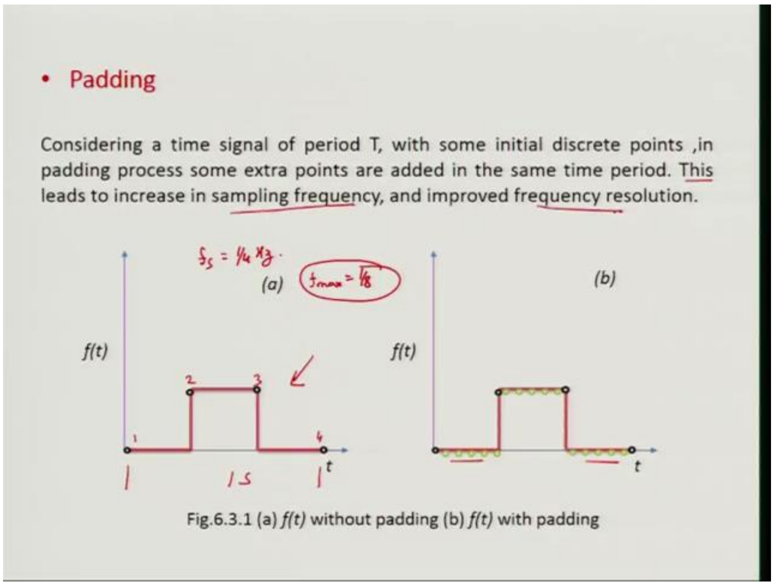
Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 60
Discrete Fourier Transform

Hello. Welcome to Fundamentals of Acoustics, today is the last day of this week. And what we plan to do is have some final important points to be discussed in context of DFT so that you can use this DFT technique effectively. So, the first term I am going to talk about or the first concept which I am going to talk about is this think called Padding.

(Refer Slide Time: 00:47)

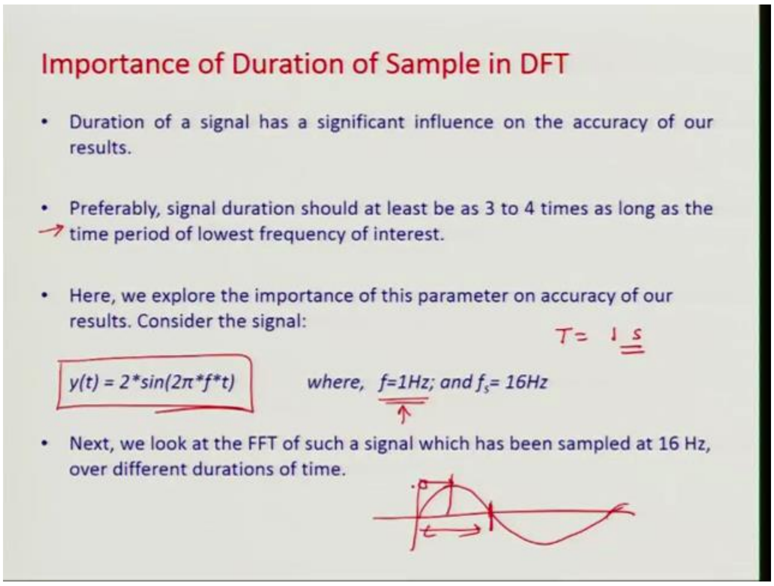


So what is padding? Let us first look at this signal. And what this signal shows is that you have square wave right or some step signal in the system. And in this case we are just measuring four points. So, we are measuring at point 1, point 2, point 3, and point 4. Now if I do its DFT or discrete Fourier transform, let say this overall time period is 1 second; so in 1 second let say I am measuring four points so our sampling frequency is 1 over 4 which means that the highest frequency which I can detect in this case will be our sampling if the sampling frequency is f s is equal to 1 over 4 hertz, then what do I get?

So, f max which I will get will be 1 over 8^{th}, so that is one thing. The other thing is that I will also loose on frequency resolution. So, what people do is that in the team same times signal they put some extra points, so these are padded points. So that is what it says. So, this type of power padding in the same time period, over the same time period if I put some pad in points then it in proves are sampling frequency and also frequency resolution. So, this is one important concept.

And then in the remaining portion of this lecture what we will discuss is; how does duration of the signal affect the goodness of our results this is one thing. So, I can record a signal for 1 second, 2 second, 3 second, and which of the results will be better how does it influence. Second thing is how does sampling frequency influence are results. And the third theme is; what is the relationship between frequency resolution and all these parameters. So, these are three important parameters.

(Refer Slide Time: 03:18)



So, importance of duration of a sample in DFT; so if the duration is longer than in general our results become more and more accurate, and we will show that. So, one important criteria is that suppose you have frequency of time period t and let say that is the lowest frequency. So for an example you may have a signal which has 100 hertz, 200 hertz, 400 hertz, 700 hertz; so 100 hertz is the lowest frequency in that signal.

Then you should have your duration at least four five times more than the time period associated with the lowest signal, because if you do not have that kind of a time period your results will be may not be accurate. So, that is what we are going to show. So, let us consider this signal y is equal to 2 sin 2 pi f t and let say that initially we just consider this case f is equal to 1 hertz. So, what is the time period when f is equal to 1 hertz? 1

second. Now our signal it will look something like this and I can either record I can acquired data up to this much time as one option, I can record data up to only one fourth of the cycle; I can make several recordings. And let see how they are results get influenced.

(Refer Slide Time: 05:17)

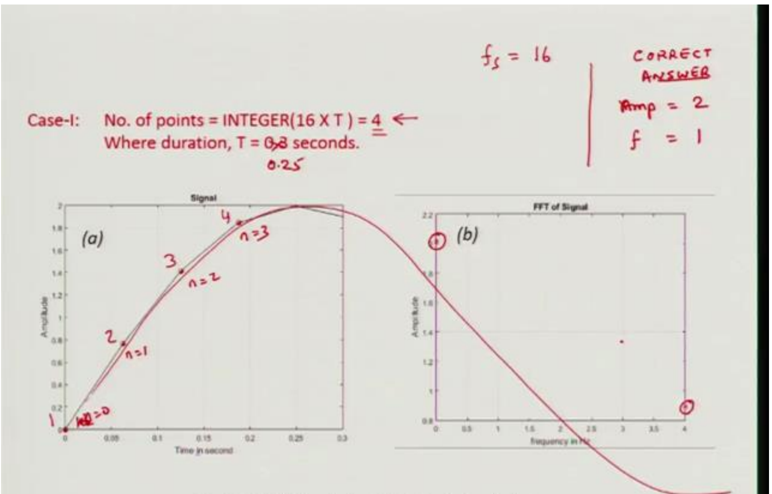


Fig.6.4.1 (a) Signal plot (b) FFT of signal plot

In the first case I am just recording four points; point 1, point 2, point 3, point 4, the actual signal is going to go like this, but I am only recording these first four points point 1, point 2, point 3, and point 4. In this case our sampling frequency is 16 we are set, our data accusation system we are set that it is sampling frequency so here we have written f s is 16 hertz. So, I am going to sample it actually not for 0.3 seconds to be more accurate it is 0.25 seconds. So, I get four points; so I get point number 1, point number 2, point number 3, point number 4.

This corresponds to k is equal to, excuse me not. So, this is n is equal to 0, n is equal to 1, n is equal to 2, n is equal to 3, and I will get four different frequency components when I do the FFT. Now theoretically the correct answer should be what is the correct answer? That the amplitude is 2 and frequency is 1 hertz; so f is 1 hertz. Now this is the FFT plot of the signal. So, let see what are a FFT shows is that we have one peak, one point at 0 hertz and its amplitude is 2. So, this is not right result. And I get another point at 4 hertz and the amplitude is about 0.9 or something like that.

So, we have just taken one forth of a cycle and we are getting very bad results.

(Refer Slide Time: 07:39)

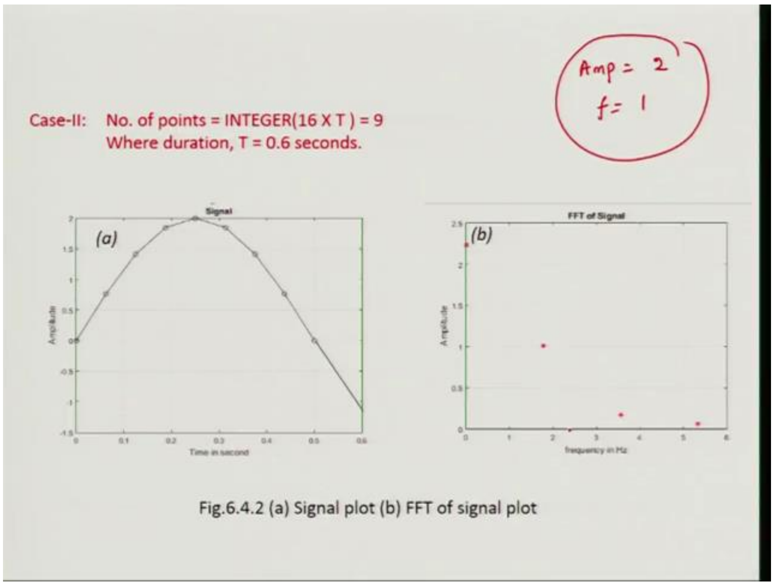


Fig.6.4.2 (a) Signal plot (b) FFT of signal plot

Now in the second case, I am taking larger number points I am taking 9 points, I sampling frequency is still 16 hertz; actual answer is amp is equal to 2 f is equal to 1. So, ideally the correct result should have only one point, now let see. I still get a peak at 2, I get a peak at 1.5, I get peak at 3.5. So, I get apset results, I am not getting any peak at 2.

(Refer Slide Time: 08:20)

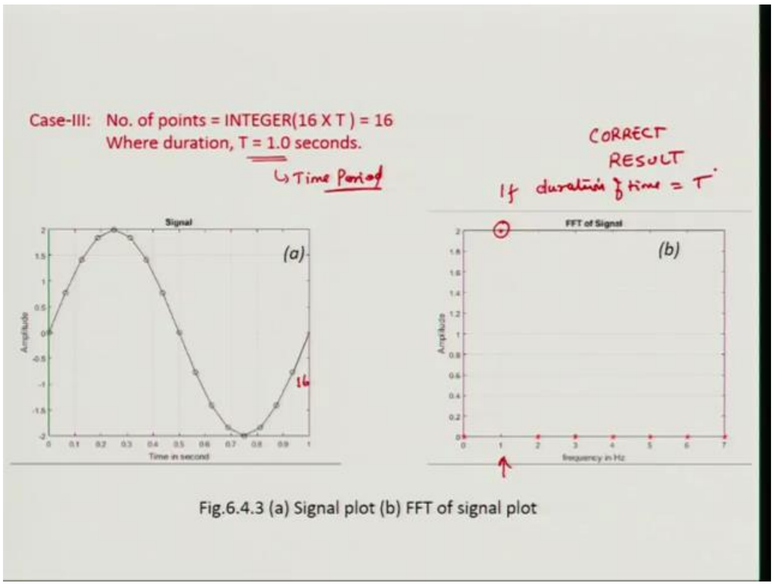


Fig.6.4.3 (a) Signal plot (b) FFT of signal plot

Now, consider this here my duration is t is equal to 1 second and this is same as equal to time period. So, I have all the 16 points which help me complete the cycle. The 16 point is this one. Now when I do; so if the cycle is perfectly complete when I do the FFT I get just 1 peak at f is equal to 1 hertz and all other points in my FFT are 0. So, I get a correct result. So, when do I get correct result, if duration of time equals time period.

(Refer Slide Time: 09:18)

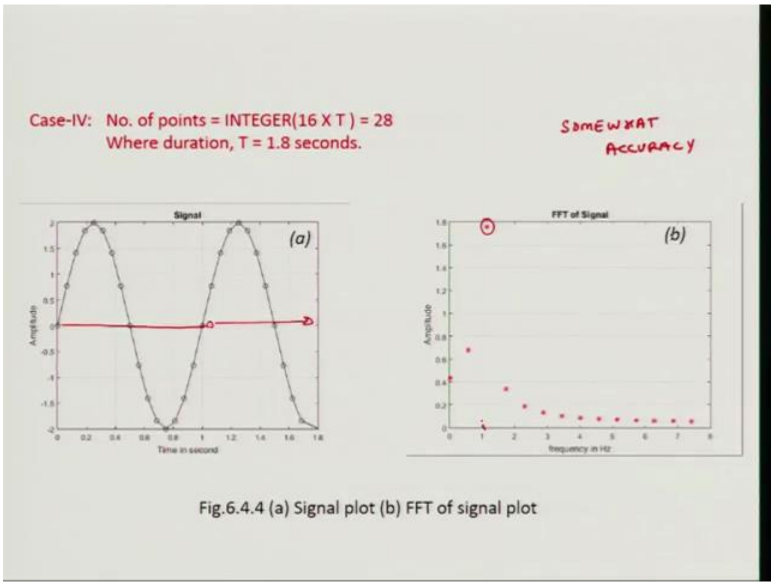


Fig.6.4.4 (a) Signal plot (b) FFT of signal plot

Now, in this case this is one complete cycle and this is maybe one and half cycles. What do I get? I get a peak at 1.8 at 1 hertz, but I get some other points also. So, this is somewhat accurate, it depends on what is our definition for accuracy. But it is certainly better than this result or this result; it is certainly better then these results.

So, the point is that if I have a large number of complete cycles I start improving my accuracy, this is another case. Here I have two complete cycles I once again get an accurate result.

(Refer Slide Time: 10:20)

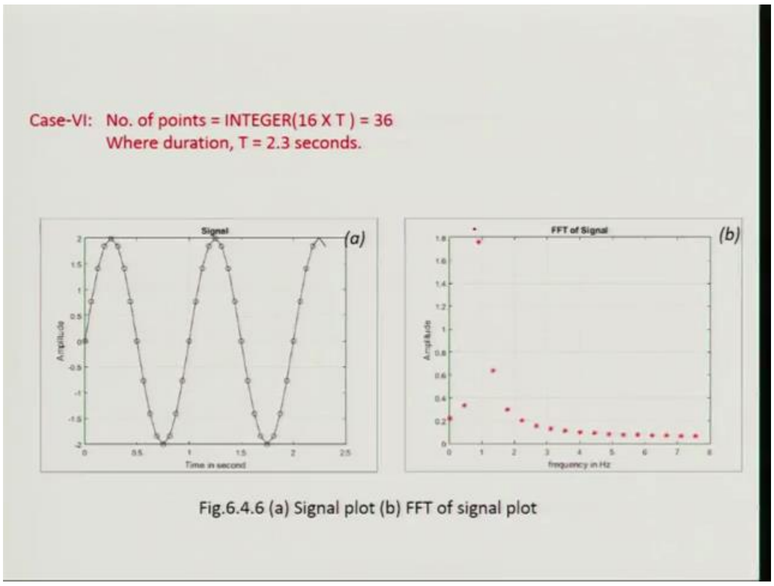


Fig.6.4.6 (a) Signal plot (b) FFT of signal plot

And what I will do is; so this is another in case. So, here I have 36 points I have 2.3 seconds of data accusation, so that is 2.3 cycles. And I see a peak at 1.8 and several other points, but this is the pre dominant peak.

So, my accuracy is improving as I increase the duration.

(Refer Slide Time: 10:45)

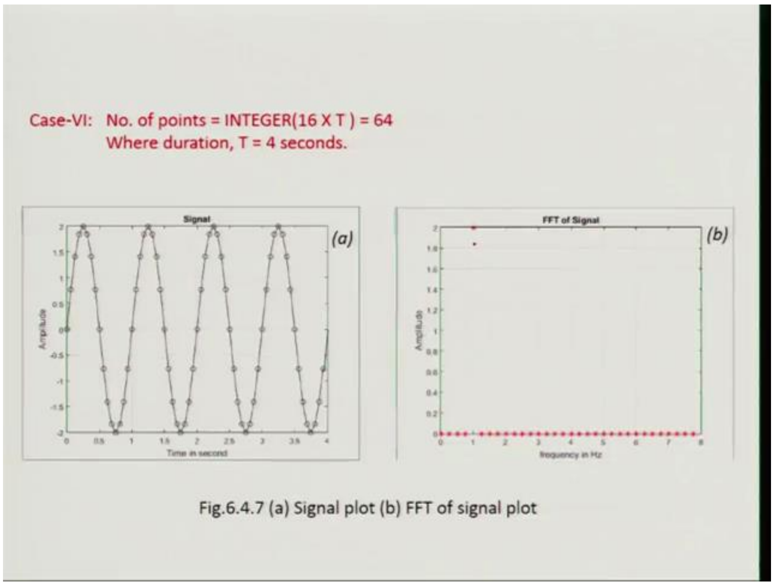
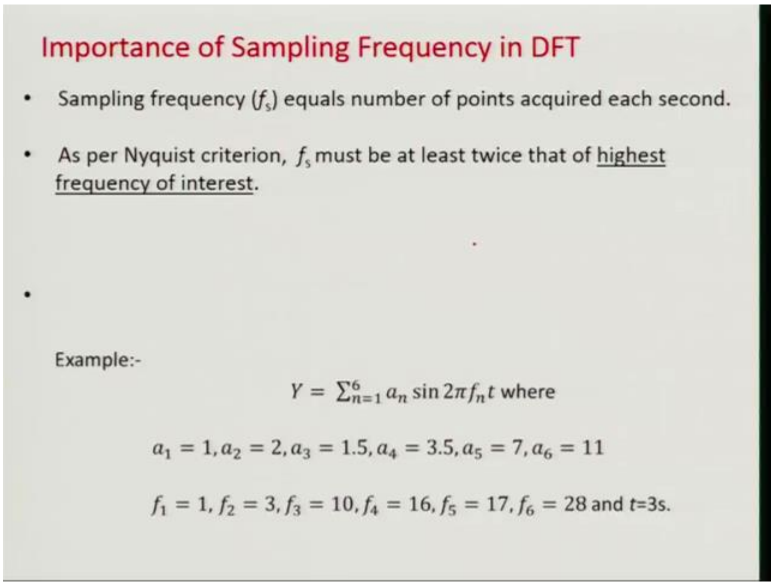


Fig.6.4.7 (a) Signal plot (b) FFT of signal plot

At t is equal to 4, again it is an integral number of cycles so I get an accurate result. So, the main theme of this thing is that if we already know what is the frequency then we can set at a duration time same as an integer number of cycles. But, usually we do not know otherwise we would why would data we do the data acquisition system. So, the point is that we have to make a guess that, the lowest frequency in my signal may be 5 hertz, its time period is 0.2 seconds, then I should at least acquire my data for four five times that time period then I have a good chance that my FFT will be accurate. So, that is the first thing.

(Refer Slide Time: 11:53)



The second important point to be understood is importance of sampling frequency.

(Refer Slide Time: 11:39)

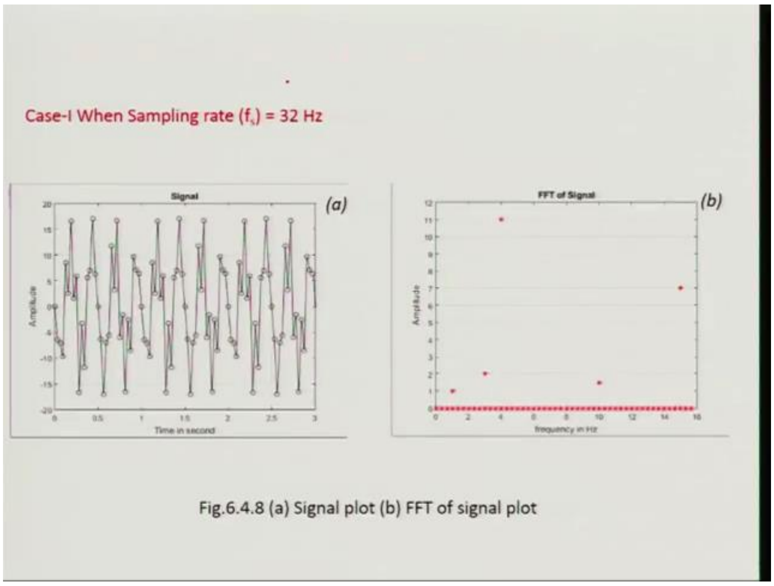


Fig.6.4.8 (a) Signal plot (b) FFT of signal plot

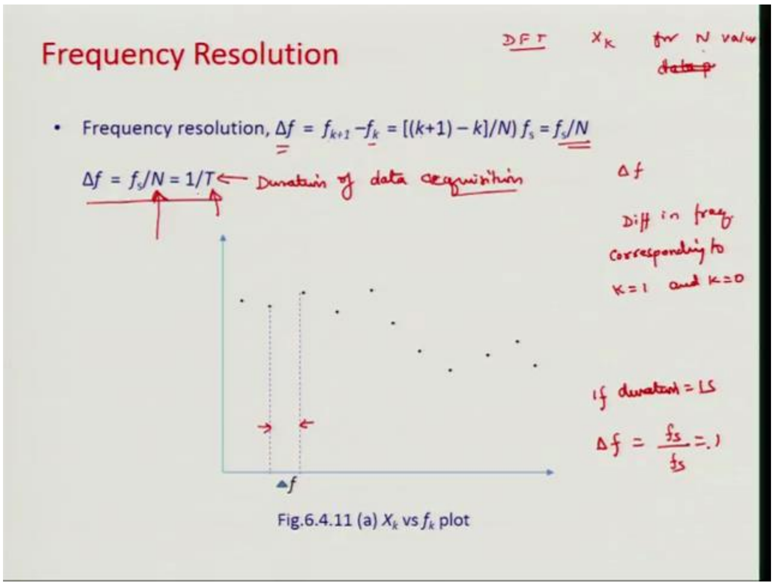
So, what is sampling frequency? It equals number of points acquired each second as per Nyquist criteria, this should be at least two times that of frequency of interest. Now look at this; so I have constructed a signal which has six frequency components; f 1, f 2, f 3, f 4, f 5, f 6. F 1 is 1 hertz and its amplitude is 1, f 2 is 3 hertz its amplitude is 3, and it keeps on going up to 28 hertz; f 6 is 28 hertz and its amplitude is 11 seconds. And I am generating data for 3 seconds, so that is fairly large which is good.

Now, f 6 is equal to 28 hertz, so my sampling frequency should be more then how much it should be to get good resolution on frequency it should be more then or at least equal to 56 hertz. If I do the sampling at lesser frequency I may get some what bad results. So, let us look at this. So, this is sampling at 32 hertz. So, I have sampled my data at 32 hertz; so the best possible. So, I will get accurate results up to what frequency? 16 hertz. So, actually I do not get any information about 16 hertz actually, I will not get any information about 16 hertz; so up to 16 hertz I get decent information. So, if I compute.

So, let us look at our numbers. So, I am sampling it for 3 seconds, what is the lowest frequency of interest 1 hertz and sampling it for 3 seconds so I gone up to at least 3 times larger is better. So, this is the FFT if I do it at 32 hertz. So what do I get? At 1 hertz the number should be 1 which is correct, the next frequency component is 3 hertz which is here and its amplitude should be 2 so it is also correct. The next frequency component is 10 hertz, but I am (Refer Time: 14:19) 10 hertz and its amplitude is 1.5, which is correct. The next frequency component is 16 hertz which is here and its amplitude is; excuse me 3.

So, I am getting some erroneous results here, I am getting these two erroneous results. But if I sample it at 64 hertz then I get exactly what I want. So this point is correct, this point is correct, this point is correct, all these points are correct. So, again sampling frequency will help us capture high and frequency component more accurately, and this is data at 128 hertz. So, again results are identical

(Refer Slide Time: 15:15)



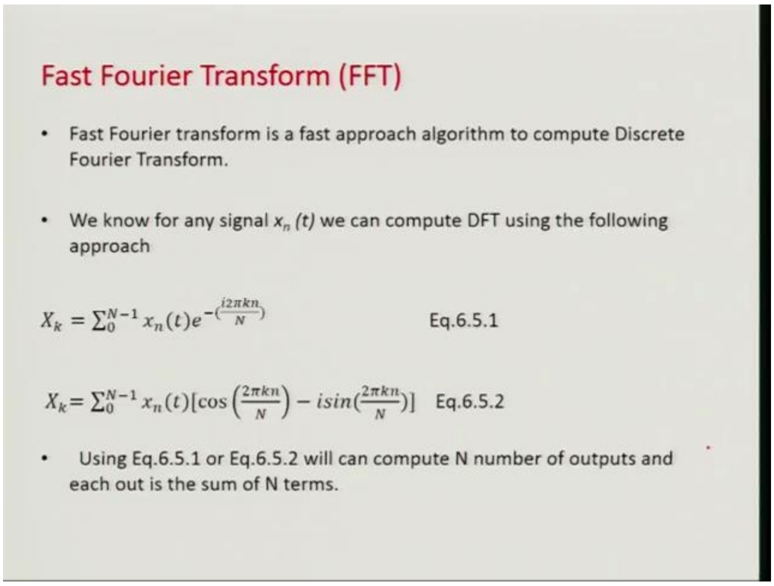
So, the last thing is about frequency resolution. So when we do a DFT, we get values of x k for N data points; I am not for values of x k for n values right so we get x 0, x 1, x 2, till x to the power x n minus 1. So the question is, what is the delta f? That is the difference in frequency corresponding to k equals 1 and k equals 0. So, what is that? So that is delta f. So what is delta f? It is the difference between f k plus 1 minus f k. What is f k plus 1? It is k plus 1; so we have explained this, this is the actual frequency. Actual frequency is k divided by n times f s, so it is k plus 1 minus k divided by n times f s and that is equal to f s over n. So, delta f equals f s divided by n equals 1 by time period.

So what does that mean? If I increase my n; I am sorry this t is not time period it is the duration of data acquisition. So, what does this mean? What it means is that if I increase my time period or the number of points then this distance delta f it will become narrow. So, I will get more frequency. So, there may be some frequency which I may miss out if n is small, but if I want to capture that particular frequency I have to narrow this delta f and I can narrow it by doing what, by increasing the number of points; which essentially means that the duration of our data acquisition system has to be long.

So if duration is 1 second exactly then what will be the delta f? So, if duration equals 1 second then delta f frequency resolution will be sampling frequency f s, and how many number of points I will get f s, so that is one. So, if I acquire data for 1 second long then I will get frequency resolution of 1 hertz. So, I will get one data point in 0 hertz 1, hertz 2, hertz 3, hertz 4, hertz and so on and so forth. And what is the maximum frequency I will move up in this situation? It will be f s divided by 2. If I increase my duration to 2 seconds then I will go up in steps of half hertz, 0 hertz, half hertz, 1 hertz, 1.5, 2 and so on. And the maximum frequency which I will go up will be, it will still be f s divided by 2.

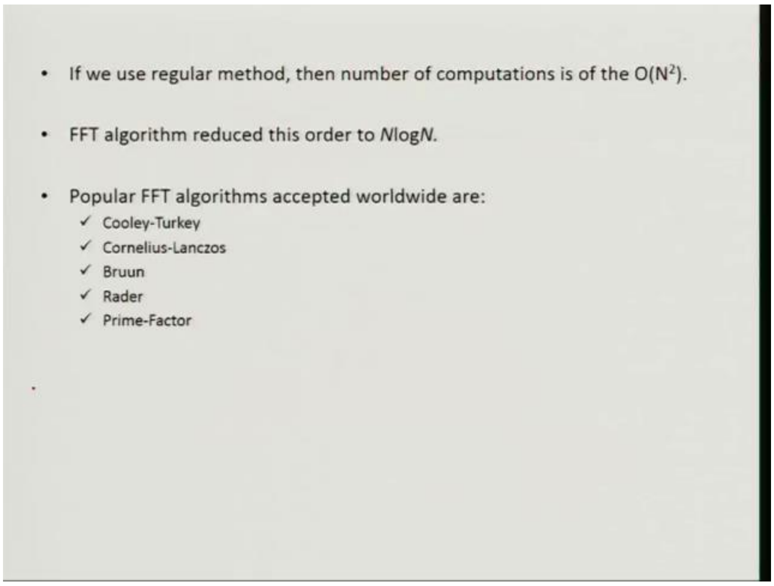
So, I cannot exceed the maximum frequency beyond f s over 2 that is the mathematical limit, but I can go in finer step if I increase my time duration. So, this completes our discussion on DFT.

(Refer Slide Time: 19:21)

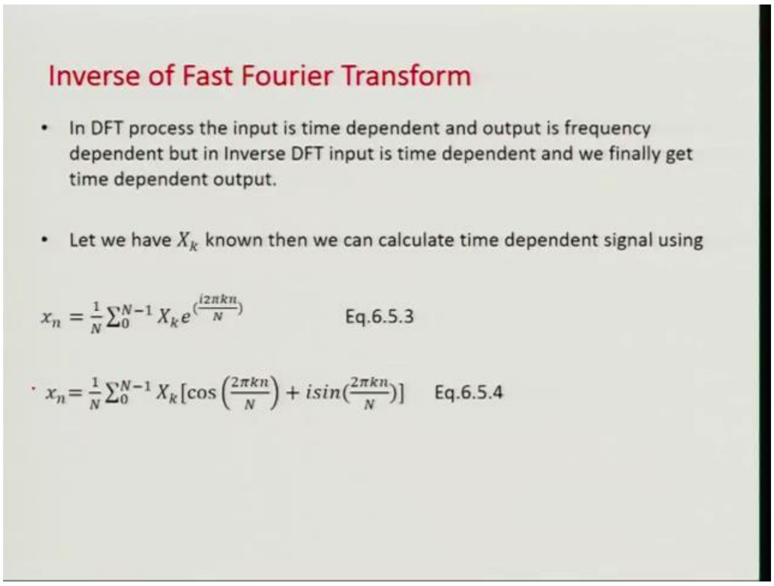


And then there is one last point that just as we had forward and inverse transforms for continuous functions, this is the expression which will help us convert time domain data discrete data; this is the expression which will help us convert time domain discrete data into frequency domain.

(Refer Slide Time: 19:54)



(Refer Slide Time: 19:56)



And if I want to convert frequency domain data discretize do data in to time domain then I use this relation. So, this is the inverse of fast Fourier transform.

So, with this we conclude our discussion on Fourier transforms and DFT. And next week onwards we will go in to another application area which will help you use whatever knowledge you gaining in this course, your actual field problems

Thank you and have a great day. Bye.