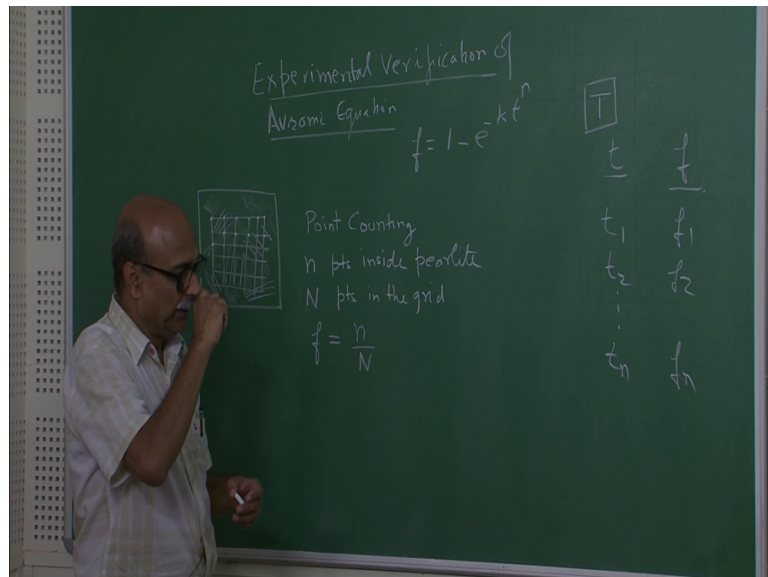


**Heat Treatment and Surface Hardening - II**  
**Prof. Kallol Mondal**  
**Prof. Sandeep Sangal**  
**Department of Material Science & Engineering**  
**Indian Institute of Technology, Kanpur**

**Lecture – 31**  
**Experimental verification of Avrami Equation**

In this lecture we will continue our discussion with the Avrami equation. And we will look at how to experimentally verify the Avrami equation.

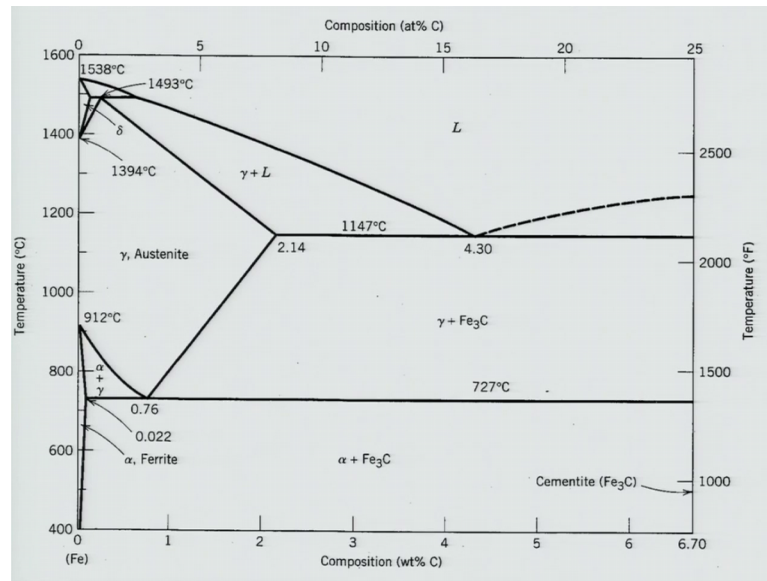
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So, today's lecture will be focused on experimental verification of Avrami equation. Which is let me just write the equation as well here f is equal to 1 minus e to power minus k t to power n. Now in order to experimentally verify such an equation, we have to conduct some experiments on a material in which we can measure for a given temperature amount transformed from one phase to another phase in different times.

So, for example, one can take a sample of steel in which we could look at the amount of pearlite transformed as a function of time from austenite phase at different temperatures. Over each temperature one should be able to determine the parameters of the avrami equation n and k. So but just before we go into this discussion. Let us quickly have a look at the iron carbon diagram.

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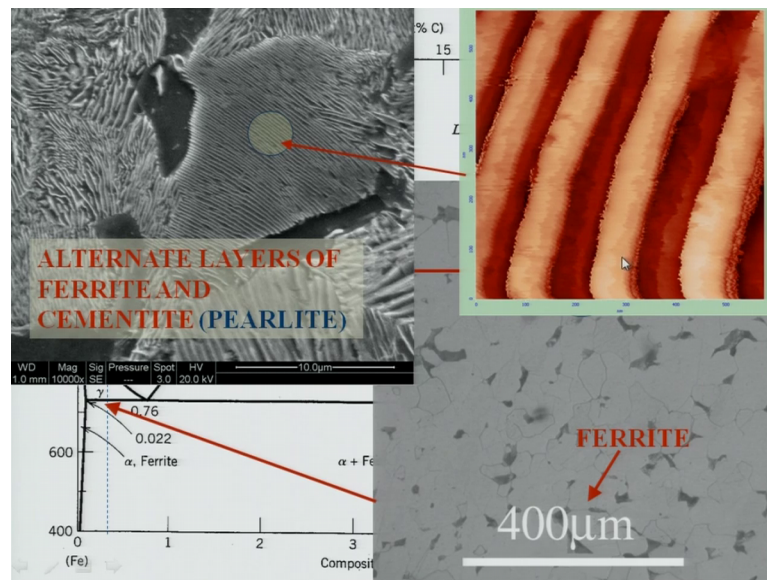


So, the and if one looks at the iron carbon diagram this is the iron carbon diagram. Here you can see that you have this is the usual iron carbon diagram which you already know.

You are let us say a steel with a certain composition, heated into the austenite field, and then it is allowed to cool down to produce the parent to produce the product phase alpha ferrite plus Fe<sub>3</sub>C cementite, which together are called pearlite. And depending on the carbon content of the steal you may form proeutectoid Ferrite you may form proeutectoid cementite or you may produce directly 100 percent pearlite.

So, let us quickly see what kind of microstructures we get. For instance if I take a hypoeutectoid composition, which is given by this broken line I take the steel to in the austenite range and then cool down then the microstructure evolved if I looked at through the optical microscope.

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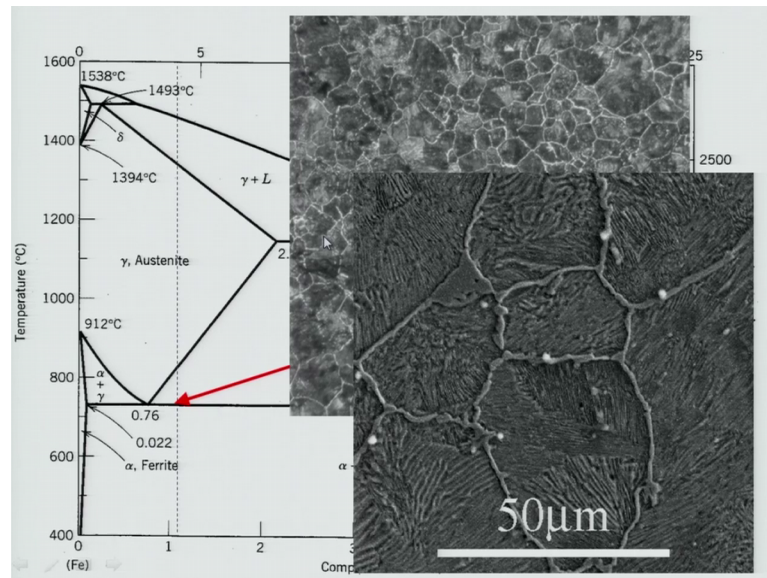


This is what kind of microstructure one would see. That you have a matrix which is essentially ferrite and there are re darkish regions as you can see, which if I zoom into one of these regions and looked at the structure under a scanning electron microscope then this is what I am going to see.

I am going to see these layers these are the layers of ferrite and cementite or what is together constitutes pearlite is the result of the eutectoid transformation of austenite to pearlite. To see these layers even more clearly one can further zoom into this and look at it under a atomic force microscope and you can see these layers very clearly. I just to put things in perspective regarding the dimensions, this length is of the order of 500 nano meter. So, you can see that thickness of each of these layers is the order of less than 100 nanometres.

Similarly, if I take a hypereutectoid composition given by this broken line which is now greater than the eutectoid composition of around 0.8 and look at the microstructure in an optical or microscope in a scanning electron microscope for better resolution.

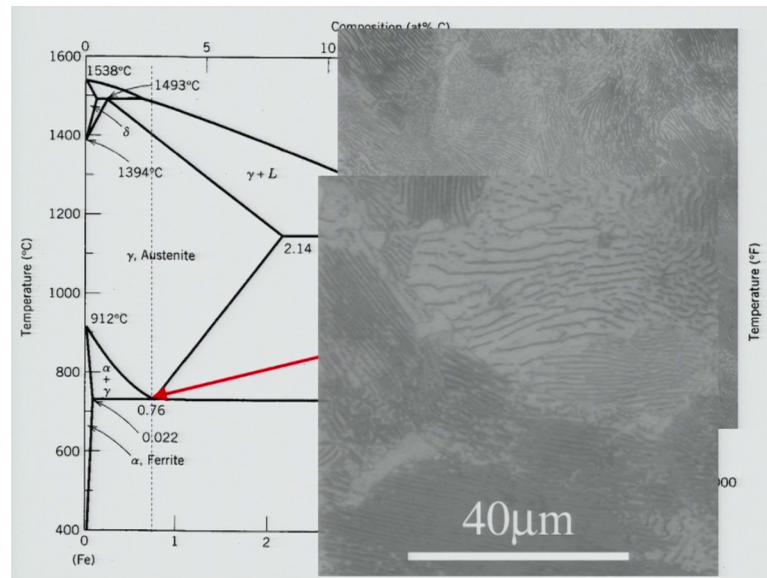
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One can see that there is a network of phase precipitated this is the cementite phase, the proeutectoid cementite that forms and this cementite does essentially heterogeneously nucleated at the grain boundaries. When the temperature reaches below the eutectoid temperature than the remaining austenite transforms to pearlites. So, all of this regions within that is all pearlite.

Now, consider a third composition and that is the composition the eutectoid composition. Now in this composition if I let it cool from the austenite I will get now a structure which is 100 percent pearlite.

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Now this is the impact this is this material I am interested in look in doing an experimental verification of the avrami relationship. So, what we can do? Well, we can heat take a eutectoid sample. Heat it to the austenite region let us say heated to about 900 degrees centigrade.

So, we will be well in inside the austenite region and then quench it at a certain temperature. For instance so, when one can quench at 700 degree centigrade. Now how does one quench at 700 degree centigrade, it is easy to quench in water when you want to produce martensite well you take a salt bath. A salt bath is the furnace in which certain composition of different salts are taken so that they have a certain range of melting point. So, we quench in a molten salt bath. So, if a salt bath is kept at 700 degree centigrade then we will quench the sample, at 700 one sample we quench we keep it for some time take it out and then quench in water.

So, what will happen? The final structure that would form would be some amount of pearlite which has formed depending on how much time the sample has been kept in the salt bath, the remaining austenite transformers to martensite on water quenching. Now we do a metallography of the sample. We take it to a microscope first of course, polish n h to reveal both the pearlite and the martensite. And then measure the volume fraction of pearlite in the sample.

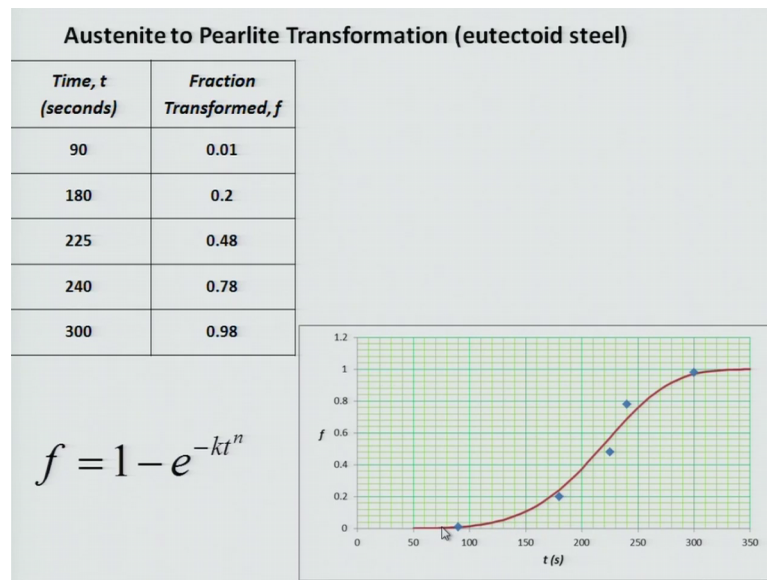
Now, how does one measure the volume fraction of pearlite in the sample? Well, we take a sample for instance let me illustrate this on the board that I am looking at a microstructure which has some pearlite regions. So, I am just drawing a schematic sketch, and rest all is martensite in this. So, we use a metallography technique called point counting. In this technique one would place a grid of points. So, a grid is something like this where you can see lines, horizontal lines and vertical lines in the grid these are the intersection points, all of wherever the lines are intersecting these are the points of intersection.

We count how many such points are in the pearlitic region. Suppose there are  $n$  points inside pearlite. And there are total of capital  $N$  number of points in the grid. Then the volume fraction of pearlite that is formed is given by this estimate or this ratio small  $n$  upon capital  $N$ . This is a well established technique for measuring volume fractions. Now you would not just do it in one area, you do it in several areas and then find a average volume fraction and that would be the representative fraction of pearlite form. Now this you would do on one sample one time second sample different time and so on until you values of volume fraction and time. So, I will get a table in the end, that a time  $t_1$  and this is all done at some temperature  $t$ .  $T_1, f_1, t_2, f_2$  and so on the  $n$ th point. So, if I have  $n$  number of samples I will have  $n$  data points. Now this data needs to be analyzed using the avrami relation.

So now the next thing one we need to look at is how do we analyze this data. So, let us first see what do we mean by analysis. And let me show you here that we want to analyze avrami kinetics, which means we need to analyze the experimental data that we have collected.



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So suppose and this is an experiment in which some data has been collected of austenite to pearlite transformation in a eutectoid steel. So, and this temperature here though it is not mentioned, the temperature of the salt bath was kept at 660 degrees centigrade.

So, at 90 seconds 0.01 trans only 0.1 has transformed one or 0.01 a fraction is a pearlite phase or simply one percent pearlite has formed. At 180 seconds 0.2 or 20 percent pearlite has found. 225 seconds you have 48 percent 240 seconds you have 78 percent, and in a 300 seconds almost we are now very close to 100 percent transformation.

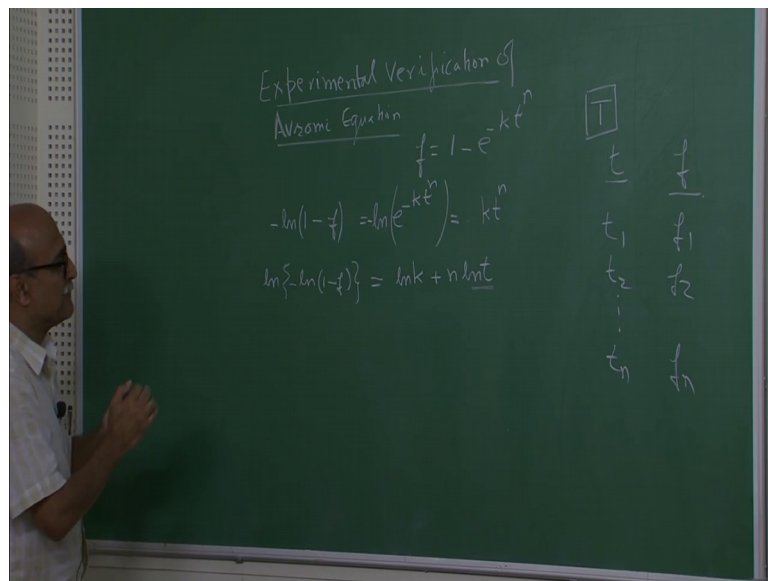
Now, the first thing you do whenever you have some experimental data the best thing you can do to begin with if you have no clue is to simply plotted on a graph sheet, and this is let us say the graph that you get of. So, this data which has been plotted here of the vertical axis is the fraction transformed in the horizontal axis is time in seconds, and this is the data point that you have. From this you may start making some conclusions. Now the objective here is if we need to analyze this data in terms of the avrami relation, I need to find the parameters *n* and *k*. And once I know what the parameters in *n* and *k* are, I would be of the avrami equation here. I would then be able to plot a continuous graph super imposing on these points as shown here.

So, this is what I am after, that once I find *n* and *k* then I can plot continuous graph through these set of points. And I can now predict what would be the amount transform at different times for the given temperature. So now, that next objective is that this how

to go ahead to look at this problem. And to look at this problem let me just go back to the board again and look at the sick avrami equation again and see what we can do with this equation. Many times and you have a non-linear equation like this, you try to see if I can if one can linearize this equation.

So, in order to linearize this equation let us rearrange this equation in this fashion  $1 - f$  equals  $e^{-kt^n}$ , and then take logs on both sides.

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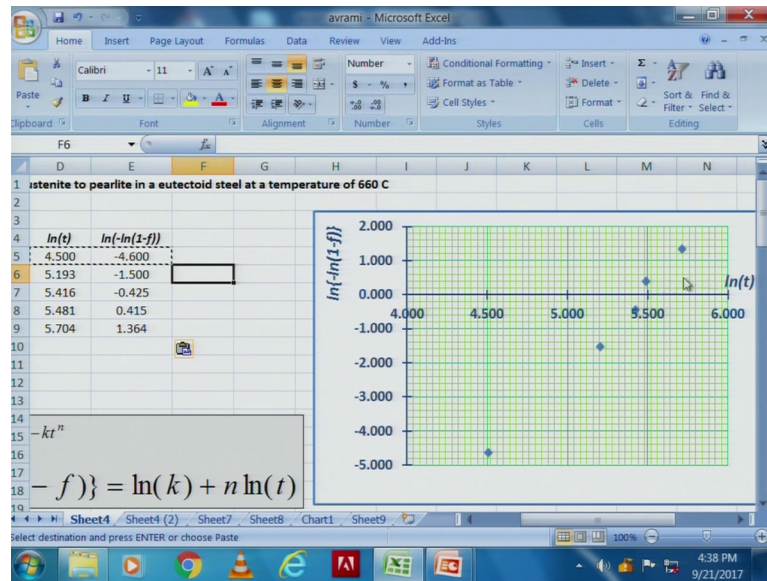


And take the log of this, and take taking the log of this would be equivalent to writing this term as minus  $k t$  to the power  $n$ . Now what I do is bring this negative sign on this side. So, I have relation like this. Take logs again of the left hand side minus  $\ln(1 - f)$ , and the right hand side  $k t$  to power  $n$ . And I will get a relation  $\ln\{1 - \ln(1 - f)\}$  equals  $\ln k + n \ln t$ .

Now, bringing the negative sign here ensures since  $f$  is less than 1,  $\ln(1 - f)$  will be a negative value, and the negative sign here will make it positive. So, I will be take the log of a positive quantity. And out here if I take the log of this will simply become  $\ln k + n \ln t$ . Now you can see that the left hand side term taken as the whole linearly varies with  $\ln t$ . So, this is what we mean by linearization and let us what let us go to a spreadsheet take this data that we have and plotted on a graph, and see how that graph looks like.



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So, here I have the same data of isothermal transformation of austenite to pearlite in a eutectoid steel at a temperature of 660 degrees centigrade, and let us take logs on both sides. So, first take log t that I need to calculate on the right hand side of the equation that I have. In fact, let me show you this equation as well so that is also in front of us. So, I want to calculate the left hand side and the right hand side which is log t, and the left hand side which is are 2 logs.

So, let me calculate the right hand side. So, let us call that has spread sheet has a function called natural log ln, and it requires me to give time here which is in a cell a 5 and this gives me the value of log 90 here. And here let me take logs minus log. So, I have to take 2 logs, and then I have to take 1 minus f and therefore, this in cell B 5 is f one bracket close second bracket closed and this gives me the left hand side term for at time t equal to 90 seconds.

Let me just put another bracket here that is all, now copy these 2 formulae downwards and calculate for all each of these experimental data points. I get a set of values for log t and log of minus log of 1 minus f. Now this here will show me the result if I plot this data. Now this data shows that now the points appear to fall on a straight line, all the 5 points a roughly on a straight line.

Obviously when you do an experiment there are going to be fluctuation. So, will they will never fall exactly on the curve now. So now, one thing I can do is now I can try to

draw a straight line through it, but how do I draw a straight line through it? One is I can use my judgment and make a make up a plot. So, you know I could have a line like this, but somebody else may draw a line like this, or a third person may draw a line like this. So, thinks that this line is better.

So, who is to figure out that which is the correct line? So, what we should try to do now is how to objectively find a the best line passing through this set of data points. Once we know the best line then the slope of this line then would be the value of the parameter  $n$  of the avrami equation, and the anyway extend this line all the way until it intersects the vertical axis that would give me the intercept. And intercept would then be nothing  $\log k$ . So, finding the best line essentially involves finding out the intercept and the slope of the 2 of the and then from that I get my avrami parameter.

So, I have to my problem at hand now is what is the best so called best values of the coefficients  $n$  and  $k$ . So, we would then I will now going to I am going to do edit t, I am going to digress a little bit and we will look at how to find the best fit parameters of the avrami equation, or how to find the best fit parameters for a straight line if I have a set of data points. So, what we do is we have already linearized this equation. Let me call the left hand side as simply  $y$ , right hand side  $\log t$  let me call it as  $x$ ,  $\log k$  which is a intercept let me call it  $a_0$  and let me call  $n$  as  $a_1$ . So, that I have a simple form of the linear equation  $y$  equals  $a_0$  plus  $a_1 x$ .

So, if I have a pair of points a set of pair of points  $x$  and  $y$ , then given these set of data points I have, I want to find the best fit parameters  $a_0$  and  $a_1$  which would give me the best fit line through the experimental data. I will discuss how to determine the best fit parameters  $a_0$  and  $a_1$  through the method of linear regression in the next lecture.