Fundamentals of Acoustics
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Lecture – 59
Discrete Fourier Transform

Hello, welcome to Fundamentals of Acoustics. Today is the 5th day of the 10th week of this course, yesterday we had started our discussion on a discrete fourier transform and we had shared with you a relation which will help you transform the discrete data in time domain into frequency domain and the expression for that was showed here.

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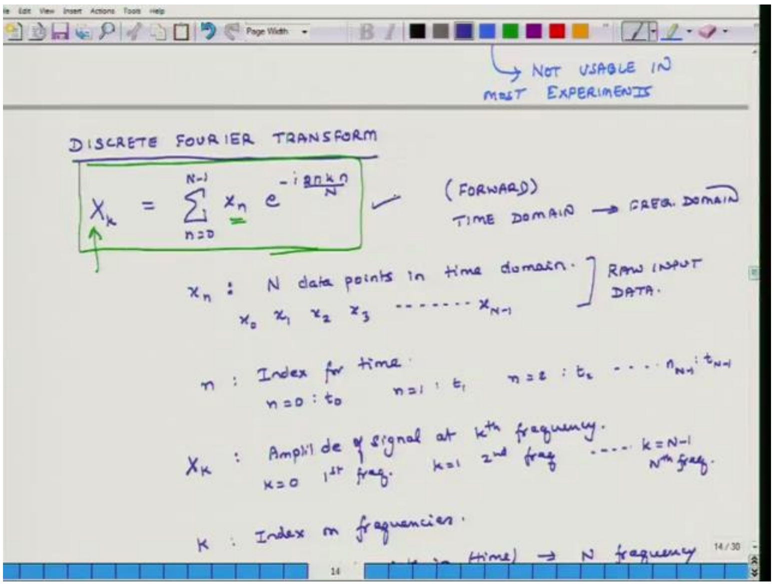

So, that is the expression and we had explained the importance of parameters N, lower case x k and upper case K.

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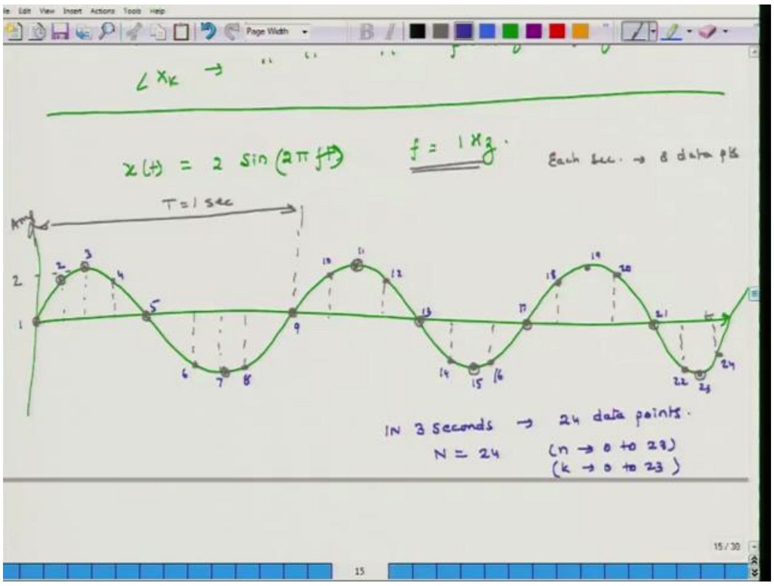

And now what we are going to do is we have a signal and let say that the signal is X equals 2 times sin of 2 pi f t and let us set f has 1 hertz.

So, the actual signal is shown in green continuous line, but modern data individual systems do not acquire continuous data because they are digital. So, instead of continuous signal, what we are data acquisition system is just getting points and let say we have recording the first 24 points that is for a period of 3 seconds, so I have these 24 points which I have marked here. Now what I want to do is, I want to use this information for 24 points and from that information I want to figure out the Fourier components of this signal that is it is representation in the frequency domain.

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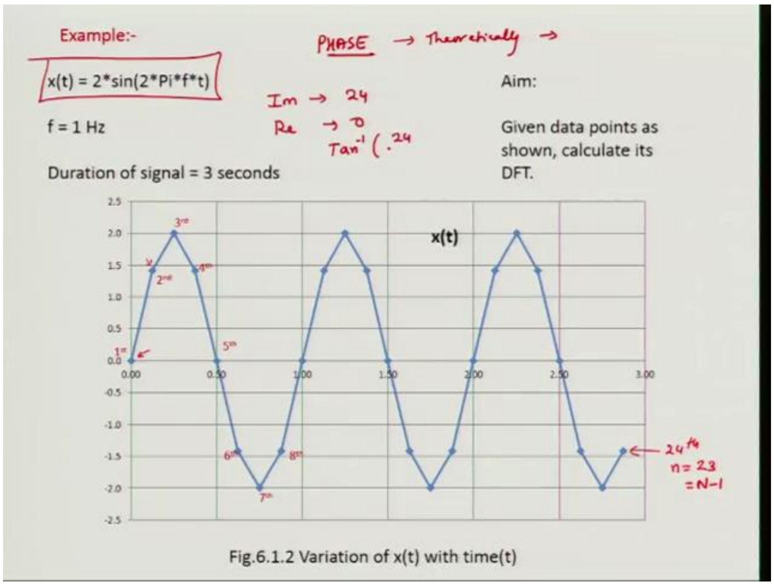

So, with that let us look at some of the slides. So, these are the points, this is point number 1 which corresponds to n is equal to 0. So, this is point number 1 corresponding to n is equal to 0, this point corresponds to n is equal to 2 and so on and so forth and this is the 24th point and the corresponding value of n is 23, which is equal to capital N minus 1. So, this is the actual data which we are getting and now from these data we have to figure out the frequency domain representation of the signal. So, what we do?

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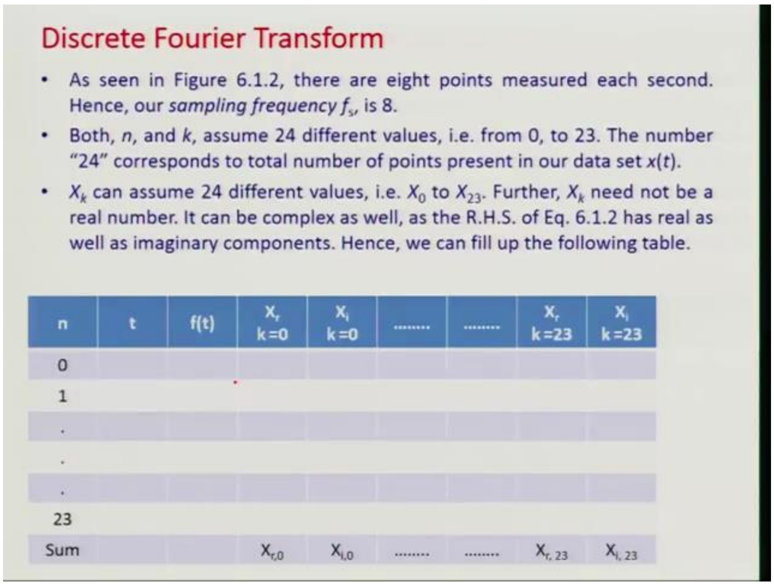

n	t	f(t)	X_r k=0	X_i k=0	....	....	X_r k=23	X_i k=23
0								
1								
.								
.								
.								
23								
Sum			$X_{r,0}$	$X_{i,0}$	....	....	$X_{r,23}$	$X_{i,23}$

So, first let us look at this table, we have explained that there are 8 points measured each second; our data acquisition system the way on which we are using it is measuring 8 points every second. So, we say that our sampling frequency f s is 8, 8 samples per second, if it was measuring 16 samples then it would have been 16 and so and so forth. The other thing to notice that both n; n is what it is the induction time and k which is the induction on frequency, they can assume 24 different values.

So, in this table n is going down and it starting from 0 1 2 3 4 till 23, and k is starts from here and it is going in this direction. So, k is equal to 0 here and I had said that when we compute X k it will have a real part and an imaginary part. So, for each value of k I have X r which is the real part and X i which is the imaginary part. So, I have 2 columns for each value of k, one is this column is the real part, this column is the imaginary part for k is equal to 0 then for k is equal to 1, I will again have a real part and an imaginary part.

So, in this table I have constructed this table which have been used to compute different values of X k is specifically there different values of X k their real and imaginary parts and we learn how to do that. So, that is what it is says here, X k can assume 24 different values from k is equal to 0 to k is equal to 23 and further X k need to be a real number it can be complex as the RHS of the equation, which we had discussed a real it can have real and imaginary components.

So, what we have in this table is a matrix and here onwards we are going to compute different values of X k and n goes down and it goes down from 0 to 23, the next column is time, the third column is f t. So, this is what there is now we learn how to do it.

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Table 6.1.1

n	t	f(t)	0	0	1	1	2	2	3	3	4	4
0	0.000	0	0	0	0	0	0	0	0	0	0	0
1	0.125	1.414	1.414	0.0000	1.366	-0.3660	1.225	-0.7071	1.000	-1.0000	0.707	-1.2247
2	0.250	2.000	2.000	0.0000	1.732	-1.0000	1.000	-1.7321	0.000	-2.0000	-1.000	-1.7321
3	0.375	1.414	1.414	0.0000	1.000	-1.0000	0.000	-1.4142	-1.000	-1.0000	-1.414	0.0000
4	0.500	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
5	0.625	-1.414	-1.414	0.0000	-0.366	1.3660	1.225	0.7071	1.000	-1.0000	-0.707	-1.2247
6	0.750	-2.000	-2.000	0.0000	0.000	2.0000	2.000	0.0000	0.000	-2.0000	-2.000	0.0000
7	0.875	-1.414	-1.414	0.0000	0.366	1.3660	1.225	-0.7071	-1.000	-1.0000	-0.707	1.2247
8	1.000	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
9	1.125	1.414	1.414	0.0000	-1.000	-1.0000	0.000	1.4142	1.000	-1.0000	-1.414	0.0000
10	1.250	2.000	2.000	0.0000	1.732	-1.0000	1.000	1.7321	0.000	2.0000	1.000	1.7321
11	1.375	1.414	1.414	0.0000	-1.366	-0.3660	1.225	0.7071	-1.000	-1.0000	0.707	1.2247
12	1.500	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
13	1.625	-1.414	-1.414	0.0000	1.366	-0.3660	-1.225	0.7071	1.000	-1.0000	-0.707	1.2247
14	1.750	-2.000	-2.000	0.0000	1.732	-1.0000	-1.000	1.7321	0.000	-2.0000	1.000	1.7321
15	1.875	-1.414	-1.414	0.0000	1.000	-1.0000	0.000	1.4142	-1.000	-1.0000	1.414	0.0000
16	2.000	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
17	2.125	1.414	1.414	0.0000	-0.366	1.3660	-1.225	-0.7071	1.000	-1.0000	0.707	1.2247
18	2.250	2.000	2.000	0.0000	0.000	2.0000	2.000	0.0000	0.000	2.0000	2.000	0.0000
19	2.375	1.414	1.414	0.0000	0.366	1.3660	-1.225	0.7071	-1.000	-1.0000	0.707	-1.2247
20	2.500	0.000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000	0.000	0.0000
21	2.625	-1.414	-1.414	0.0000	-1.000	1.0000	0.000	-1.4142	1.000	-1.0000	1.414	0.0000
22	2.750	-2.000	-2.000	0.0000	-1.732	-1.0000	-1.000	-1.7321	0.000	-2.0000	1.000	-1.7321
23	2.875	-1.414	-1.414	0.0000	-1.366	-0.3660	-1.225	-0.7071	-1.000	-1.0000	-0.707	-1.2247
FFT (sum)			0	0	0	-7E-16	-4E-15	-3E-15	1E-14	-24	3E-15	5E-15
Mag of FFT			0		7E-16		5E-15		24		6E-15	

So, this is what we do. So, this is an illustration how for each value of k we can compute X k. So, first n is equal to 0 to 23 it has been listed here, then at a given value of time, when n is equal to 0, time is 0, when n is equal to 1 what is the value of time we are measuring 8 points each second right. What does that mean? It means that each second is divided into 8 different parts, which means that corresponding to n is equal to 1, this is 1 over 8th, t is 1 over 8th which is 0.125 seconds, for n is equal to 2 it is 0.25 seconds and so on and so forth. So, the final value of t corresponding to n is equal to 23 is 2.85 seconds.

Now this formula was already given and what was this formula. So, for each time I have to know what is it is value. So, what was the value? It was 2 sin 2 pi f t and f was 1 hertz. So, for this value of time I have calculated the value of function it is given here. So, all these data is given from this given data which we can measure experimentally we have to compute capital X right X k is X 1, X 2, X 3, X 4, that is what is shown here.

So, these 2 columns correspond to k is equal to 0, these 2 columns correspond to actually it should not k, but X k this column corresponds to X 1, this corresponds to X 2, this corresponds to X 3 and so on and so forth. So, what do we do? So, we will explain that in the next slide. So, let say we want to compute X 0, we already know the values of f 0, f 1, f 2, f 3, from this column.

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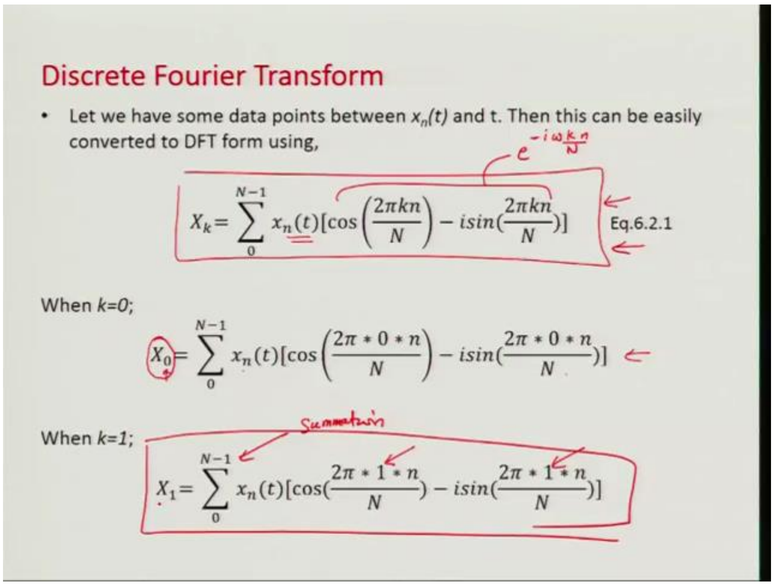

So, what is X 0? This is the general relation, X k equals X n which actually changes with time, times what is this thing? This is e to the power of minus I omega k what is the k n divided by N. So, I can express it as a cosine term and the sin term, I can express this e I omega k n as a sin and the cosine term.

So, what I have done is I have expanded this and in this relation I have put k is equal to 0. So, when k is equal to 0 this expression becomes X 0 equals sum of X n cosine 2 pi times 0 times n divided by capital N minus I sin 2 pi times 0 times small n divided by capital N; how did I get this? I use this equation and then this equation, I have put k equals 0 and then for different values of n I have summed this entire expression for different values of n small n and n changes from 0 to N minus 1. So, in that way I get X 0. If I have to when calculate X 1 again we go back to this equation and I plug in k equals 1. So, when k is equal to 1 I have 1 here, and using in this expression I do the summation this is the summation and I get X 1. So, in this way I have been able to calculate X 0, X 1, X 2, X 3, and so on and so forth. So, these 2 columns are for X 0, these 2 columns are for X 1, these 2 columns are for X 2 and so on and so forth and as I said each X will have a real and an imaginary part. So, I have listed those here.

So, what I have done is here this is the individual component corresponding to each n, what is the individual component? So, if N is equal to 1 then when I put n is equal to when in this expression I get the first component right. So, these are the individual components and this is the some of the entire column. So, this is the sum of the entire column corresponds to this operation and. So, what and then finally, I get the magnitude of a 50. This magnitude of a 50, this magnitude is basically this term square plus this term square and the square root of 2 is square root of the sum of the squares, this is the real this is the real component, this is the imaginary component, if I square them add them up and take the square root that is the magnitude of X k.

Similarly, for X 1 square of this plus square of this and then add them up and take the square root I get this is X 1 the last row; this is for X 2 this is the real portion, this is the imaginary portion, this is the square root of sum of their squares so that is X 2 and similarly this is X 3. So, in X 3 what do we see? This is the real portion, this is the imaginary portion and the sum of them is giving as X 3 and so on and so forth. So, what does it tell us? X 0 equals 0, X 1 equals 7 E minus 16 which is 0, X 2 equals 5 E minus 15 which is 0, X 3 equals 24, X 4 equals 0 and so on and so forth.

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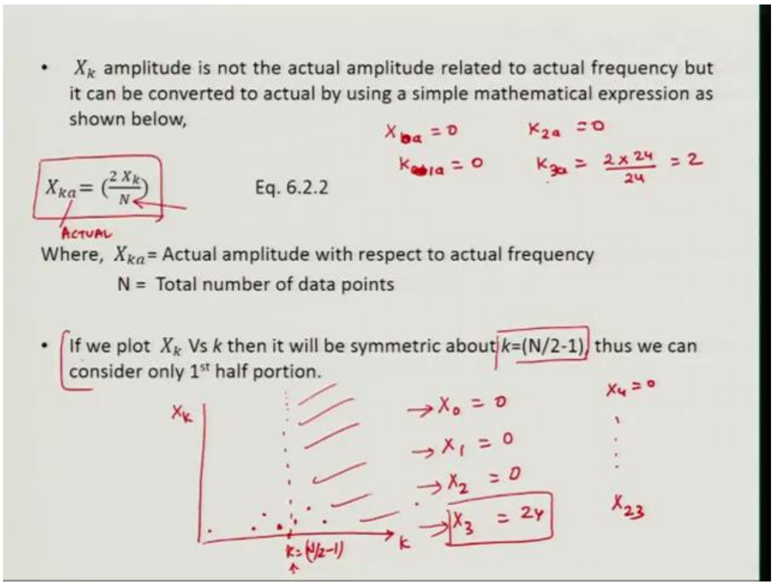

So, what did we get? We got X 0 is equal to 0, X 1 equals 0, X 2 equals 0, X 3 equals 24, X 4 equals 0 and it went on till X 23 because there are 24 points, what does this mean that the first frequency has 0 amplitude, second frequency has 0 amplitude, third frequency has 0 amplitude, but fourth frequency has an amplitude of 24.

Now, this is not the real amplitude as I mentioned earlier, we have to converted into some real units. So, what is the real amplitude? The relation for real amplitude is given here. So, that is equal to X ka, k a means actual, so this is actual, this equals 2 times X k this is the relation 2 times X k divided by N. N is.

Student: Number of points.

Number of points; which mean that X 0 a equals 0, X 0 no; X 1 a equals 0, X 2 a equals 0, but X 3 a equals.

Student: 2 (Refer Time: 13:50).

2 into X k. X k is 24 divided by number of points 24 is 2, but we still do not know, but this X 3a corresponds to how many hertz. All we know is that this is the fourth frequency in the system, first frequency actual amplitude the 0, second frequency actual amplitude to 0, third frequency actual amplitude to 0, fourth frequency amplitude is 2 that is all we know.

Now, we will figure out how do we compute the frequency of the system. But before we do that this is an important thing to note, it is says if we plot X k versus k. So, I am plotting X k here and I am plotting k on the X axis, then at this location k is equal to N over 2 minus 1. So, let say this is k is equal to N over 2 minus 1 this function it will be symmetric. So, if it rises here it will rise here, if it goes down here it will go down here. So, at this location X k is equal to N by 2 minus 1, the plot of X k versus k will be symmetric and typically we take data only from the first half or the second half because all that data is symmetric.

So, what we have learnt till so far is 2 things; how do we calculate different values of X k we have learned that and from those X k we can compute the actual amplitudes of individual frequencies, but what are those actual frequencies we do not know that. So, in the next few slides we will learn that as well. So, this is the thing for the remaining portion.

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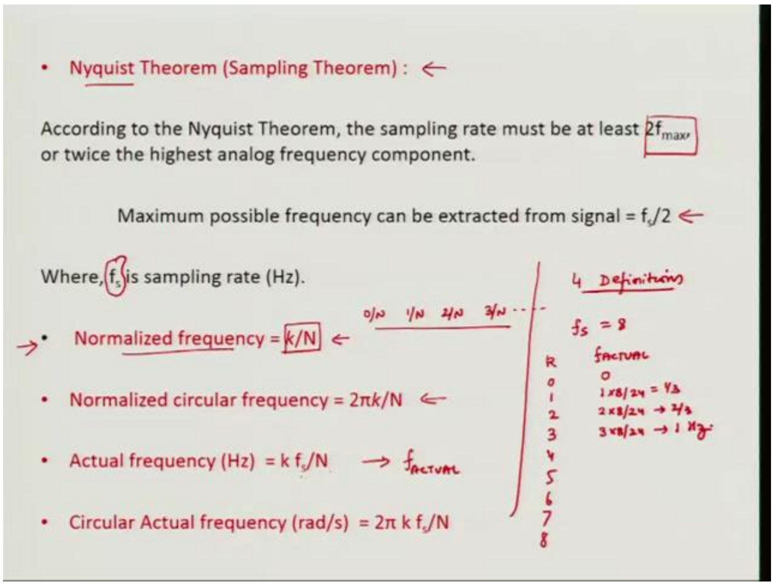

So, before we talk about frequencies let us talk about this Nyquist theorem. So, this is a theorem developed by mister Nyquist and he says that if there is a signal and let say that signal has lots of frequencies, it may have 1 hertz, 2 hertz, 30 hertz ,40 hertz, 20000 hertz and if I have to capture all the frequencies which it has. So, I let say that the highest possible frequency in that signal is 20000 hertz.

Then I should sample it at a frequency which is twice the maximum frequency in the signal 2 f max. So, if I think that my signal it may have frequencies as I high as 15000 hertz then my sampling frequency it should be at least, it should be at least 2 times that which is 30000 hertz. So, that is what it means. So, another way to look at is that if I am sampling my signal at frequency at value f s, f s is sampling frequency when the maximum possible frequency which I can extract from it is f s divided by 2, this is there. So, this is an important parameter f s because when we calculate the actual frequencies we will use this.

Now, in DFT, we had actually 4 definitions of frequencies, first definition is known as normalized frequency. So, how many frequencies, frequency components we get if there are n points.

Student: (Refer Time: 18:00).

We will get n number of frequencies right corresponding to k is equal to 0, k is equal to 1, k is equal to 2 and so on and so forth. Now the normalized so for the k th frequency, normalized frequency is defined as K divided by N. So, it will be 0 by N, 1 by N, 2 by N 3 by N and it will go up to.

Student: (Refer Time: 18:30).

N minus 1 by (Refer Time: 18:32), the second frequency is we do not bother too much about this it does not make may a lot of physical sense; the second frequency definition in DFT is normalized circular frequency. So, if you just multiply this K over N by 2 pi this is your normalized circular frequency and the actual frequency is k times f s divided by N, it is k times f s divided by N. So, in our case what is the value of f s?

Student: 8.

In our case f s was 8. So, k, 0, 1, 2, 3, 4, 5, 6 ,7, 8 and so and so forth; actual frequency let us call this f actual. So, let us compute f actual, when k was 0 f actual means 0 hertz when k was 1 it means 1 times 8 divided by 24 is one-third hertz, when k was 2 times 8 divided by 24 it means two-third hertz. When this was 3 it is 3 times 8 divided by 24 it means 1 hertz.

So, the moment we know k using sampling frequency and the total number of points which we have collected, we can calculate actual frequencies and we have already learnt how to calculate the actual amplitude using this relation. So, we know now how to calculate actual amplitude and we found that actual amplitude for the fourth frequency was 2 and that corresponds 2 frequency which equals 1 hertz and that is exactly what are original r result should be, because what was the original signal? The original signal was 2 sin times 2 pi f t. So, the frequency was 1 hertz which we have calculated and the amplitude was 2 and the last thing is phase, we can also calculate the phase of this signal, now what is the phase of this signal phase theoretically, it is what?

Student: (Refer Time: 21:20).

So, let us look at. If this also the imaginary component was 24, real component was 0. So, tan inverse 24, no actually sorry it is minus 24 if we look at.

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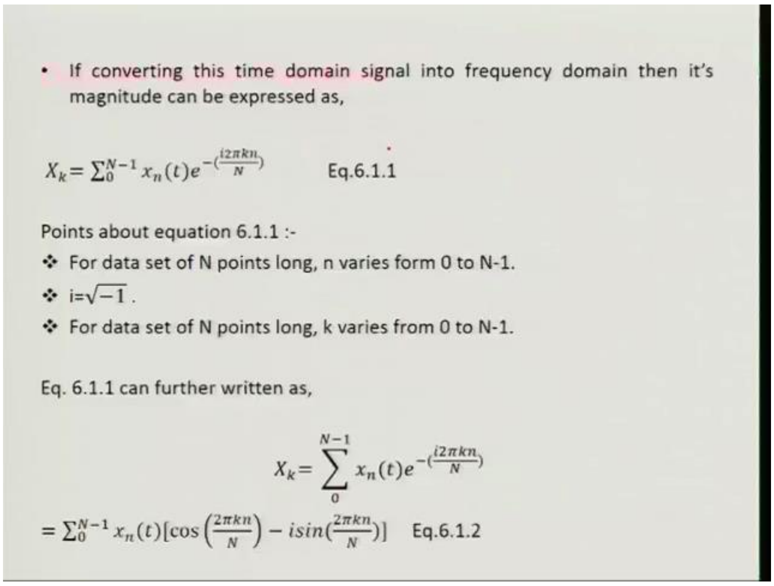

Student: Minus 24.

Where is that? This is minus 24; imaginary component is minus 24, real component is 0. So, from this you can compute the phase also. So, now, you know how to compute phase for each frequency component, how to compute it is amplitude? The amplitude can be calculated by first constructing the table and then using this X k a equals 2 X k divided by N and then the corresponding frequency can be calculated by using this relation.

So, I think that all this information is sufficient for today's lecture, we will meet once again tomorrow and we will discuss some other features of DFT so that when you actually acquire data, you know how to correctly acquire data and how to use it effectively. So, with that we close the discussion for today and we will meet once again tomorrow.

Thank you.