Fundamentals of Acoustics
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Lecture – 08
Important Mathematical Concepts-Complex Time Signals

Hello, welcome to Fundaments of Acoustics. This is a 12 week course, today is the second day of the second week of this course and as mentioned earlier in this week, we are going to review some of the important concepts which will be very often used in our course and specifically today we will be discussing about complex time signals.

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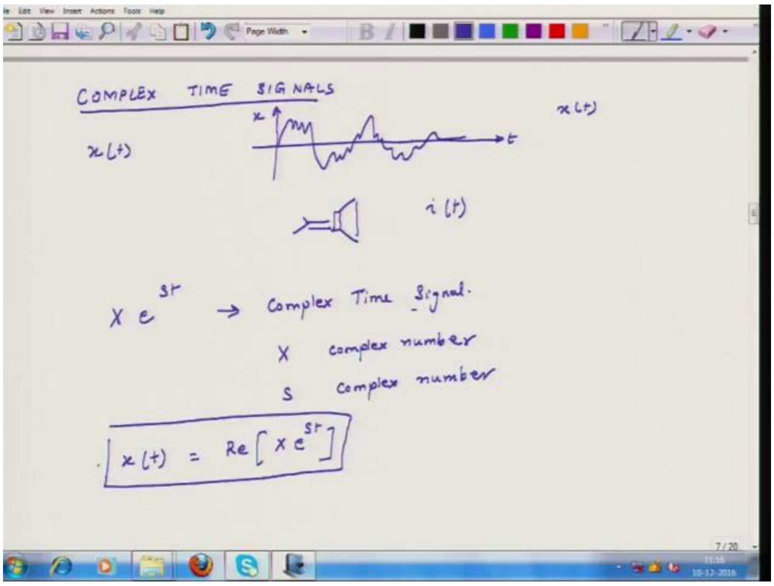

Complex time signals, so there could be a real time signal x t for instance the pressure in air it may be changing like this. So, this is my time axis and this is the amplitude of pressure x then x t is the real time signal because it is real which corresponds to pressure.

Another example of a real time signal will be if you have a loud speaker and you are providing it with some voltage and there is current going in, so i t which is the current that is a real time signal. Now complex time signals are analogs of these types of signals, but in the complex domain. So, complex time signal in general it can be represented as x e to the power of st. So, this is the complex time signal. Here both x is a complex number and so is s which is also a complex number the relation between a complex time signal and a real time signal is that x t is equal to the real portion of this complex time signal e s

t. So, if I know a complex time signal and I want to figure out its actual physical component I take its real component and that is what is x t.

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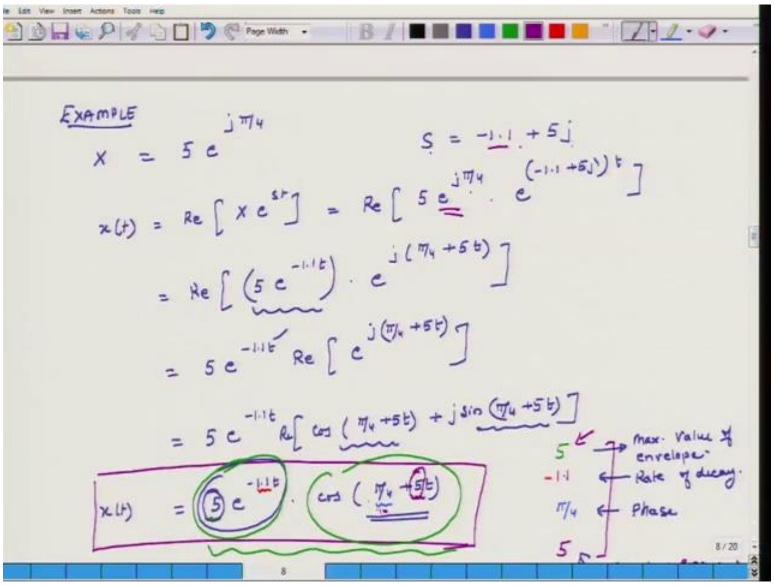

Now, let us look at an example. So, example, x as I said it as it could be a complex number. So, this in this case is 5 e to the power of j pi over 4 and S equals minus 1.1 plus 5 j, 5 j; by the way this x is often known as complex amplitude of the signal. So, a complex time signal has a complex amplitude and that is x and s is called complex frequency of signal complex frequency. So, complex time signal has complex amplitude and a complex frequency and this complex frequency because it has real and imaginary components. So, complex frequency is having real portion what should I use a let us say p plus j omega now this p is not necessarily pressure. So, this is the real portion of complex frequency and this is the imaginary component of the complex frequency. So, going back to the example x equals 5 e to the power of j pi over 4 that is my complex amplitude and s which is complex frequency is minus 1.1 plus 5 j, x t is real of x e to the power of st which is equal to real of 5 e j pi over 4 times e to the power of minus 1.1 plus 5 j times t.

Now I reorganize my complex time amplitude terms. So, I take the complex terms I mean imaginary components separately. So, I have 5 e to the power of minus 1.1 t this is the real portion and times e to the power of j pi over 4 plus 5 t. So, this is the complex portion. So, this is entirely real. So, I can just bring it out. So, I have 5 e minus 1.1 t real of e to the power of j pi over 4 plus 5 t which can be written as 5 e minus 1.1 t and I know real of cosine pi over 4 plus 5 t plus j sin pi over 4 plus 5 t. So, this is the real portion, this is the imaginary portion. So, x t is 5 e minus 1.1 t times cosine of pi over 4 plus 5 t. So, that is the real time signal and we have extracted that real time signal from the complex time signal.

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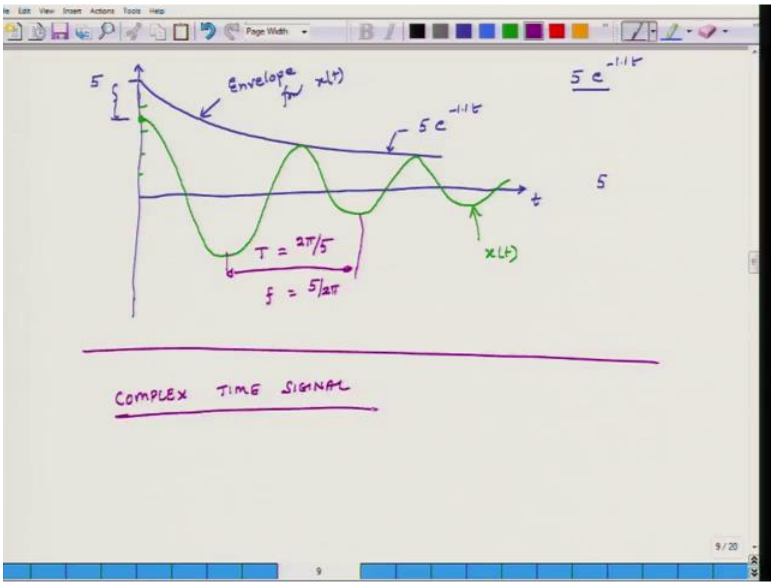

Now, let us try to understand the physical signification of each of these terms. So, what are the different terms? We have 1.1 which is this number or minus 1.1, then I have 5 which is this number, then I have pi over 4 and finally, I have these 3 terms are there 151.45, there is another 5 there is another 5. So, the other 5 is this way 5. So, we will try to understand the physical signification to do that we plot a graph. So, on the x axis we are going to plot time and on the y axis, we will plot 2 graphs, the first graph, we are going to plot is for this entire thing. So, we will plot a graph for 5 e to the power of minus 1.1 t at t is equal to 0.

The value of this entire thing is going to be 5 and as t increases from 0 to infinity this term it decays to 0. So, the graph is going to look like this. So, this value is 5 and this graph, this is the plot for 5 e to the power of minus 1.1 t, the other graph we are going to plot is for this entire term x t, for this entire term x t. So, at time t is equal to 0 this portion is going to be 5 and this portion is going to be cosine of pi over 4 a cosine of pi over 4 is 0.7 is 07 1 over square root of 2. So, 5 times 0.707 is roughly 3.5. So, let us say

1 2 3 4, so this is and this is 5. So, at time t is equal to 0, its value is going to be somewhere here and this is going to do this. So, this is the plot for the entire function which is x t.

Now, let us look at the role of each of these terms. So, the first thing is that if this pi over 4 was not 0, if pi over 4 was not there and it was instead of pi over 4. So, in this original function instead of pi over 4 if we had 0 then at time t is equal to 0 instead of pi over 4 we would have had cosine 0 which would be 1 times 5. So, this green curve would have coincided with the blue curve. So, that is why what it means is that the green curve can never cut across the blue curve. So, this green curve, this blue curve is known as envelope, this is the envelope for x t it can never exceed it if I change this phase phi over pi over 4 and I make it 0 then it will at some points it will be equaling it, but it can never cross that line. So, that is the envelope. So 5, this number 5 not this purple 5, but green 5 which is this number this is the maximum value of envelope that is what it means this number is not going to cross that number.

1.1, negative 1.1, if it was positive then it would grow which does not make sense, but in the lot of dammed system things actually decay with time. So, negative 1.1 is rate of decay the higher this number suppose I make it minus 2 then this decay would be sharper and so on and so forth, rate of decay.

The third number is pi over 4 and pi over 4 comes with cosine term and based on the value of this pi over 4 if I pi over 4 becomes 0, then this difference it becomes less. So, the value of this pi over 4 is nothing but the phase of the signal, it is the phase of the signal and the last term, this purple 5 which is this 5 this 5 represents angular frequency and this is r omega. So, f is what omega over 2 pi. So, the frequency will be 5 by 2 pi hertz and the time period which is the distance between 2 peaks, this is the time period. So, time period is what? Inverse of frequency so that 2 pi over 5 and frequency is 5 over 2 pi.

Going back, the complex amplitude, this is the complex amplitude, it is amplitude; the amplitude of this complex amplitude is what? It is the maximum value of the envelope and then the complex amplitude has a magnitude and it also has a phase, the phase when we play with the mathematics, it shows up as actually the phase of the system, phase of the signal and then complex frequency again has 2 parts - a real part and a an imaginary part the real part of the complex frequency is the rate of decay and the imaginary part of the complex frequency corresponds to the angular frequency of the system and from that we can calculate the frequency and the time period of the system. So, this is what complex time signal is all about, this is what the meaning of complex time signal is.

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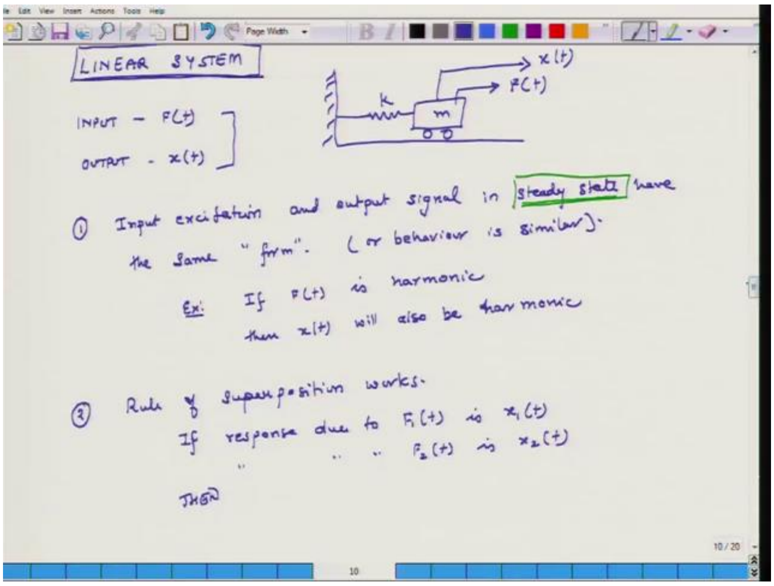

The next, we are going to discuss is a linear system because all the discussion in this particular course will be about Linear Acoustics. We are not going to talk about non-Linear acoustical effects. So, we have to understand what is a linear system and I will not go too much into the mathematics of what is the linear system is, but I will try to explain it in terms of the physical characteristics of a linear system. So, for reference purposes we will consider a mass spring system. So, there is a spring there is a mass connected and I am exerting a force of f t on the system and because of this force there is a displacement is the masses exhibiting. So, I will mass spring system, I am pulling it using a force and this force is not a static force, but it changes with time. So, sometimes I pull it in positive direction sometimes push it and because of this the system is the mass is moving back and forth, it is moving back and forth.

In this case, what is my input? What is my input in this system? It is the force and the output that is what I am interested in trying to figure out is the displacement. So the first attribute of a linear system. So, a linear system has several characteristics and the first attribute of a linear system is that input excitation and output signal in steady state have the same form or they behave or behavior is similar. I will explain that, what does it mean that if example if f t is harmonic then x t will also be harmonic it will also be harmonic, but this will not happen always this will be the situation when the system is in steady state, what does the steady state mean that if I have this spring mass system when I initially pull it, the system will shake from its initial position, it will have some transient vibrations, but those transient vibrations are going to die and once the system goes into steady state. So, it moves back and forth uniformly and then in that situation when it is in a steady state then if the excitation is harmonic then the output will also be harmonic if the excitation is exponential then the output will also be exponential if the ex excitation is of a rectangular wave then. So, will be the output. So, this is 1.

Second, rule of super position works, what does that mean that if response due to f 1 t is x 1 t and response due to f 2 t is x 2 t then. So, what is happening is that if I am exciting this force in first case just because of due to f 1 and the response is x 1 and in the other situation I am exciting it by f 2 and the response is x 2.

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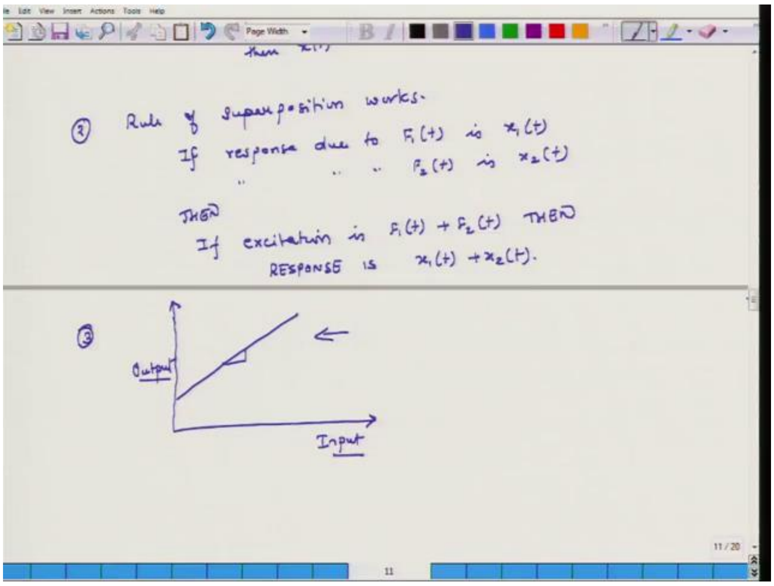

Then in the third case, if excitation is f 1 t plus f 2 t then response will be x 1 t plus x 2 t. So, this is there. And the third thing is the third thing and that is why we call these systems linear is that if I plot f or in general input and here I am plotting output then this relation is going to be a straight line. So, if I double my input output will also increase proportionally it may not necessary double there will be some slow process associated with it, but it will there may be a dc offset also, but the slope is going to be constant slope of the line which maps output at input it will be constant it does not change as I change my input that and that is why we call this systems as linear.

So, these are some of the 3 important attributes of linear systems one is that input excitation and output signal in steady state have the same form and form I use it in a very loose sense basically to say that the behavior of input and output will be similar second super position works and third thing is if I plot output versus input then it will be a straight line. This concludes our discussion for today and tomorrow we will start talking about transfer functions.

Thank you.