Heat Treatment and Surface Hardening (Part-II)

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Lecture - 13
Expressions for equilibrium of two phases - II

Let us continue the concept what we have been doing for the last few lectures. And in lecture 12, we have come to the particular point that how to get equation for liquid solution as well as solid solution. And that equation contains delta G value in pure condition for both A and B. And also it contains the entropy of mixing term along with temperature term. So that means, which is nothing but delta G mix. So we will have our lecture 13.

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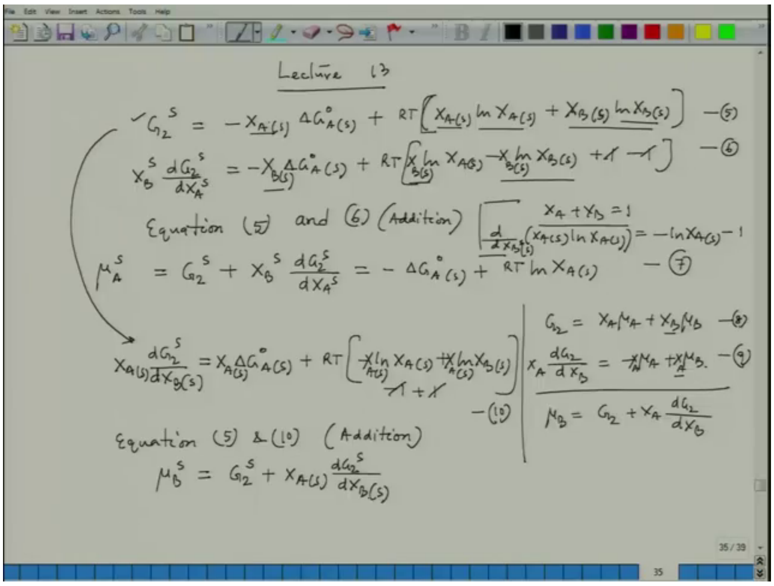

And in this lecture we will try to see the equilibrium condition along the common tangent for solid line and liquid line. In order to do that we will just go back to the picture what we have drawn, in our last lecture 12. This is the picture what we have drawn. And then finally after doing lot of calculations, lot of equation development we have reached to this particular situation. And also you have seen that in order to find mu A for a solution I can make use of this equation, of course we can also develop equation how to find the chemical potential of B, and let us start that part. I know G 2 solid is equal to minus X A solid plus.

Now G 2 S G X A S is equal to minus delta G 0 A S plus R T l n X A S minus l n X B S and then, from this part it would come 1 and from this part it would come minus 1 both would get cancelled. So, if I multiply with X B S then it would be minus. So, this is equation let us say 5, and this is 6. If I continue from the equation number what we have done in the previous lecture. So, there we ended equation number 4, but now we have to start with equation number 5 and 6. So, if we add equation 5 and 6, then we get G 2 S plus X B S G 2 S; you see that these 2 if we add this 1 and this 1 so it becomes 1, this is minus delta G A 0 S. Here also, if we add this quantity and this quantity they would cancel, and if we add this and this quantity they will become 1. So finally, we would get R T l n X A S, so this is equation 7.

And this is nothing but mu A S. Similarly, I would get mu B S; that means, chemical potential of B in solid solution the similar line so I can try to get another equation for mu B. And just to follow up what we have done in lecture number 11, if G 2 is equal to X A mu A plus X B mu B then X be equal to minus mu A plus mu B, and then if I multiply with X A, and then add this is let us say 8 equation this is 9, if we add them we would get mu B equal to G 2 plus X A G 2 d X B. Since this one and this one if we add they, will be 1.

Soon then, d G 2 S X B S equal to delta G 0 A S plus R T minus l n X A S plus l n X B S this would be minus 1, and here it would be plus 1. So, it would get cancelled because derivative of l n X A solid would term in to minus 1 if we multiply this term again because X A plus X B equal to 1.

So now, d d X B of X A solid l n X A solid could be equal to minus l n X A S minus one. Then if we multiply this 1 with X A solid, so this is X A solid this equation is let us say 10, see if we add equation this is addition, equation 5 and 10 addition we would get mu B as equal to G 2 S plus X A S G 2 S S is equal to R T l n X B S.

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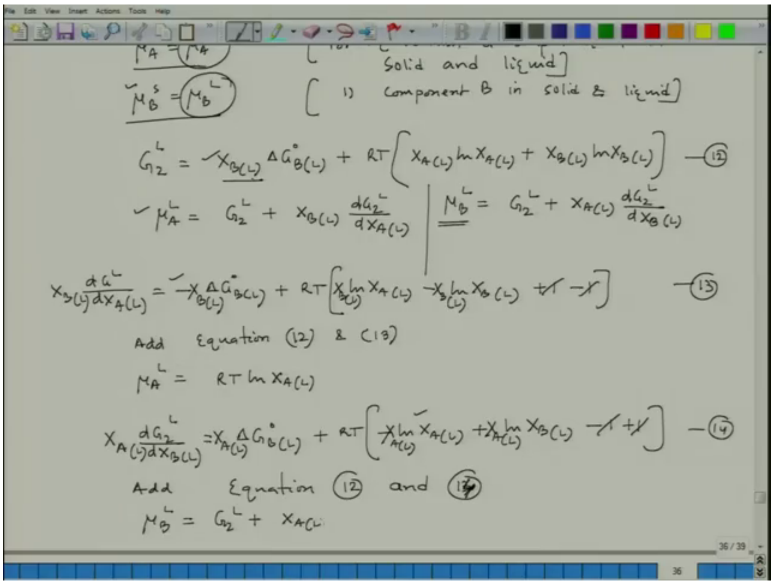

So, this is equation number 11, for equilibrium for component A in solid and liquid is nothing but mu A solid equal to mu A liquid. So, what we have calculated is nothing but mu A in solid phase, and mu B in solid phase. Now you have to also calculate mu A in liquid phase, now for equilibrium same of component B in solid liquid it will be mu B solid equal to mu B liquid and we have calculated these 2 quantities, we have to calculate this as well as this, and for this we have to make use of G 2 of liquid solution

now we can write the G 2 of liquid solution is equal to X B liquid delta G B 0 I can simply write liquid plus R T X A liquid l n X A liquid plus X B liquid l n X B liquid and it would follow the same way of calculating mu A as well as mu B in the liquid phase in case of mu A which is nothing but G 2 L plus X B liquid d G 2 l T X A L whereas, for in case of mu B liquid it would be G 2 L plus X A L dg 2 l d X B L.

Now, d G 2 L L equal to since this is in terms of B, so it would be minus delta G 0 B L plus R T l n X al it minus l n X B L plus 1 minus 1 this is coming because of derivative of logarithmic term, and as well as there is a multiplication factor which is X by L on X B L

So, if I multiply this 1 with X B L the way we have done it here, then this is X B L X B L X B L; that means, in the liquid phase. So, this equation let us say I term it as 12 and this is 13, if I add equation 12 and 13, we would get mu A liquid equal to, so this would get cancelled this term and this term would get cancelled. So, we would get R T l n X A L.

Similarly if I try to find out mu B L, so this is G 2 L d G X B L equal to delta G B 0 L plus R T minus l n X A L plus l n X B L minus 1 plus 1, and then if I multiply with X A L this is X A L then this term, and then if we add, this is equation 14 add equation 12 should be 14. Then we would get mu B L equal to delta G B L so, 0 L plus R T l n X P L.

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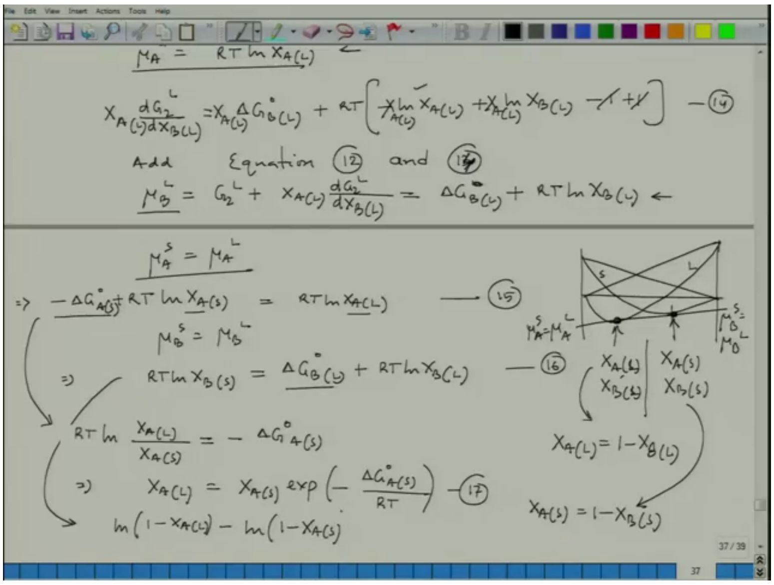

Now, I can use this equilibrium conditions. So, now, my equilibrium conditions are mu A solid equal to mu A liquid, I can get mu A liquid equation as this one. So, mu A equation R T l n X A L is equal to I have already calculate mu A S which is this 1. So, I can put that equation as minus which is minus delta G A plus R T l n X A S this is equation which is let us say equation 15. And similarly for another equilibrium mu B is equal to mu B L that case it would be mu B L we have calculated it is nothing but mu B S if I consider mu B S is this 1. So, this is R T l n X B S equal to and mu B is L is basically this one, which is delta G B L plus R T l n X P [laughter], just cross checking mu B S is this mu B mu A S is this, and then mu A L is this and mu B L is this. So, I am getting equation 16.

So, now these 2 equation if we see carefully, we have 2 unknown because if we know delta G m for pure A and delta G m or P or B, then we can easily calculate delta G 0 of a solid and delta (Refer Time: 09:10) 0 B for liquid.

Now, here if we see this term, X A S and X A L if I try to plot that miniature plot, so this is mu A S equal to mu A liquid this is mu B S equal to mu B liquid. Now I am pointing these 2 lines, these 2 points, at this point I can have mu A solid mu B solid this particular composition, and this cortical and this is for I think this equation if I go back to that particular plot, this is for solid, this is for liquid.

So, these 2 compositions I can term in the form of X A or X B both the time they are liquid, this some touch some time it becomes a little confusing, and this is X A solid X B solid. Now I can term X A liquid so, these compositions; and these compositions, I can term X A liquid equal to 1 minus X B liquid. And this 1 I can write X a solid equal to 1 minus X B solid, so actually we are having 2 unknown 2 equations.

So, now we can solve this particular, and to solve this we have to just express either liquid or solid composition with reference to a particular component either A or B let us do it from the point of liquid pure A ok, from the point of A.

So, now if I try to does a little bit of simplifications here, so l n X A liquid by X A solid I can write it minus delta G 0 A S, then I can write X A L equal to X A S exponential minus delta 0 A S divided by R T, this is equation 17. Similarly from this equation I can write l n 1 minus X A liquid minus l n 1 minus X A solid equal to minus delta G B 0 l then.

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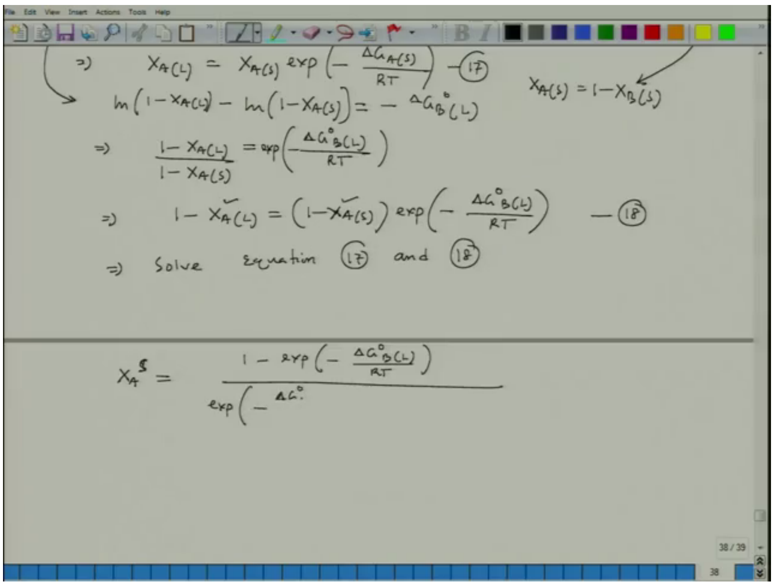

I can write 1 minus X A L equal to delta G 0 B L by R T minus exponential by 1 minus X A S then it would become 1 minus X A L equal to 1 minus X A S exponential minus delta G 0 B L by R T so, we have equation 18.

Now interestingly you can see that equation 17 and 18 can be solved because we have 2 unknowns, this is 1 unknown and this is another unknown. So, we can solve this solve equation 17 and 18 and the solution, if we see that solution becomes I can solve this, so now, I can get X A S I can get equal to 1 minus exponential minus delta 0 B L divided by R T exponential minus del 0 A S minus. Similarly X A L equal to 1 minus X A S. So, we have both these compositions we have found out what is my X A composition and X A comp X A composition for solid and liquid state.

So, let us stop here. We will continue in our next lecture, that why these two solutions for compositions at two different points, where the phases are attic are in equilibrium leads to a phase diagram and that to binary phase diagram.

Thank you.