Heat Treatment and Surface Hardening - II
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Lecture – 38
Evolution of TTT and CCT diagram from f vs. t plots

In this lecture I am going to introduce a very important concept; the concept of time temperature transformation diagram or in short TTT diagram.

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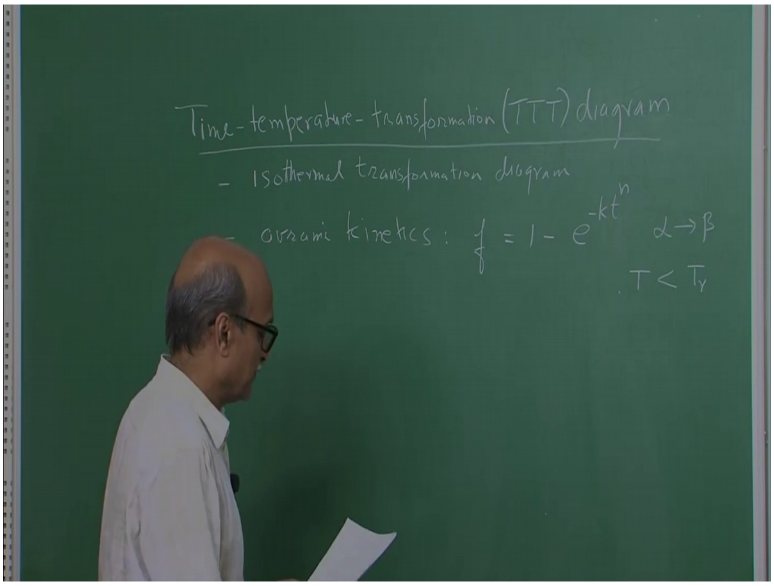

Time-temperature-transformation or TTT diagram, in short or alternatively it is also called as an isothermal transformation diagram. Now what does this tell us? It tells us very neatly in a very in a single diagram that under isothermal conditions of different temperatures what would be the kinetics of transformation. So, if we have data for the kinetics for the transformation then we can create such diagram.

For example if we know the avrami kinetics of a particular system where say a avrami kinetics tells us fraction transformed as a function of time for a given temperature. Now if I have the kinetics for many different temperatures then I can put this entire kinetics into a single diagram which very simply tells me how fast things are going to change at different temperatures and how long they are going to take and so on. So, let us see how we can relate the avrami kinetics, and from the avrami kinetics I will show you how we can create such diagrams. So, starting point is the avrami kinetics which is fraction transform which is f is equal to 1 minus e to power minus k t to power n now we are going to consider once phase transforming to the product phase.

So, alpha transforming to beta this could be for example, iron FCC transforming to iron BCC as we lower the temperature or it could be austenite undergoing a eutectoid reaction producing pearlite alternative layers of ferrite and cementite or any other transformation. So, let us see let us consider a temperature; obviously, we are going to consider temperatures less than the transformation temperature where alpha can transform to beta. So, let me sketch first the avrami kinetics for a given temperature.

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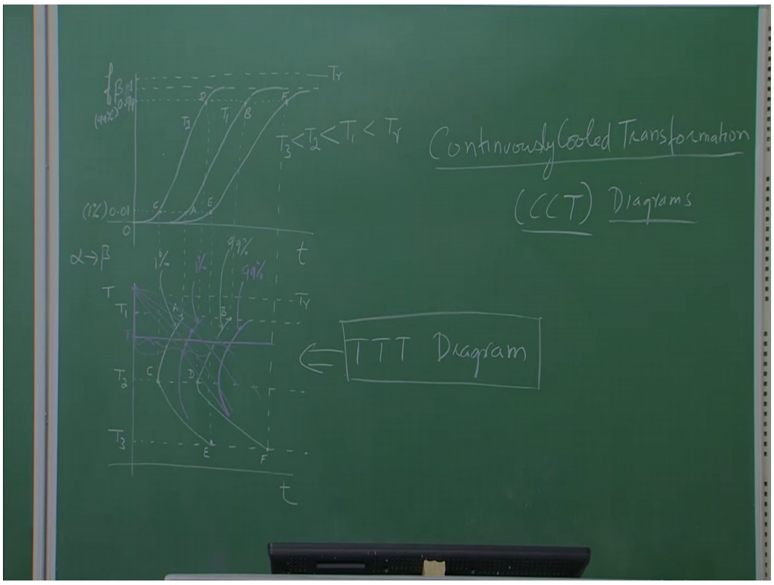

So, I have my fraction transformed beta phase, this is my time axis and let us say at some particular temperature this is fraction 1 this 0 and let us say this particular transformation kinetics given by this curve is at a temperature of T 1 where T 1 is less than the transformation temperature T r.

But we keep this T 1 quite close to the transformation temperature. So, we are first looking at the kinetics that are relatively high temperature with reference to the transformation temperature, and then we will consider other temperatures lower than T 1 and see what happens there. Now in this let me mark out 2 points, one is let us say when the fraction transformed as 0.01 or simply 1 percent beta phase is formed and let me mark one more point 0.99 or simply 99 percent transformed. In a corresponding diagram and here in this corresponding diagram my vertical axis is going to be temperature.

And the horizontal axis is going to be time my T r let me also mark the transformation temperature T r, similarly let me mark the transformation temperature T r with reference to the transformation temperature T 1 let us say is somewhere here. So, this is the temperature T 1, this 0.01 percent transformed is many times consider as the start of the transformation because it is really difficult to actually figure out exactly where the transformation really starts because you cannot measure very small quantity. So, very often we say that one person is a point where the transformation start and similarly 99 percent is where the transformation is essentially over.

Now, let me reproduce these 2 points in the temperature versus time plot here, let me call these 2 points as A and B. So, I reproduce these points on this corresponding graph and call these points here as this point as a and this point as b. So, these are basically in this temperature time plot a is representing 0.01 transformed b is representing 0.99 fraction transformed now let us consider another temperature less than t one. So, let us consider a temperature t 2 now as would be expected you remember that as we are reducing the temperature or increasing the under cooling initially the rate of transformation will increase it will go through a peak and for much higher temperatures again the transformation kinetics will start to reduce.

So, let us say t 2 is a temperature where the transformation rate is actually increasing or in fact, near the peak then what will happen to the avrami kinetics the avrami kinetics would be much higher which means this curve will shift to the left. So, let me plot a second curve showing the kinetics at temperature t 2 and this is at a much higher rate. So, therefore, it is shifted to the left this is at temperature t 2 these 2 points on this represent one percent transformed and 99 percent transformed now we copy these 2 points as well let me first mark these points as c and d and let us copy these 2 points here and let us say t 1 is somewhere here sorry t 2 temperature is somewhere here.

So, if I copy point c point c gets copied here I will call this c as well in this diagram similarly, reproduce d point also on this and let us say this comes somewhere here. So, these are point c and d on the at temperature t 2 corresponding to this particular kinetic behavior or the rate of transformation behavior now let us consider third temperature which is less than t 2 and let us call that as t 3 here that we are gone sufficiently low the under cooling is large such that the rate of transformation has fallen down. In fact, it the rate of transformation is even lower than the rate of transformation at t 1 then the third curve of avrami ah behavior at temperature t 3 would now be to the right of this temperature t 1 and hence this becomes my third transformation curve let me extend this line this point corresponds to 1 percent transformed.

And if I extend this line this point corresponds to 99 percent transform. let me call these points as point e and point f and I will reproduce these 2 points as well on the temperature time diagram and let us say that this temperature t 3 is here. So, reproducing point e would be here reproducing point f and this point would come here. So, if I look at these points carefully particularly the points a c and e here a c and e these are 3 points which correspond to the condition where only 1 percent transformation is complete then the points b d and f b d and f correspond to the condition where 99 percent of the transformation is complete and you can imagine that I can draw many other fractions and I can keep transferring them.

And then I can join all the 1 percent points together by a smooth curve I can join all the points which are corresponding to 99 percent of the transformation complete together by a smooth curve. So, I can join a c and e then this will come out like this I join b d and f corresponding to 99 percent complete would correspond to a curve something like this. So, all the points on the curve a c e so in fact, the even the inter measured points are corresponding to the points where 1 percent transformation is complete all the points on the curve b d f correspond to 99 percent complete this is what is called as the time temperature transformation diagram or the t t t diagram. So, now, what is this diagram telling me?

So, let us I have this alpha transforming to beta below the transformation temperature then if I look at if I look at let us say I am initially at a temperature above t t r and I suddenly cool down very rapidly almost in 0 time that I reach a temperature t one and then I hold there then I know the this much time will be required for 1 percent transformation to take place and this much time would be required for 99 percent transformation to be complete similarly if I cool down from above the t t r temperature to a temperature t 2 here then only this much time is required for the transformation to take to for 1 percent transformation to take place and while this much time will be required for 99 percent transformation to take place.

Similarly, if I go down to t 3 and isothermal hold there I require this much time for the transformation to complete 1 percent and this longer time for 99 percent transformation to take place similarly I could well hold on to some other temperature t and hold there then this much time for 1 percent to complete this much time for 99 percent transformation to complete. So, what we have manage to do here in this t t t curve we have taken all of this avrami data that we can experimentally obtain by as we already discussed that how we can do this by quenching samples in constant temperature baths like a salt bath and then hold the sample for different times and then do quantitative measurement of the micro structure to obtain volume fraction as a function of time.

Then we can obtain n and k data for different temperatures and from that we can get these curves and these curves we can draw one single diagram which very readily tells us how the kinetics are going to take place at different temperatures. now this in a briefly is actually what the time temperature transformation diagram now what would happen if I just cool down very rapidly that I never touch these curve as you can see that these curves are anywhere going and as we are going lower and lower temperature you require anyway very long time for transformation take place. So, if I cool down. So, fast that I do not touch even this first starting curve a 1 percent transformation then I can expect that no transformation should take place well I can expect that no diffusional transformation can take place in some systems.

And particular example of that is the steel system the iron carbon system where even if you do not touch these t t t curves the austenite would then transform into a metastable phase called the marten site on this curves those starting marten site start lines and marten site finish lines are also put which I will show little later now one another important thing that I want to talk about is that here these kinetics we can get from this diagrams only if we are able to cool down very rapidly and after that we have to hold iso thermally though these diagrams are important there are another set of transformation diagrams which find even greater importance in industry because large number of heat treatments

That are done in industry are not heat treatments like this that you are at high temperature you cool down very fast and then hold at a constant temperature. In fact, most of the heat treatments though this is done in many cases, but very many larger cases industrial heat treatments would involve not cooling in this fashion and then isothermally holding it would be more like cooling continuously. So more like cooling like this So, what can we do if we have a cooling characteristics which is a continuous cooling all the way down to room temperature can these diagrams help us than the t t t diagram because a t t t diagram are valid only if we have a cooling regime or the heat treatment regime of this kind.

Well the t t t diagram can still help us where they can help us is that from the t t t diagram we can generate another set of diagrams which are called as continuous continuously cooled transformation diagrams or in short c c t diagrams. So, in the case of the continuously cooled diagrams let me just redraw this little bit below suppose I have cooled like this it is possible then from the t t t diagram to generate these c c t diagrams which I will show a little later that suppose I want to know at what point along this cooling curve 1 percent phase would have transformed well I can calculate and it would be at a point somewhere here a little lower and little to the right of this point.

Similarly, if I had a cooling rate like this then I would have transformed 1 percent somewhere here another cooling rate I would have transformed somewhere here and in this in this way I can join another set of curves this one corresponding to 1 percent transformed similarly I would have got another one here corresponding to 99 percent transformed while correspondingly these white curves corresponding to the t t t diagram this is 1 percent and this is 99 percent. So, I would have got similar looking shapes the shape of a c or sometimes called as a c curve similar c shaped curves I would have I i would get for the c c t diagrams as well and these are very very useful diagrams when you are designing heat treatments. In fact, both of them the t t t as well as the c c t diagrams are useful.

When one is designing heat treatment to obtain desired micro structure and to obtaining desired mechanical properties or other properties. So, now, the next thing what I will do is look at how to convert the t t t diagram to a c c t diagram and in order to do that we have to understand few things regarding this conversion regarding the kind of transformation that would or rather the type of transformation determined by the avrami equation it could be fairly simple for certain types of transformations and it could be mathematically more complex for other types of transformation. So, let us look at converting t t t diagrams to c c t diagrams converting t t t to c c t diagrams.

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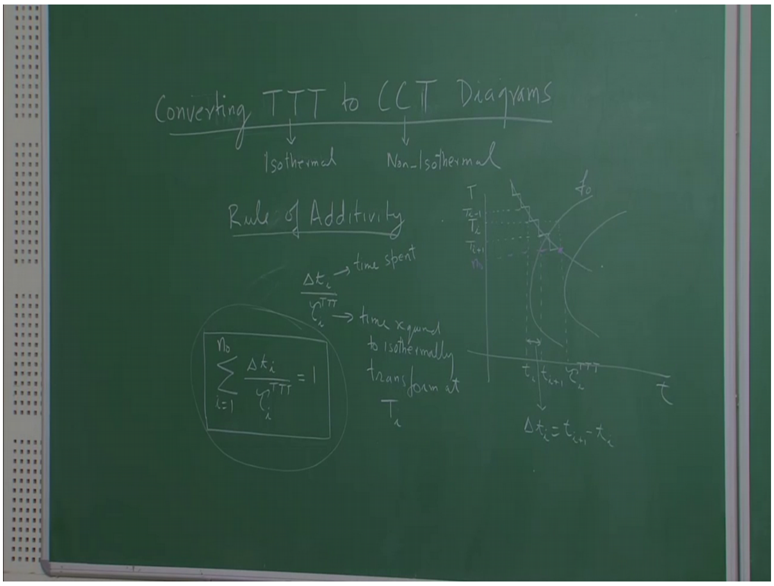

So, t t t when the isothermal heat treatments is being done c c t when the material or the sample is continuously cooled. So, this is isothermal and this is non isothermal there is a widely used rule for converting t t t to c c t. So, first we will understand the rule and then we will see under what circumstances under what conditions we can use this rule and this is called as the rule of additivity. So, the rule of additivity or called as the additivity principle can be looked at in the following manner let me just draw a schematic of t t t diagrams let us consider a continuously cooled material.

And what we do is we approximate this continuously cooled behavior into small isothermal steps that is I discretise this continuously cooled curve in this fashion. So, I discretises it into large number of steps and there are small isothermal steps as you can see let us consider this to be let us say the ith step and let me consider the temperature here as t I let me consider the temperature the previous step as t I minus 1 and the temperature for the next step as t I plus 1 corresponding to these to let us say t I there is a time t I and corresponding to t I plus 1 there is a time t I plus 1 the difference between these 2 times is a length of the isothermal step and let us call this as delta t I which is nothing, but t I plus 1 minus t I now at this temperature t i.

If I had held this sample at temperature t I how long would I take let us say for the fraction to transform by let us say some fraction f naught it could be 1 percent it could be 99 percent it could be 50 percent whatever, but some fixed fraction. So, at fraction f not if this is the curve for the fraction f not transformed how much time it would take if I had to isothermally transform at t i. So, I quench directly to t I and then hold it well that should be clear the time would be this much let me call that time as tau and superscript t t t that this is the isothermal time at the I th step or basically at temperature t i. So, now, the additive principle is as follows that I take at t I the fraction of the time spent delta t I delta small t I fraction of the time over to the total time required for the transformation at t I is delta t I divided by tau t t t i.

That is this denominator is the time required to isothermally transform at temperature t I while delta t I is the time spent in this particular isothermal step of the continuous cooling. So, now, the rule of additivity states that if I sum up all these small fractions. So, delta t I upon tau I t t t I going from the first step starting right from the beginning of the transformation temperature to let us say some step n 0 such that n 0 is that many step required such that the sum of all of these small steps become equal to 1 then we would say that along this cooling curve at the n 0 th step the transformation of fraction f 0 would have taken place this is the simple rule of additivity.

After that I can simply add up all of these time increments of delta t 1 plus delta t 2 plus delta t 3 all the way to delta t and not that would give me the time required at which fraction transform would be f not. So, I add up all of this and then as a result of all of these addition I will end up at some point here that this could be well the n 0 th step. So, this would correspond to the fraction transformed f not if I go and trace along this cooling curve now I will show you the calculation later, but will take up one example to actually calculate certain points along different cooling curves using a spread sheet to see how the shape of the c c t curve.

And how and at what position the c c t curve gets shifted, but before doing that what I would like to now show is that what under what condition this additivity principle is applicable it is not applicable in general it is applicable only in some specific conditions. So, let us quickly have a look at those conditions. So, the condition under which the rule of additivity So, validity

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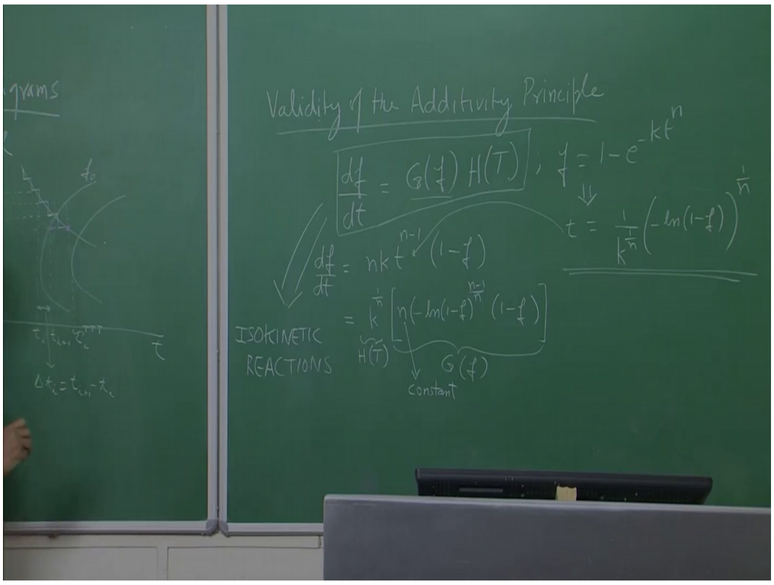

Of the additivity principle validity of the additivity principle is very easy to state that the additivity principle is valid for all transformations.

Where the kinetic follows the following relationship d f by d t the rate at which fraction transformed as per unit time d f by d t the rate of transformation should be a function like this capital g is 1 function which is a function only of the fraction transformed at that instant of time h is another function and this is purely a function of temperature if the rate kinetic create the rate of transformation follows such a general equation where these 2 can be separated as 2 separate functions then it can be mathematically shown that the principle of additivity is valid now I just want to examine is avrami relationship following this principle of this condition or not.

So, let us start with avrami equation avrami equations is one minus e 2 power minus k t to power n I take the first derivative. So, d f by d t equals n k t to power n minus 1 times 1 minus f this we have already done in an earlier lecture now what I can do is I can from this equation I can write down I want to eliminate time from this equation. So, from here I can write down an expression for time an expression for time would simply be as follows time is equal to one upon k to power one upon n times minus l n 1 minus f to power 1 upon n now this is simply rearranging the terms in avrami equation and writing it.

So, first take logarithms and then write down an expression for time this time then I can substitute here to get my rate d f by d t as follows I am go to directly write this and then you can actually expand and actually show that this is indeed the case this can be writ10 as k to power one upon n times n times minus l n 1 minus f to power n minus 1 upon n times 1 minus f n the square brackets closes here now this particular term let me call this if I if I look at this let me call this as this is purely a function of f it is also a function of n by the way, but I am just writing it as a function of f this let us say that k way k can vary with time temperature if k varies with temperature then this becomes a function of temperature.

So, in this relation now I am try to write it in this form now when can I write it in this form I can write it in this form when n is a constant. So, n is independent of temperature and only k is a quantity that varies with temperature. So, all those avrami curves that I had shown for different temperatures are could result if k is a function of temperature while n is a constant now is this really true well in many system this could actually be a approximately true in the sense n represents a mechanism of a of the transformation you know whether the mechanism is heterogeneous homogenous whether there is site saturation or not site saturation.

If you remember if you go back to the starting lectures we had shown that if there is site saturation you get a certain value of n if you get if you have constant kinetics you get a certain kind of certain value of n why k could vary. So, we could make statements on n. So, if we assume that this entire set all the at many different temperatures of the t t t diagram that I have if mechanism is a same then I can say that n is a constant and if n is a constant then this is purely a function of f an independent of temperature and k is a only one then that is left to change with temperature hence this one is a function of temperature under this condition then the avrami kinetics and in fact, all kinetics which follow this are called iso kinetic reactions ok.

So, now what we will do now is look at a example where we assume that n is a constant and k varies with temperature. So, that we have an iso kinetic reaction of 1 phase transforming to the second phase and look at how we can in excel itself in a spread sheet how it is possible to convert a t t t diagram to a c c t diagram here this is a little involved. So, I have already.

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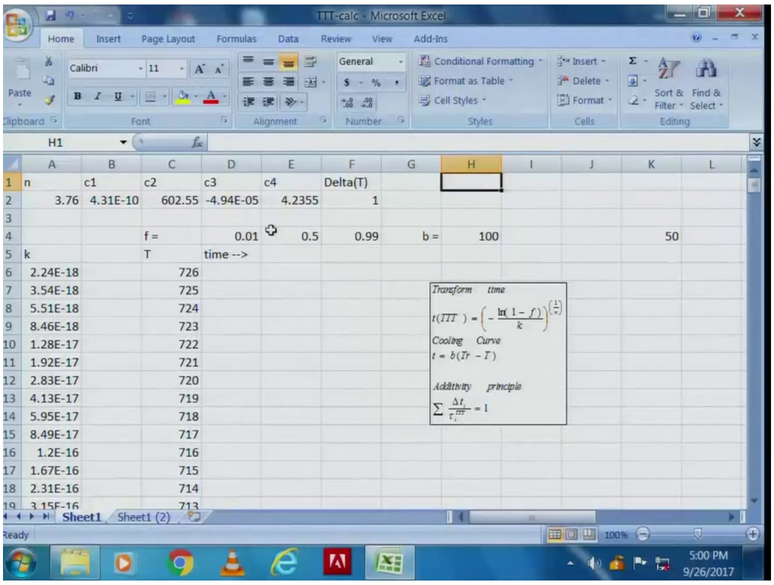

Have a lot of data I gave already input this is for a particular plain carbon steel here we are looking at austenite to pearlite transformation and what I have here is in this column I have temperatures starting from just below the eutectoid transformation temperature and then going down to over a entire large range of temperature out here is the value of k which varies as a function of this temperature.

If we want to just quickly look at how k varies with temperature well I can plot incidentally this k values have been obtain from data of t t t diagrams now this k if I make a scatter plot and let me just remove this formula box here for a minute I insert I already sorry I I click on this select data in the plot add a graph and first add x values my x value are going to be temperature and then add to the y series the k values and click ok and let us look at what kind of a graph I got well wait a second I did I plot till no these are temperature I have plotted only up to 692 I must increase my temperature range in the graph let me just go to select data edit somehow got click to forty it should have gone to 356.

So, this is going from c 6 to 356 and the temperature values and the k values are going from a 6 to 356 if I get this now you can clearly

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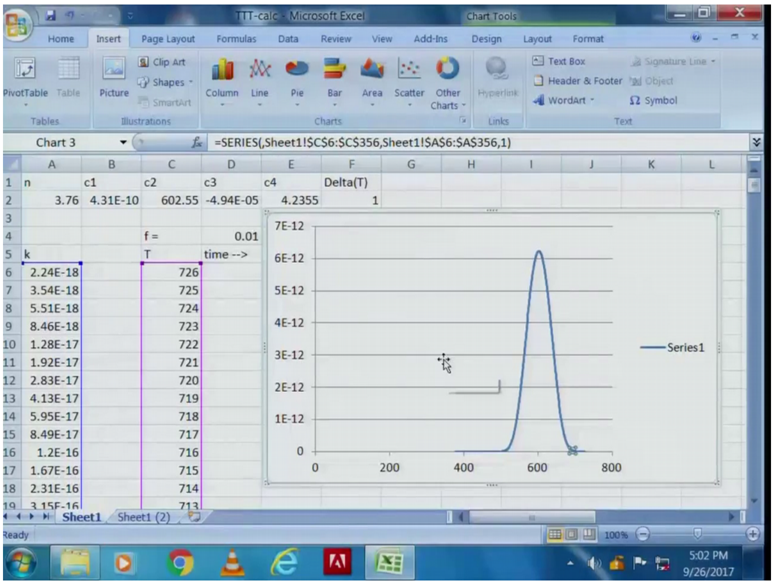

See how k varies with temperature. So, you can see that as we are reducing the temperature the k initially increases goes through a peak and then comes down this is also clearly showing that as k is increasing our transformation kinetics is increasing the rate of transformation is higher as we already know and as k reduces the kinetics reduces and this is to be expected. So, this is the behavior of k with temperature now let us come down just a second I just kept this out of the way and bring this formula box here to help us.

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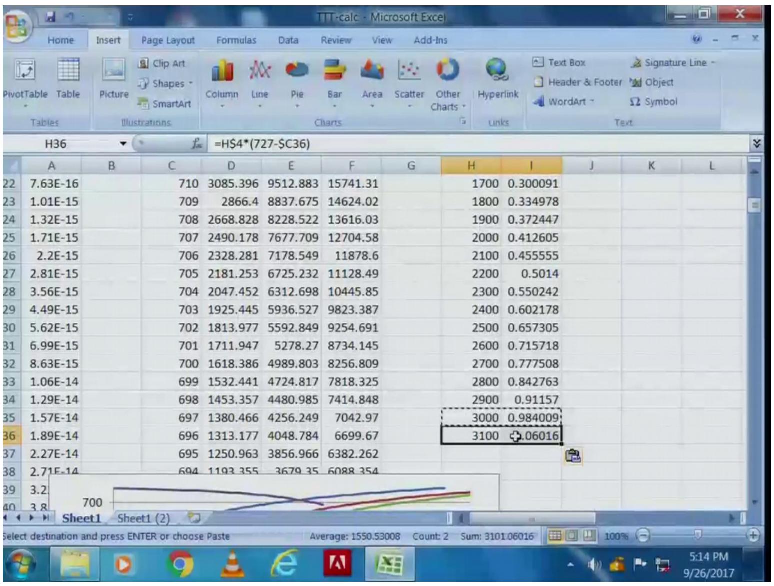

Now, what I want to do I have now k values I have the temperature values now I want to calculate the time isothermal time for at different temperatures.

That at if I hold the sample at 726 how long is it going to take for the transformation to complete 0.01 or 1 percent or 50 percent or 99 percent. So, let us begin with the time calculation now if you look at the first equation this is gives me the time of transformation or what I have just written on the board tau t t t and this is nothing, but rearrangement of the avrami equation and I will just simply quickly put down this relationship natural log of 1 minus the fraction transformed I am interested in is 0.1 and I will anchor it to row 4 divided by the corresponding k value at the temperature of 726 and this at this would have to be if you look at this ah relationship.

This will have to be raise to 1 upon n. So, cap representing to power 1 divided by and I have the value of n here in column a 2 which is 3.76 and I need to anchor it. So, that it does not change when I copy it to different cells and I get a value of 14 5 4 8 14548 second. So, a long time now I will just copy this time just a second copy and paste up to 376 and paste it to get now different times as you can see that initially large time and the time is reducing and if we keep going down time is reduced small values and then now time start increasing again as expected as we increase keep increasing the under cooling. So, as we under cooling is increasing time is initially decreasing for the transformation and then increasing all I have to do now to calculate the times for 50 percent and 99 percent I will simply copy this column.

Now, oops I must have made little mistake here this is yeah I should anchor a 6 in such a way that the column does not change, but the row should be allowed to change. So, I must put the dollar sign in front of a I will just recopy this formula again in this cell and that takes care of this and then now let me copy this entire column and then paste it here. So, now, we have the times for 50 percent transformation and then similarly if I paste this one here this gives me 99 percent transform now if I would to plot this as now a t t t diagram; that means, it will have to be temperature versus time well I already have.

So, as soon as I calculated the data the graph shows me the 3 curves the blue one is for 1 percent transformation the red one is for 50 percent transformation and the green one is for 99 percent transformation now this curve represents this data the red line represents this data and green line represents the 99 percent data. Ok so, now, I have the t t t curves

I want to convert them into c c t diagram for this I need to impose a cooling rate. So, let me keep a very simple cooling behavior of the sample. So, of the cooling curve that I am going to describe here is time is equal to some constant b times the transformation temperature minus t. So, as so, this is a linear relationship between temperature and time.

So, I am just assuming a straight line and if I put this formula here for a given value of b So, let us say b is 100 then the time would be h 4 now this should be anchored. So, that the row does not change multiplied by the transformation temperature well eutectoid ah temperature 727 minus t. So, so this would be t r minus t. So, hence I should start from let say 726 anchors this also such a way column does not change and I get a value of 100 and I just copy this value down here and paste it here. So, I get these various temperatures or other various times at different temperature. So, my cooling behavior and if I want to show the cooling rate well this is how the cooling rate shows up now this cooling rate does not look like linear.

And the reason is because we are looking at very large time scale. So, the x axis or the time axis as well put as a log scale and that is why this does not look like a linear curve if I now want to find out how much time it is going to take for the transformation to end I must calculate on this side. So, let me just take this out and use now the additivity principle. So, the additivity principle means I have to add up delta t I upon this. So, now, if you look at it the first interval would be 100 the next interval would also be 100 because it is 200 minus 100 then next one is 300 minus 200 and. So, on first one is 100 minus 0. So, if I have to put this formula in here. So, I will put this to be equal to well the first one is actually 100 I can take it like this divided by time at this particular temperature.

How much time it takes for the transformation to complete and this time and we are going to do it for 1 percent transformation well this time is only this much So, this brings me to a very small fraction as you can see at the temperature of 726 only a small fraction has gone next formula would be add the previous fraction. So, that I can get the summation taking place plus what is delta t I well delta t I is h 7 minus h 6 multipl not multiplied divided by time for 1 percent and let me anchor the column. So, the dollars sign at d and that is it this becomes 0.014 and all I do now is simply copy this. So, if I look at it basically it is showing increasing fraction and let us see where it becomes one well it becomes almost one here at 3000 seconds.

Because if I copy this formula here well it goes a little beyond So, as an a approximation we can say it takes 3000 seconds for the transformation to complete now what I will do is we can do this now for different values of b. So, all we would have to do is simply copy these columns to value of b as 50 then 25 10 and 5 and I already have a calculated sheet to show you the entire calculation in one short and you will see this these are for different values decreasing values of be these are the cooling rates and they are all terminating at the point where the summation term goes to almost 1 hence if I join all those with the smooth curve this becomes my c c t line

So, what this is telling me is this line outside here is 1 percent transform for and this is a corresponding c c t line for 1 percent transform. So, as you can see there is a significant shift of the c c t curve from the t t t curve towards the right to longer times and they are shifted to a slightly somewhat lower values of temperature as well with this is how we can convert all t t t diagram to c c t diagrams and with this I come to an end to this lecture.