Fundamentals of Acoustics
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Lecture - 09
Important Mathematical Concepts-Transfer Function

Hello. Welcome to Fundamentals of Acoustics. Today is the third day of this week of this particular MOOC course, and what we will be discussing today is essentially about Transfer Functions.

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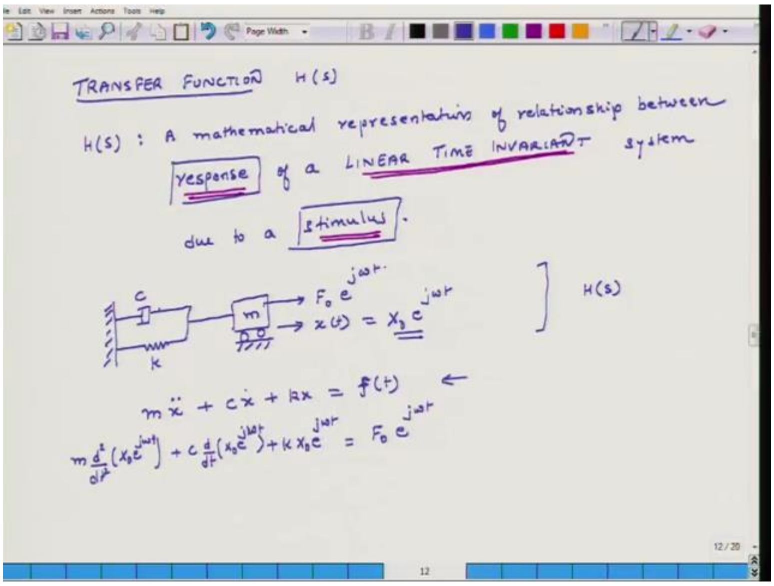

So, the theme will be transfer functions and we will begin this discussion by defining what is a transfer function, and then I will explain it. So, typically the transfer function is designated by letter H, and it is a function of this variable s. Lot of time this s is nothing, but the frequency, the complex frequency of the system. And in even more situations it is not just a complex frequency, but actually just the actual frequency of the system. So, H s or the transfer function what is it? It is a mathematical representation. So, what does this representation give? It represents of relation or relationship between response of a linear and time invariant system. And I will explain what a linear time invariant system is due to a stimulus. So, that is what it is.

So, transfer function which is H s is the mathematical representation of relationship between response of a linear time invariant system due to a stimulus. So, there are three terms which you may be wondering want they mean first is; what is the linear time invariant system? So, any system which is linear and we have discussed what is linear in the last class, so that that is what linear means.

But the system has to be linear as well as time invariant; and what time invariant means is that with time the system should not be changing. So, you know the configuration of the system should not be changing. If suppose there is a register, capacitor, inductor, all these things are in the system as time is changing its not that one register is getting replaced by another register and things are changing right. So, that kind of a system which does not vary with time the overall architecture of the system is not changing with time that is a time invariant system. And then it has also has to be linear which means that the relationship between its input and output has to be a straight line.

So, for that kind of a system if you excite it with some input and the input is I am calling it as a stimulus. So, the input is the stimulus and because of the input there is some output in the system and that I am calling as response. So, a mathematical relationship between input and output for a linear time invariant system is known as Transfer Function.

So, I will give you an example. We will again look at spring mass and in this case I am also going to add a dash spot. So, this is mass, this is the spring stiffness, this is the dash spot, and I am going to excite it with some force F naught e j omega t. And let us say that because this is a system which is linear we assume that this system is linear because the spring is linear spring, the dash spot is linear, and we also say that the system is time invariant which means the values of m k and c they are not changing with time; and also the configuration does not change with time.

Then this is a linear time invariant system and I am interested in finding out what is the value of displacement; x of t. Now because this is a linear system we had said that the steady state response for this system will be of the same form as the excitation. So, excitation is F naught e j omega t. So, x t will be x naught e j omega t. I can make this assumption only if the system is linear in nature, if it is not linear then this may not necessarily be true.

Now, the governing differential equation for this system is m x double dot plus c x dot plus k x is equal to f t. So, if you want to figure out what is the proof for this differential equation you will to have go back and look at a book on vibrations and this is the very basic equation, but if you do not understand how I got it then you please go back and look into some book on vibrations. But this is the governing differential equation of the system, and in case f of t is F naught e j omega t and x is x naught e to the power of j omega t. And what is my goal? My goal is to find transfer function for the system.

So, what I will do is replace f t by F naught e j omega t. So, it becomes F naught e j omega t. And I also replace x by x naught e j omega t, so what I get is m d 2 over d t square x naught e j omega t plus c d over d t x naught e j omega t plus k x naught e j omega t. So all I have done here is I have replaced x with x naught e j omega t and f with F naught e j omega t; and x double dot means second derivative in time, x dot means first derivative in time.

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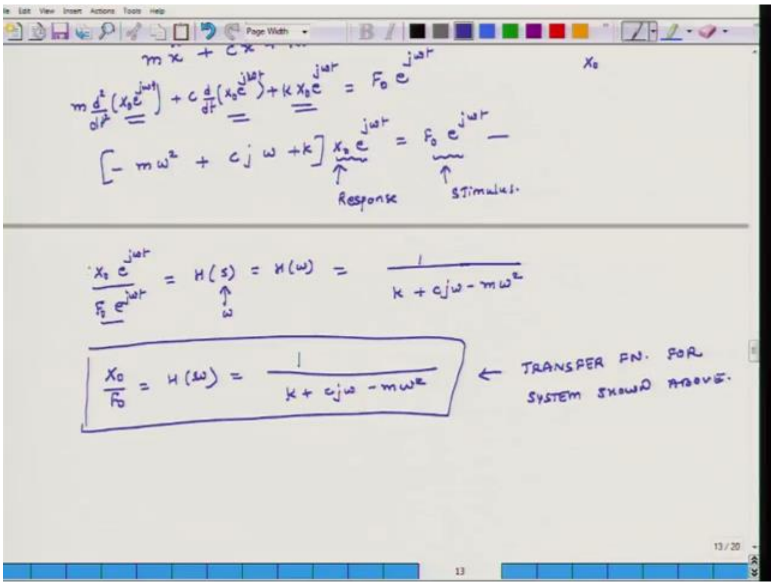

So, x naught is constant it may or may not be real, but x naught is constant. So, when I differentiate it what I get is minus m omega square and plus c j omega plus k times x naught e j omega t is equal to F naught e j omega t. So what I have done is, I have collected e j omega t from these three terms and brought it out of the bracket; and also in each of these terms x naught is there x naught is the constant so that is also outside the bracket. So, this is what my governing differential equation has changed to. So, initially it was a second order differential equation, now it is simple algebraic equation.

Now what is our goal? Our goal is to find the relationship between the stimulus or the input and the response which is the output. This is my response, actually this entire thing is my response and this is my stimulus. So, the relationship between these two is x naught e j omega t divided by F naught e j omega t; this is what H of s, but in this case s is nothing but omega, so this is equal to H of omega. And this is equal to 1 over k plus c j omega minus m omega square. So, x naught over F naught e j omega t cancels out is the transfer function and the value is 1 over k minus not value the function is omega square.

So, this is the transfer function for system shown above. And this transfer function tells us the relationship between the stimulus; the stimulus is F naught e j omega t and the response, the response is x naught e j omega t. It is the relationship between stimulus and the response.

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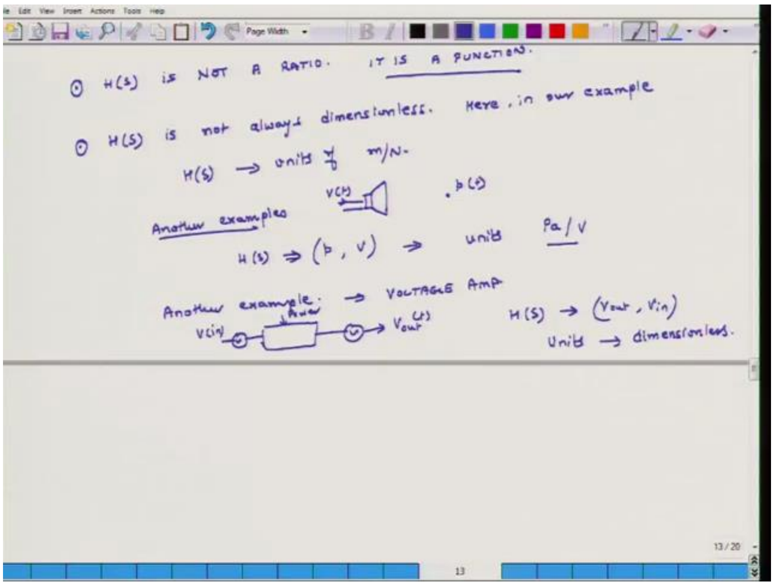

Couple of important observations first; H of omega is not a ratio it is a function, ratio is a number; it is not a ratio it is a function. See this x naught is changing with respect to omega; so it changes with respect to omega, as omega changes this x naught and F naught that relationship is changing. So, lot of times people say that H of s is just the ratio of output and input, but input and output if they are two numbers 4 and 3 then you can make it a ratio. But if output is changing with respect to some parameter or input is changing with respect to some parameter then when you divide these two entities you do not have a number you have a function. So, it is a function, this is important to understand.

So, it is mathematically wrong to call it as a ratio that is first thing. Second thing, H of s is not always dimensionless. In this case here in our example H of s has units of what meter per Newton. Why, because the output is in meters displacement input is in Newton's. And when you look at each of these terms 1 over k, k is Newton's per meter inverse of that is. So, all of these have meters per Newton in terms of dimensions.

Now, in some situations H of s may be dimensionless for instance, an electrical amplifier and suppose it is a voltage amplifier so input or the stimulus is voltage output is voltage. In this case the transfer function will be dimensionless, but it may not always be case in all the systems; in some systems where input and output has similar units there it may be dimensionless, but in several systems it may have dimensions and units.

Another example where it is having dimensions; suppose, I have a loud speaker, and because of this loudspeaker there is some pressure being generated here. And I am running this loudspeaker by exciting it with some voltage; v as a function of time, and I am interested in pressure as a function of time. So, in this case my transfer function will be what, it will be related to pressure and voltage; it will be a function of pressure. And voltage and in this case the units will be; so if the stimulus is voltage and the response is pressure then the units will be Pascal's per volt.

Again this is having dimensions, but as I said if you have another example. So, here it is of a voltage amplifier. So, here I have a box and there is input voltage and there is output voltage v out, this is another function of time this is v in as a function of time, and then of course there is some power source external power maybe it is a d c power, a c power whatever. Then I am interested in finding out transfer function for what for v out and v in. In this case units will be dimensionless.

So, it depends on the situation the transfer function may have dimensions or it may not have dimensions, but lot of times people think it is just input output. So, it is dimensionless that is not necessarily the case that is there. One last thing, what we had done here in this case we had developed the transfer function by considering this differential equation; governing differential equation right. And we said that this is the solution of this differential equation. We assume this as a solution and we assume that x naught e j omega t is the response, so that is the solution and then we plugged in x naught e j omega t and then worked out the transfer function.

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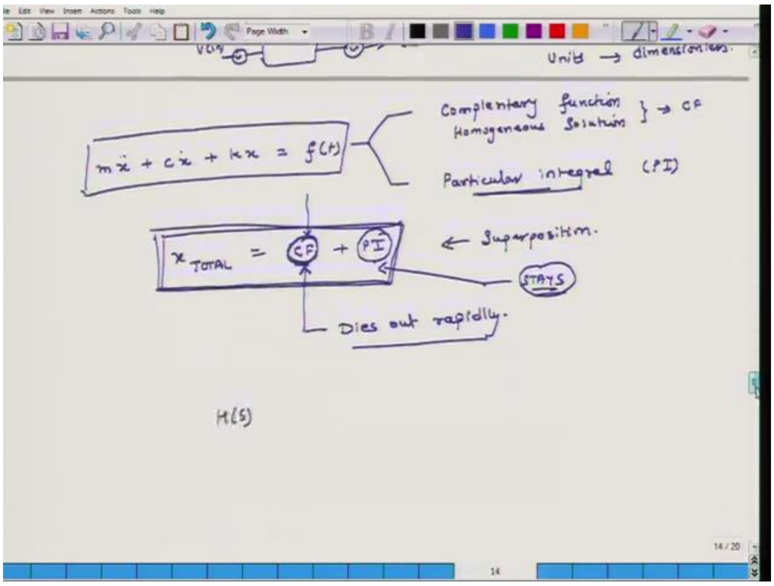

But there is something additional you have to think about. So what we will do is, we will look at this equation once again: m x double dot plus c x dot plus k x is equal to f of t, this is the original differential equation. Now you know from mathematics that this differential equation when we try to solve it, it has two components; the solution of this second order differential equation it has two components. The first one is our complementary function. So this is the first part of the solution and that is known as Complementary Function.

If we put f t to be 0, then m x double dot plus c x dot plus k x will have some solution right; it will have some solution that solution is called Complementary Function. And the other one is it is also known as; this complementary function is also known as Homogeneous Solution. So, I can call this as complementary function, CF; what is CF? It is the solution of this governing differential equation when f is 0.

The second part of the solution is; particular integral, PI or particular solution. What is this particular solution? That is the solution we developed in this case. Why is the solution called particular? Because it corresponds to a particular type of forcing function; See the general forcing function was f t here we said f t is nothing but F naught e j omega t, I know what is the nature of f t. Corresponding to this f t I tried this solution and it worked out. So, that is why this solution is particular integral or particular solution. So, this is particular integral.

Now this is a linear system if f is 0 then I have complementary function, if f is non-zero n of F naught e j omega t then I get a particular integral. Then by rule of superposition what is the overall solution? X total is equal to complementary function plus particular integral. This is from superposition. If it was not linear then we cannot say that for sure, but this is from superposition. Typically this complementary function, it dies out rapidly. For instance, when I have a mass spring system and it is addressed and I excite it with the force it will have some vibrations in the beginning which will die out and then it will go into a steadier state situation; a steady state situation.

Similarly, if I have inductive a circuit with inductor, capacitor, and register and then I turn on the switch. Initially there will be some unsteady solution you know, but then after sometime the current will become steady state in the system. So, what are their original definitions? I had mentioned that this transfer function is in context of the steady state of the situation. So, this stimulus is there and the response when It is in steady state then we develop the transfer function. So, this dies out and this particular integral this stays; this stays there.

So, when we are developing the transfer function we actually do not consider complementary function, because this dies out it is not part of the steady state solution of the system. It is only the particular integral which stays there because they are forcing function is moving back and forth continuously e j omega t, so because of that the displacement also remains in steady state. So, in transfer functions calculations we usually ignore the complementary function, we only worry about the particular integral.

But, mathematically speaking if I have to solve this for x this is the actual complete solution of the system, but most of the times this part dies away in a short bit durational time. So, we only worry about the particular integral. So, this is small but very important point to understand as we start thinking about transfer functions that we are only interested in solving for the particular integral.

So, this completes our discussion on transfer functions. And that is pretty much what I wanted to cover today. Tomorrow onwards I will start talking about pole zero plots and bode plots.

Thank you and have a great day. Bye.