Fundamentals of Acoustics
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Lecture – 07
Important Mathematical Concepts-Complex Algebra

Hello. Welcome to Fundamental of Acoustics. This is the second week of this course, and today is the first day. In this entire week we will be reviewing some of the important mathematical concepts which you may have come across in your previous years, during B. Tech or B. Sc or M. Tech programs.

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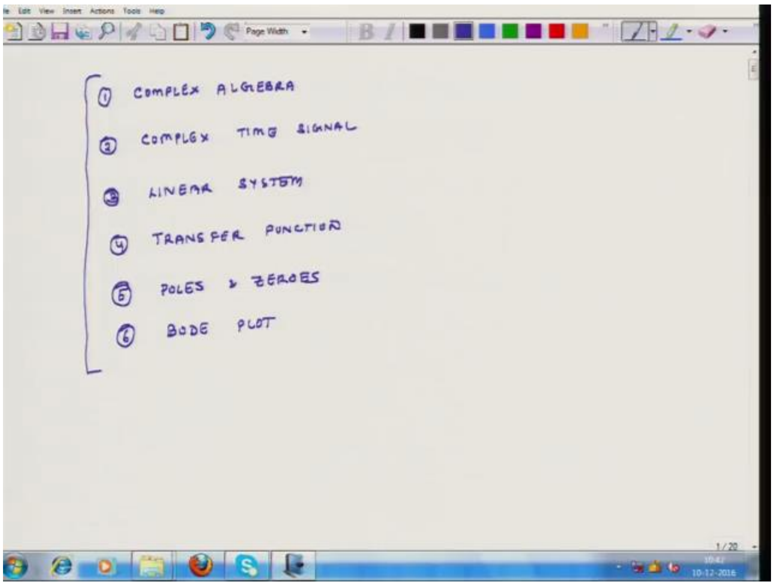

So, we will be essentially reviewing six concepts and these concepts are: complex algebra, the second concept is about complex time signals, in third concept is going to be about what is the linear system. Then we will discuss something about transfer functions, and this is the very important concept people who have an electrical engineering background may know more about it; people with mechanical background may not necessarily know much about it, but this is an important concept and we will discuss this. Then we will discuss what are poles and zeros, and finally we will discuss what is the bode plot.

So, hopefully we should be able to cover all these six concepts over this week, and then we will actually start discussing about acoustics in the next class, in the next week.

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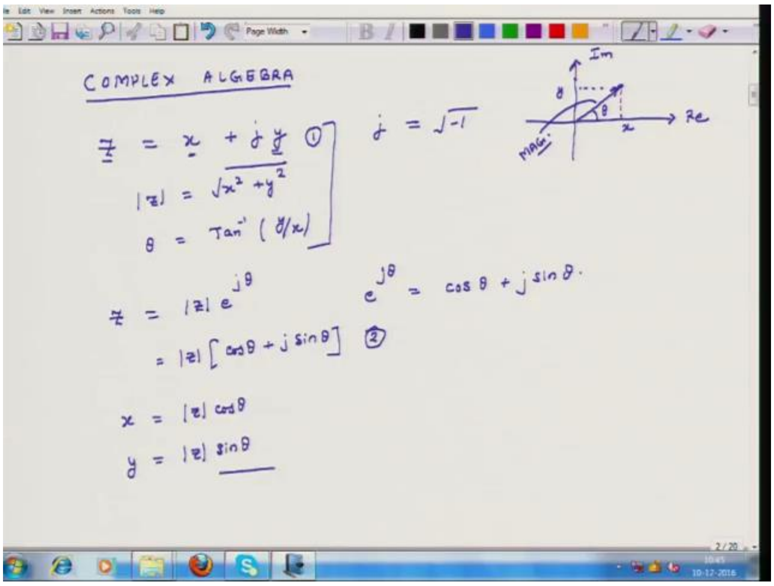

So, we will start our discussions with Complex Algebra. Let say there is a number Z it is a complex number, and it has a real part x plus and imaginary part y, and here j. So, typically we use the term i, but i will be used for other cases for current in this course so we will use term j which designates square route of negative 1. So, that is the complex number, it has a real part and it has an imaginary part. And the imaginary part is multiplied by j which is the square root of negative 1.

So, I can depict this number in a complex plane. So, complex plane has a real axis and an imaginary axis and this number can be depicted by a point or a vector. And, along the real axis its value is x and along the imaginary y x its projection is y. And this length of this vector is its magnitude. And it has an orientation related to theta, so the magnitude of Z is x square plus y square the whole thing square root. The value of theta is Tan inverse of y over x. I can also represent Z in a polar system. So, in polar coordinates Z equals its modulus Z times e to the power of j theta.

Where, e to the power of j theta using that Euler's relation is what is cosine theta plus j sin theta. So, this is equal to modulus of Z cosine theta plus j sin theta. So, let us call this equation 1, let us call this equation 2. And if we compare equations 1 and 2 then x equals modulus of Z times cosine theta and y equals modulus of Z times sin theta. So, that is the overall frame work.

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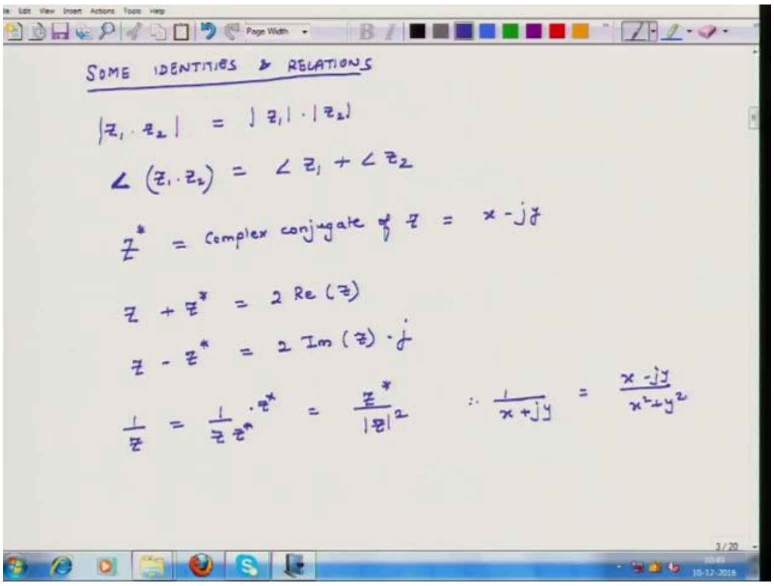

Now, we will look at some of the identities; some identities and relations. These are some of the important identities will use it very often in this course. So, if I have two complex numbers Z 1 and Z 2 and I want to find the magnitude of their product. So, I am going to multiply them and I want to find their magnitude, then this is equal to magnitude of Z 1 times magnitude of Z 2.

Similarly, if I have two complex numbers Z 1 and Z 2 then phase is designated as this symbol of Z 1 times Z 2 is nothing but phase of Z 1 plus phase of Z 2. We will also define a term Z star and this is called Complex Conjugate of Z and this is nothing but x minus jy. So, Z is x plus jy then complex conjugate is x minus jy. Another identity is that Z plus Z star is twice of real of z, and Z minus Z star is twice of imaginary portion of Z times j. So, more identities; if I have complex number Z then its inverse 1 over z, I can write it as 1 over Z z star times Z star and the denominator is nothing but the magnitude of z, so this is Z star over magnitude of Z whole square. Therefore 1 over x plus jy equals x minus jy divided by x square plus y square.

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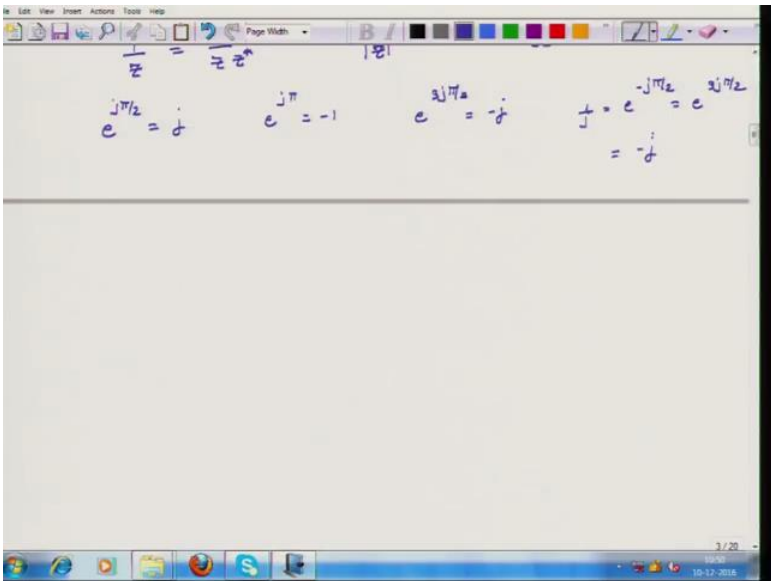

Finally, we will write e to the power of j pi over 2 equals j, e to the power of j pi equals minus 1, e to the power of 3 j pi over 2 equals minus j and 1 over j equals e to the power of minus j pi over 2 and this is same as e to the power of 3 j i over 2; and it is also equal to minus j.

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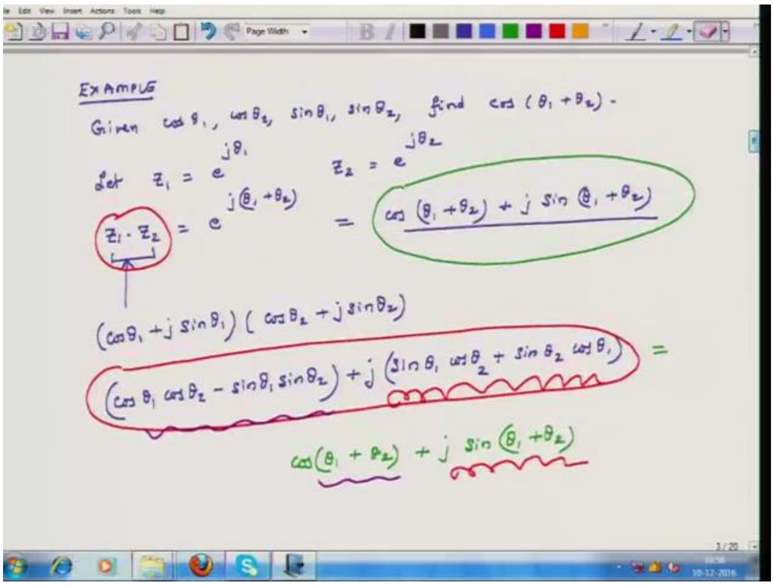

Let us do an example using complex algebra. So, the question is given cos theta 1, cos theta 2, sin theta 1, sin theta 2; find cosine of theta 1 plus theta 2. So, if I have two angles and I know their cosines and sins can I find the cosine of the some of the angles or do we calculate it.

So, we say that let Z 1 equals e to the power of j theta 1, Z 2 equals e to the power of j theta 2, then Z 1 times Z 2 is e to the power of j theta 1 plus theta 2. And if I expand it I get cosine theta 1 plus theta 2 plus j sin theta 1 plus theta 2. Now, so this is one part; now we will expand on this side. So, Z 1 is cosine theta 1 plus Z sin theta 1 and Z 2, so I have to multiply Z 1 and Z 2 so this is cosine theta 2 plus j sin theta 2. So, I multiply these two I get cosine theta 1 cosine theta 2 minus sin theta 1 sin theta 2 plus j sin theta 1 cos theta 1 plus- oh this is I am sorry. So, this is plus sin theta 2 cosine theta 1.

So, this corresponds to this side and this entire thing is equal to this term in green. So, that equals cosine theta 1 plus theta 2 plus j sin theta 1 plus theta 2. So, the left side and the right side are equal and they will be equal only if they are real parts and imaginary parts are individually equal. So, this part has to be equal this and this part has to be equal this part.

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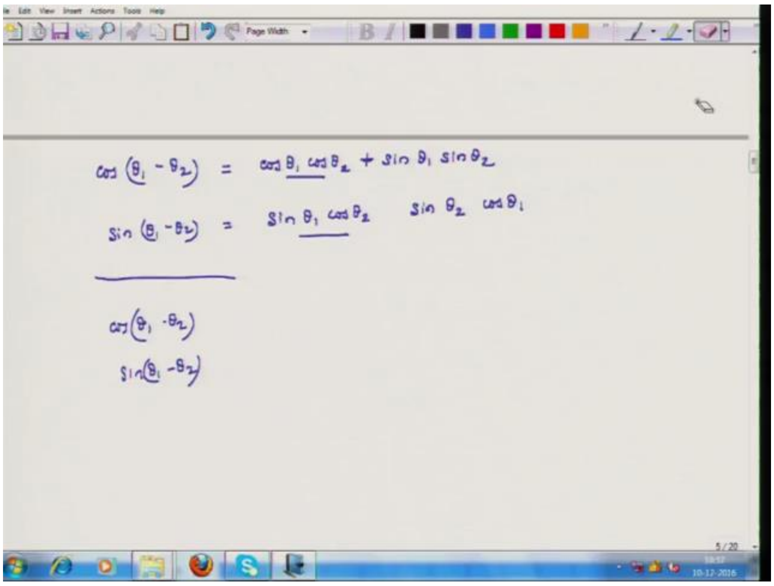

So, from this equality I can write cosine theta 1 plus theta 2 equals cosine theta 1 cosine theta 2 minus sin theta 1 sin theta 2. So, this gives me the value of cosine for some of two angles and the other equality when I compare the imaginary parts I get sin theta 1 plus theta 2 equals sin theta 1 cosine theta 2 plus sin theta 2 cosine theta 1.

So, similarly we can also develop relations for cosine theta 1 minus theta 2 and sin of theta 1 minus theta 2, you can do that. Now only thing we have to do is we can re-use these relations and there we replace theta 2 by minus theta 2. So if we do that replacement what happens, this positive sign gets replaced by negative and cosine of theta 2 and cosine of minus theta 2 is same so this does not change, but sin of theta 2 and sin of minus theta 2 are inverse of each other, so this negative sign becomes a positive; sorry this is theta 2. And similarly this does not change for this positive sign becomes a negative.

So, using this complex algebra it is a very powerful tool to develop all these trigonometric relations relatively easily, otherwise we have to make graphs and drawings and from that we have to prove cosines of and sins of two sums of two angels are corresponding to these relations. We will do another example and through that we will learnt that these are element of cyclicness in these complex functions or complex numbers.

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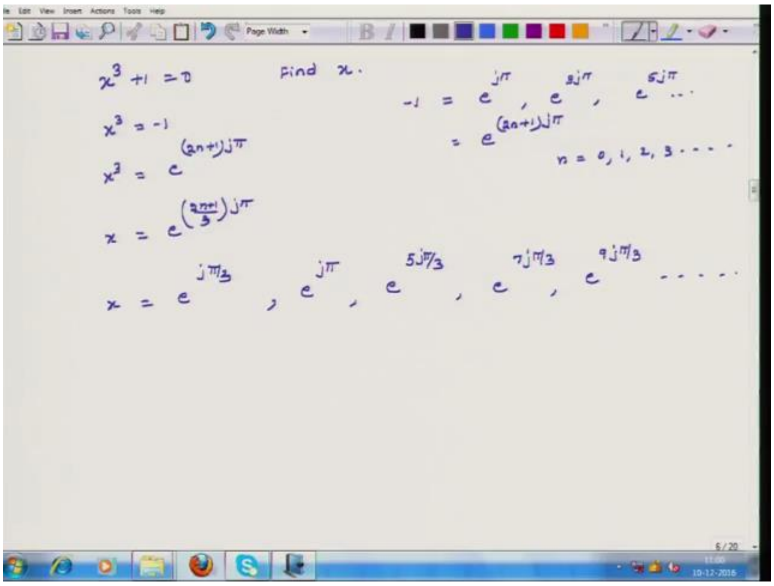

So, consider this equation x plus 1 equals 0. And our aim is to find x. So, I can write x cube equals minus 1 and we know that minus 1 equals what, it is e to the power of this is one possible value e to power of j pi or e to the power of 3 j pi or e to the power of 5 j pi. In general I can write it as e to the power of 2 n plus 1 j pi.

So, x cube is equal to e to the power of 2 n plus 1 j pi. So, x equals e to the power of 2 n plus 1 divided by 3 j pi. So, what that gives me is, so here n equals 0, 1, 2, 3, these are integer numbers; they can also be negative actually does not matter. So, x equals, so I put n is equal to 0 and this case and the first solution is e to the power of j pi over 3. When I put n is equal to 1 then the second solution is e to the power of when n is equal to 1 then 2 plus 1 divided by 3, so e to the power of j pi. When I put n is equal to 2 then I get e to the power of 5 j pi over 3. When I put n is equal to 3 then I get e to the power of 7 j pi over 3.

Next solution is 9 j pi over 3 and so on and so forth. So, I am getting an infinite number of solutions.

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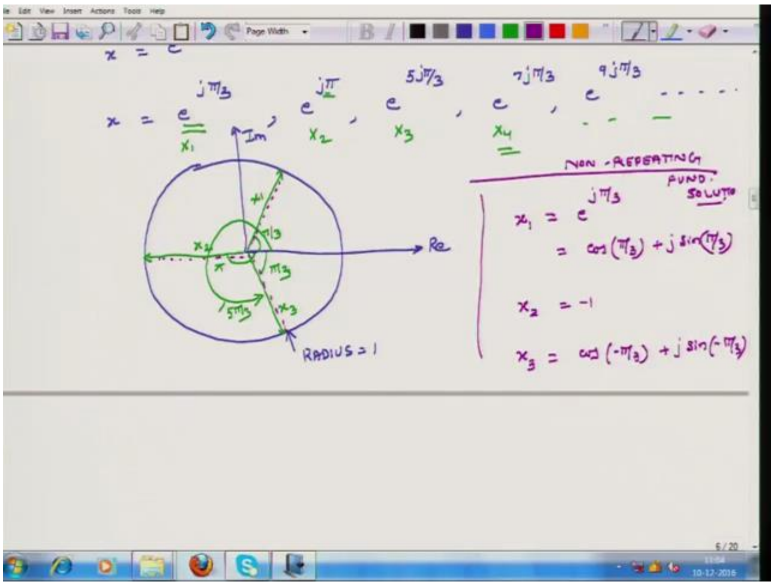

Now, let us look at what does this these solutions represent. So, consider a circle. So, let say this is the real axis and this is the imaginary axis. One thing you notice that the magnitude of all these solutions is unity. So, we will make a circle whose radius is 1, so radius is unity. The first solution is e to the power of j pi over 3 which means that the first solution is at an angle of 60 degrees or pi over 3 radius from the x axis or the real axis. So, that is my first solution. So, let us call this x 1, this is x 2, this is x 3, x 4 and so on and so forth. So, this is pi over 3 radius.

Let us look at the second solution; it is oriented at an angle of pi degrees. So, this is my second solution. So, this is x 1, this is the second solution x 2 and it is oriented at an angle of pi radius. The third solution is oriented at an angel of 5 pi over 3 radius or negative pi over 3 radius. So, it is like this. So, this is pi over 3 or if I am measuring it like this then this 5 pi over 3, so this is x 3. Let us look at the fourth solution; the fourth solution is oriented at an angle of 7 pi over 3. So when I plot it, it just overlaps this solution x 4. Solution x 5 over laps x 2, solution x 6 over laps x 3 and so on and so forth.

So, the fundamental non-repeating solutions are x 1 equals e j pi over 3 or I can say it as cosine pi over 3 plus j sin pi over 3; that is one root of this equation. Second root is x 2 equals minus 1 because e to the power of j pi is negative 1, and the third root is x 3 equals cosine of minus pi over 3 plus j sin of minus pi over 3 radius. So, these are the non-repeating fundamental solutions; all others are repeats of these solutions.

This concludes our overview of complex algebra. In the next class we will discuss a new concept known as Complex Time Signals. So, I hope you learnt something new or you are able to recap some of the information on complex algebra in today's lecture. Please review this because we will be using complex algebra very heavily in this course, and it is important that you have a good review and you warm up to important principles related to complex algebra. So, have a great day and I look forward to seeing you tomorrow.

Thank you.