this is what is the melting temperature eutectic. Now if we extend that, this is what is become delta X. So, as you see here delta X is basically a constant K 2 into delta T correct; that is what you see here. So, we can write down that. So, therefore, V is equal to K 1 into K 2 V l delta T by lambda. Now we also know note that it is delta X is basically, is a function of lambda; why, because as you see when the lambda is very large, then del its equal to delta X 0, this is what it is. So, whatever lambda X, because; that means, whatever lambda will become to lambda star, then it is equal to delta X 0.

Now, when lambda is a finite value, then this value will decrease. So, therefore, we can write, we can write down delta X is equal to delta X 0 multiplied by 1 minus lambda. Now if I do the things properly, then this V will be equal to K 1 K 2 D l delta T 0 multiplied by that factor 1 minus lambda star by lambda divided by lambda; that is what you can write down correct. So, now, we can actually do the Maths properly. If I take the equation from here to there, that V, V is equal to K 1 K 1 K 2 D l delta T 0. We can write down that is equal to 1 minus lambda star by lambda divided by lambda.

Now, if we do the Maths little bit properly, we will find out V lambda square is equal to K 1 K 2 D l delta T 0, and a factor lambda minus lambda star. So, this can be treated as a constant C 1. So, that is why V lambda square is basically equal to C 1. So, once you know the V for a particular level of under cooling, we can calculate what is the lambda or rather its other way if we know lambda, we can calculate V at a particular of under cooling; that is what it says.

So, for the eutectic growth this relationship is very important. Although this relationship has, can be obtained in a very general mathematical manner, using Jackson Hunt. What he has done, I told you in the last class, but we can simply assume many things, and get that equation very easily. So, similarly one can actually get this also, constant C 2, but

you must see one thing, the basic equation which is related to V lambda and delta T is given at the top, and in that equation only variable, which you know is delta T. The standard cooling, which can be measured in a experimental. Things which you do not know is basically a (Refer Time: 22:30) is the lambda and. So, therefore, only knowing delta T 0 will not give us the answer of V, and lambda independently correct, because this is single equation. We have two variables V and lambda for a particular value of delta T 0, or delta T under cooling.

So, question is this. If I have to calculate V and lambda independently, then I must have another equation relating V and lambda; that is not available to us, unlike dignity growth here it is not available. So, what we normally do. We try to assume that eutectic growth happens at the minimum level of under cooling. So, we try to minimize the V velocity with respect to, a maximization, with respect to the delta T; that means, the growth is maximum at the under cooling level. So; that means, we need only finite level of under cooling for the growth to happen. Now that is not correct. So, therefore, is better actually.

We can actually look at the microstructure after solidification, find out the lambda, and then you see delta T 0. We can find out what is the possible V A velocity of the interface, so that is possible, but remind that, that one thing, the lambda which was there, then the growth may not be retained. When you solidify, it completely, because there will be cortioning of the alpha beta lambda lambda. So, that is leads to change of lambda or actually change of our estimation of the lambda. So, this one estimation will lead to under estimation of the value of solute.

Obviously, one estimation means higher value will lead to under estimation of value of V. So, therefore, it is not possible to measure exactly the velocity of the interface, calculate that using this equation, what better method is, to do is that taken a system, in which we can actually observe, when the eutectic is growing at a certain velocity, we can observe these lamely, and then measure the value of lambda perfectly, when it is in contact to the liquid, and use that value at a particular level of under cooling to calculate V. So, finally, it boils downs to the important aspect that eutectic growth is, still it is not a fully solved problem. We can actually only see that eutectic growth is a under deterministic problem in the solidification literature.

You know as I told you the measure under cooling part is the delta T D. So, delta T has two parts; delta T D is the diffusional part and; obviously, delta T, the curvature part which is basically known as delta T R. Why curvature part, because interface is highly curved, and because of the curvature of the interface, that must be under cooling lead to that and this. Out of these, two are the major aspect of under cooling, which are required for the interface to grow one.

Sometime people also add another part, which is known as the interfacial under cooling, but this value is basically required, when the eutectic growth is. This value, is basically corresponding to the automatic jump from the liquid to solid. The under cooling, normally it is very low, when interface is kind of a rough. So, the atoms can easily move jump and do that, but when the interface is very faceted; like in silicon aluminum, silicon type eutectic. This value is very high, and one must add this particular component to the total under cooling equation to do that.

So, in actual experiment we cannot measure each of these. We can estimate later on theoretically, we can measure total under cooling and plug in there, in the velocity to (Refer Time: 26:31) lambda relationship, and get the velocity in the form of some values. So, as you see here the eutectic growth is much more complex than, this can be thought of as compare to dendrites eutectics more widely also equally widely observed in the microstructures.

So, therefore, it is important that we know their growth very well, but unfortunately those theories, has still not developed. There is a, you know requirement of develop a new theory for understanding eutectic growth in future. Probably you will get to know these theories well, but for the time being, you need to only understand how eutectic grows in a liquid, whether can we use thermodynamic and kinetic aspects, to understand there or to basically obtain mathematical equations of their growth rate and other aspects. So, in the lecture I am going to discuss about these little bit about the, what happens in the casting, and then move ahead for the solid to solid phase transformation.