Fundamentals of Acoustics
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Lecture – 06
Adding Decibels

Hello, welcome to Fundamentals of Acoustics. Today is the last day of this particular week of this course and what we will do today is we will do some mathematics on decibels. Essentially what we will learn is how do you add up decibels. So, suppose there are 2 sound sources and one sound source is producing one particular sound pressure level and then there is another sound pressure level attributal through the second source then what would you expect the overall noise to be? So, that is what we are going to talk about today.

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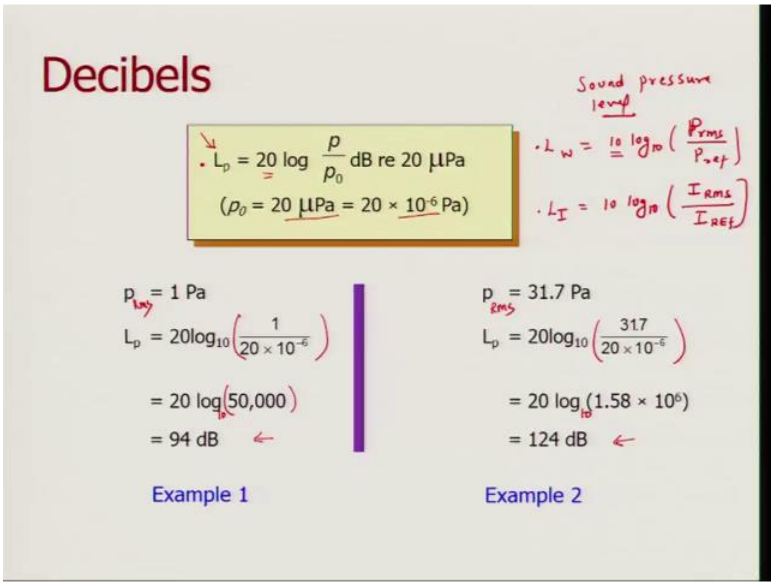

The theme is how do you go around adding decibels? We have defined decibels in 3 ways in context of sound power level, sound intensity level and sound pressure level.

The definition what you are seeing here is the definition what you see here is for L p that is sound pressure level. For sound power level which is L w, the definition is 10 log and base 10 and P rms. So, it is capital P divided by P reference. Similarly sound intensity level is 10 log 10 I rms that is the rms value of intensity divided by I ref. So, that is my sound power level sound pressure intensity level, but this is sound pressure level and in case of pressure level, instead of the factor 10 I ref factor 20 because in the original definition, it is 10 log of p square divided by reference pressure. So, this is one important reference, we have to remember and if the sound is propagating in air, in the reference pressure is taken as 20 micro pascals which is this number. For starters, what we will do is we will calculate decibel levels in 2 cases consider that the rms value of pressure is 1 pascal then the sound pressure level is 20 log 10 of 1 divided by 20 micro pascals. So, that is essentially log of 50000 times 20 and that works out to be about 94 decibels.

Another example let us say your rms value of pressure. So, this is also rms, rms value of pressure is 31.47 pascals then L p is 20 log 10 of 31.7 divided by 20 micro pascals. So, that comes out to be 124 decibels. Now, the question is if suppose both these sound sources exist in a room and we want to use micro phone and want to sense what is the total power level, then how do we go around calculating it?

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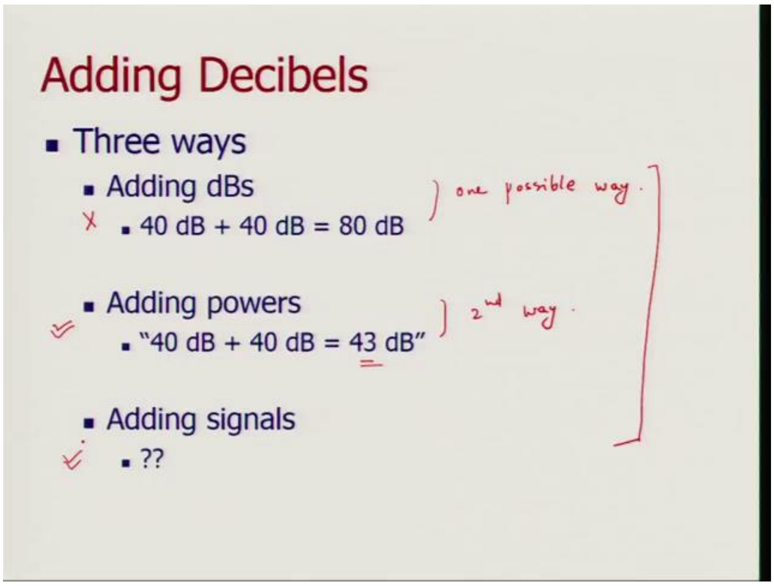

Now, they could be 3 or may be more than 3 possible ways of adding up decibels. First one is that you add up suppose, there are 2 sources; each source is contributing 40 decibels. So, you add up the decibels 40 days d B plus 40 d B is 80 decibels, this is one possible way. The second possible way is that you add up powers and we will see the details of this, but if you add up the power associated with each source then it comes out as 43 decibels.

The third possible way is that you add up the amplitude of the signal, you add up the rms values of the pressure and then from that you compute what is the total power level. So, the question is that which of these 3 ways is the correct way, do we add up decibels, do we add up power, do we add up signals? We know for certain that this is certainly not the right way. So, in some situations this method works out as the correct method in other situations this is the appropriate method to be used.

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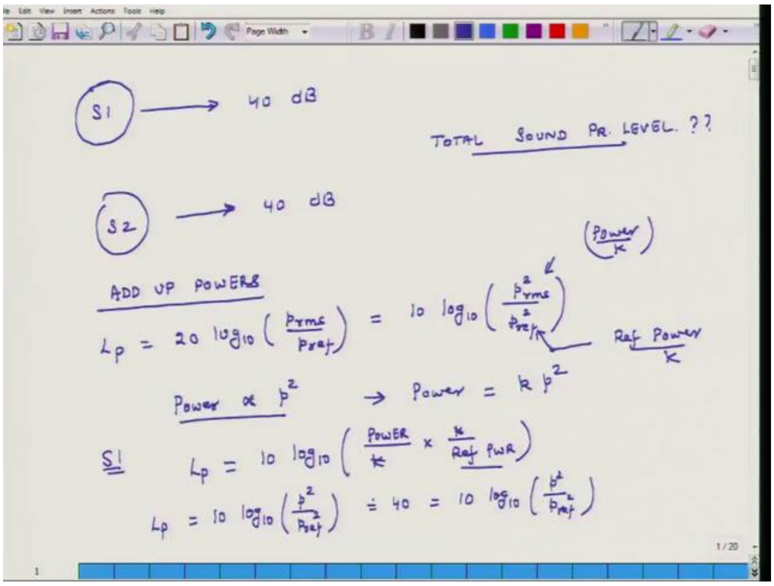

We will do some math. So, let us consider 2 sources, source 1, source 2 and this is producing let us say 40 decibels, this is also producing 40 decibels. Then what is the total sound pressure level that is the question. So, we will first use the first approach that we use powers. So, we add up powers we add up powers. So, know that L p equals 20 log of 10 p rms by p ref and this is equal to 10 log of 10 p square p ref square, now power is proportional to square of pressure. Why do I say that consider kinetic energy? It is mass times velocities square by 2. So, energy in this case, kinetic energy is proportional to p square of velocity then let us consider a spring and we compress it, the energy contained in the spring is stiffness k times deflection square k x square over 2.

There also energy is proportional to the deflection, the square of the deflection which is x. So, in a lot of these, not lot, all these cases whatever is our fundamental entity, the square of that is proportional to the energy or power. So, for that reason I am saying that power is proportional to a square of pressure or I can say that power is equal to some constant, some constant times p square. So, we will do the computation for source 1. So, what we will say is this p rms square is what? It is equal to power by k. So, I can say that for source 1 L p equals 10 log 10 divided by power of source 1 by k and then this I can also call it as reference power divided by k.

This is equal to k by reference power. So, these k guys go away and this is what I have left. So, the ratio of power and reference power is same as ratio of p square rms and p square reference that is what it means. So, this understanding is important. So, essentially I have to add up what; that means, is I have to add up the square of rms values in both the cases. So, that is what it means. So, now, we will go back then L p equals 10 log 10 p square by p ref square and I know that L p is equal to 40 decibels. So, this equals 10 log 10 p square by p ref square are equivalently 10 to the power of 4 equals p square by p ref square.

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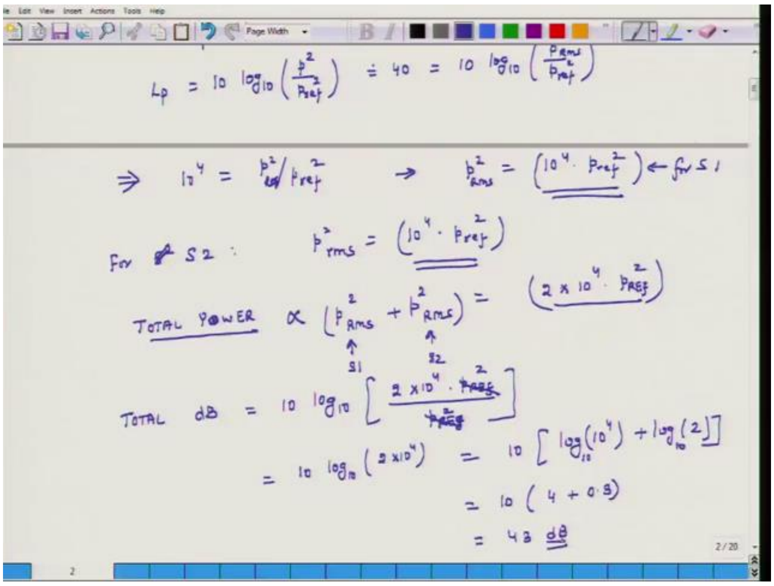

This means, p square equals 10 to the power of 4 times p ref square, this is rms value just to be sure, it should not be confused. All these are rms values which means that the power of source one is proportional to 10 to the power of 4 times reference pressure is square because power is proportional to is square of pressure. So, this is for source 1 for s 1 similarly for S 2, S 2 - p square rms is also 10 to the power of 4 times p ref square why is at the same value because initially we had defined the source 1 and source 2 have both are producing 40 decibels.

Source 1, the square of pressure is 10 to the power 4 times p ref square and also the same thing is 2 for 2 for the second source which now, power is proportional to this term for second source and power is proportional to this term for the first source. So, total power why I am I adding up total powers. So, I have 2 sources one is emitting p 1 watts of power another one is emitting p 2 watts of power total power in the room is p 1 plus p 2. So, total power is what it will be proportional to p total p rms square plus p. So, this is from source 1 this is from source 2. So, that is what it is, 2 times 10 to the power of 4 times p ref square.

This is, when I add up total power. So, this total power is proportional to this term. So, essentially when I have to add up power I have to square the rms pressure and add them up. So, if that is the case, then my total d B is equal to 10 log 10 and then I put this number in the system 2 times 10 to the power of 4 times p ref square divided by p ref square. So, these terms cancel out. So, what I am left with this 10 log 10 of 2 into 10 to the power of 4 and that is equal to 10 log of 10 to the power of 4 plus log of 2. So, this comes to be, all this is all base 10. So, this is 10 times 4 plus 0.3 log of 2 is about 0.3. So, that equals 43 decibels.

If there are 2 sources and both the sources are emitting same amount of power then the total number of decibels in the room, it goes up by 3 decibels, it goes up by 3 decibels, if there are 2 sources, if there are 4 sources then power total power; amount of power in the room goes up by a factor of 4 and for each doubling the amount of decibels increment is 3. If it is 4 then it is 6 decibels.

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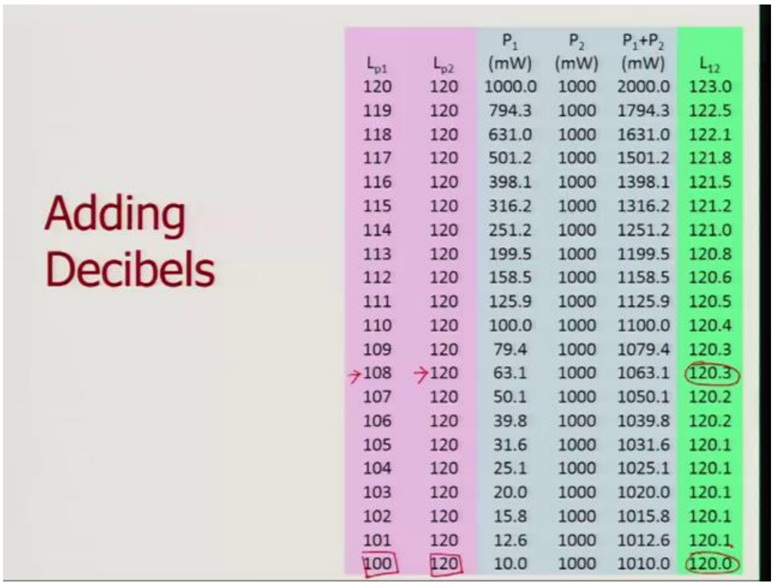

L_{p1}	L_{p2}	P_1 (mW)	P_2 (mW)	P_1+P_2 (mW)	L_{12}
120	120	1000.0	1000	2000.0	123.0
119	120	794.3	1000	1794.3	122.5
118	120	631.0	1000	1631.0	122.1
117	120	501.2	1000	1501.2	121.8
116	120	398.1	1000	1398.1	121.5
115	120	316.2	1000	1316.2	121.2
114	120	251.2	1000	1251.2	121.0
113	120	199.5	1000	1199.5	120.8
112	120	158.5	1000	1158.5	120.6
111	120	125.9	1000	1125.9	120.5
110	120	100.0	1000	1100.0	120.4
109	120	79.4	1000	1079.4	120.3
108	120	63.1	1000	1063.1	120.3
107	120	50.1	1000	1050.1	120.2
106	120	39.8	1000	1039.8	120.2
105	120	31.6	1000	1031.6	120.1
104	120	25.1	1000	1025.1	120.1
103	120	20.0	1000	1020.0	120.1
102	120	15.8	1000	1015.8	120.1
101	120	12.6	1000	1012.6	120.1
100	120	10.0	1000	1010.0	120.0

So, that is what this table shows. So, that is what this table shows. So, what you have is 2 individual sources, source 1, source 2 and here we are actually computing pressures. So, the first source is 120 decibels second source is 120 decibels and when I add them up; I end up with 3 decibels extra in presence of 1 and 2. So it is 123 decibels.

And what this table is for different combinations. So, 119 decibels, second source is 120 decibels, the total amount of power is 122.5 and so on and so forth. So, I think now you understand the mathematics of this and for any row you should be able to calculate the total number of decibels present in the room, when 2 individual sources are correct are present for instance, if you have source 1 generating 180 decibels another source is generating 120 decibels, then the total number of decibels in the room is not 180 plus 120, but it comes to be 120.3.

One observation you will see here is that if one source is for instance look at this last row, first source is 100 decibels, second source is 120 decibels. So once source is significantly weaker than the other source then the total number of decibels present in the room does not change significantly and it does not reflect in the first place of decibels. So, it still remains 120 decibels. So, this is important to understand that if there is a very weak source and a strong source then the overall sound level more or less remains unchanged because of the presence of the weak source, the presence of both the sources will be felt when both sources are somewhat equally strong, that is what these determine.

Now I have mentioned that we could add in this possible way and this we had said that this is wrong, this is one way we have added.

Where we are adding up powers and by adding a power, we add up 40 decibels and 40 decibels and we get 43 decibels, here we are adding a power. There is another scenario where we do not necessarily have to add power, but rather we add up signals. So, that is what I wanted to talk about.

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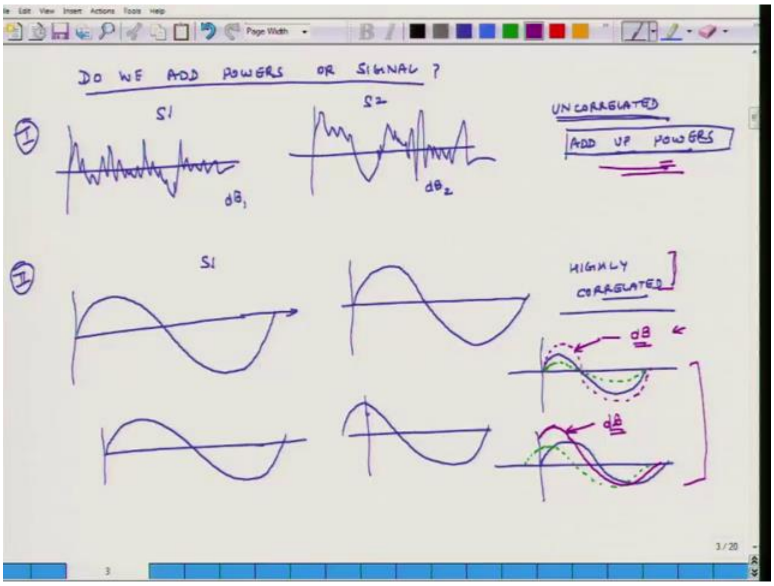

The question is do we add powers or signal. So, let us look at 2 scenarios. So, let say that the nature of sound for source 1 is this and the nature of sound for source 2 is something like this, in the other scenario the first source is a nice sine wave. So, this is S 1 and the second source is also a nice sine wave same frequency and there also they also happen to be a phase. These signals, this is scenario 1 and this is scenario 2.

Signals of this nature on scenario 1, they are uncorrelated signals and in the second situation there are highly correlated, what does correlation mean? What correlation means is that when signal 1 goes up and goes down, there is no correlation between the movement of signal 1 and signal 2. So, when 1 signal is going up, you cannot say for sure whether the signal 2 is going to go down or up or whatever, there is no relationship, it means signal 1 and signal 2 and example of 2 uncorrelated signals may be there are 2 persons in the room and both of them are talking.

There is no guarantee that 1 person when he starts, he starts with a particular frequency and the frequency is going up, the same is happening to other person's speech, there is no relationship or correlation between 1 person's speech and another person's speech. So, these types are signals are uncorrelated signals; the second scenario would be that there are 2 sources and when one source goes up the other source also goes up and so on and so forth. So, there and also the frequencies are similar very close. So, these are highly correlated signals and example could be that there could be 2 motors in the room running at the same r p m and they are producing sound because and their producing the sound and it is running at the same r p m. The correlation between one amplitude and another motor's amplitude is very tight. So, these are correlated; highly correlated signals.

So, in the first scenario, we cannot make any judgments about the amplitude. So, in this case, we add up powers. So, if this is dB 1 and this is dB 2 then the total amount of dB can be calculated by adding up powers and we add up powers and we get the answer and that answer is correct in a statistical way. It is not correct from one point of time to other point of time, but if you take a long time average of the overall power level, you will get a fairly good estimate of the total some power level.

In this case, what we can do is we do not have to necessarily add up powers, but rather we add up the signal. So, this is let us say first signal and the other signal may be like this. So, then the total signal I will just add them up and from this signal, I calculate decibels this answer will be different than this answer, but in case of correlated signal, this gives me a much better estimate of the sound power level. Then I use the term correlated signal. It does not mean that both these waves; they have to be in phase, they do not have to be necessarily in phase for instance, one signal could be like this and the other signal could be like this.

In this case also, there is correlation, I know that when one signal is rising then the other signal may be falling; I know the relationship between the rise and fall of both the signals. So, here also what I can do is I can add up both the signals. So, this is one signal and the other signal is like this and then I add up both the signals. I do not know may be we will get something like this that. So, the point is that I can calculate decibels from this add up added up signal. So, here I am adding up pressures when I am having correlation, but if I do not have correlations. So, I do not know how pressures can be added up, but what is an aggregate quantity? So, I add up powers.

In uncorrelated signals, we calculate decibels by using powers, in correlated signals, we calculate total decibel by using addition of pressures. So, that is what I wanted to cover in this particular lecture and this concludes our discussion for this week. We will start the next week by continuing our discussion on some of the basic concepts and in the specifically in the next week, we will start covering 3 or 4 broad areas. One is complex algebra, the second thing we will try to learn or relearn in case, we have already learnt it in your earlier classes is about bode plots and the third concept we will learn about is, what is a transfer function? So, with that we conclude the discussion for this week. Have a great day and I look forward to seeing you next Monday.

Thank you, bye.