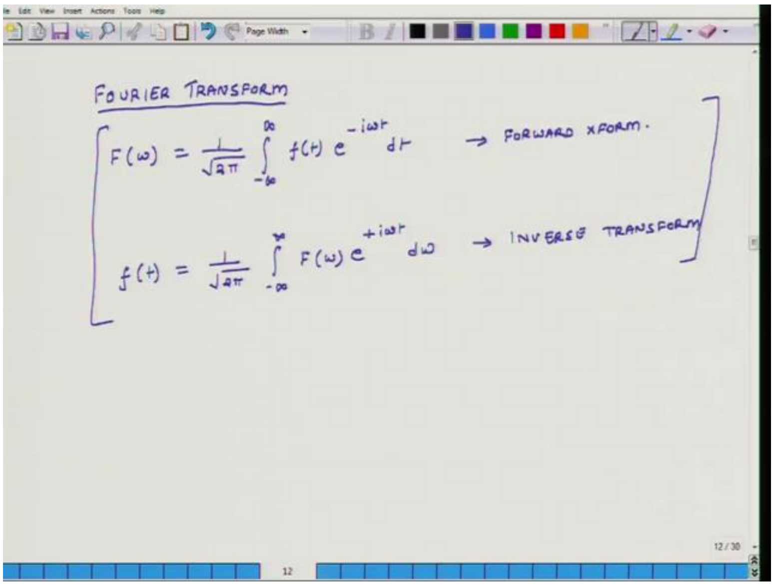
**Fundamentals of Acoustics**
**Prof. Nachiketa Tiwari**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture – 58**
**Discrete Fourier Transform**

Hello; welcome to Fundamentals of Acoustics, today is the 4th day of this 10th week of this course and what we plan to do today is introduce discrete fourier transform which is commonly abbreviated as DFT, but before I do that I just very quickly wanted to mention which I forgot to mention in one of the last 2 class is that we had said that there are 2 types of wave transforms, one is the forward transform and the other one is the reverse transform and I had given you all the relation for the forward transform, but I missed on providing you with the relation on reverse transform. So, next 4-5 minutes I will just discuss that and then we will move on.

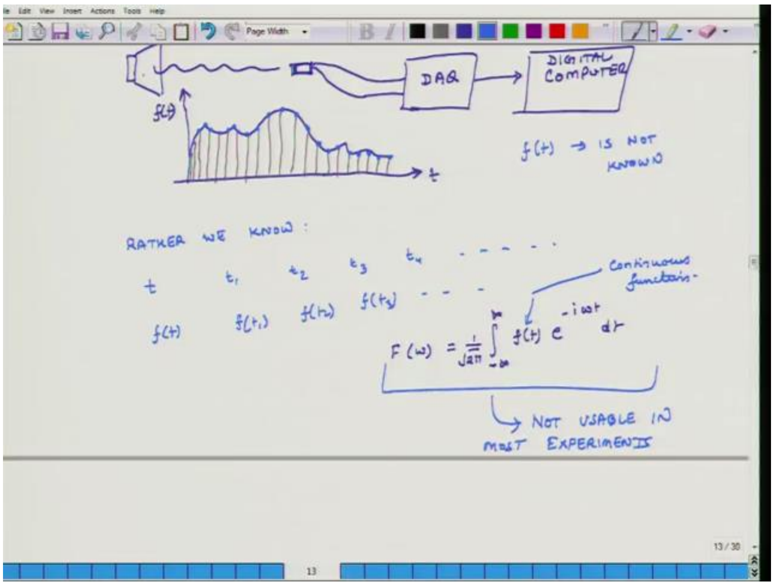
(Refer Slide Time: 01:07)




So, Fourier transforms; if I have to convert time domain data into frequency domain then I use for forward transform. So, I say F of omega equals 1 over square root of 2 pi minus infinity to positive infinity, f t e to the power of minus I omega t d t. So, that is my forward transform and the inverse transform is. So, what does the other transform do? If I have frequency domain data, then from that information I can get back to my time domain and that I can do using this relation and here instead of F t I have f of omega capital F of omega and instead of exponent to the power of minus omega t, I have e to the power of plus I omega t and I integrate with respect to omega. So, this is my inverse transform.

So, these are important relations and using these you can convert any continuous or peace wise continuous signal if it is in time domain into frequency domain and vice versa. So, the only condition is that the signal has to be piecewise continuous integrable signal at also should be integrable. So, what does that mean? That it is integral over the entire domain should not become infinite, it should be a finite number.

(Refer Slide Time: 03:23)

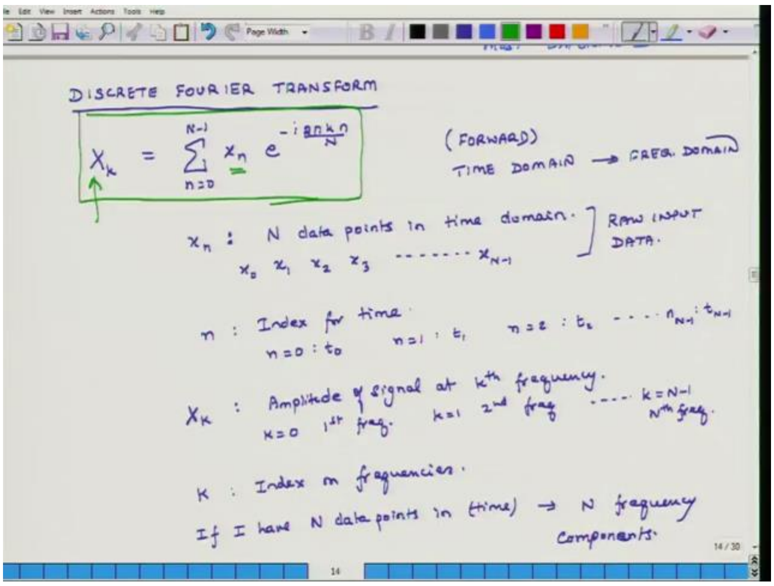



So, now next we will talk about Discrete Fourier transform, Discrete Fourier transform lot of time it is abbreviated as DFT. So, first let us talk about the why do need discrete Fourier transform? So, considered a loud speaker and its emitting some sound and let us say I have a micro phone here. So, that is my micro phone and this micro phone is recording the signal and the signal it goes through a data acquisition system. So, I call it that data acquisition system and then from here my data goes into a computer and this is the digital computer. So, the signal which is coming from the speaker to the micro phone, it is a continuous signal. So, the signal continuous, but what does the data and the signal which is coming out. So, which is the entering the micro phone and which is leaving the micro phone it is a continuous signal, but most of the modern data acquisition systems and the digital computer in combine, what they do is that they do not capture continuous signal rather they captured data at every small intervals of time. So, the computer does not capture this blue curve, rather what does it do? It computes or not computes it measures the value of the signal at discrete at intervals of time.

So, this is my time axis. So, let us it measures very one-tenth of a second. So, I will get a lot of points and what I will not have is a continuous curve. So, f t is not known rather what we know is, rather we know. So, we do not know f t as a continuous function, rather what we know is this table. So, t at t 1, t 2, t 3, t 4 at different intervals points of time I know the value of ft. So f 1; f t 1, f t 2, f t 3 and so on so forth; I do not know the continuous function, I just know points at different intervals of time after every phase interval of time. So, if I have to do a Fourier transform for just points I cannot do it because my Fourier transform expression is what? My fourier transforms says that F omega equals 1 over square root of 2 pi integral of from minus to plus infinity f t e minus I omega t d t.

So, here I have to know f t as a continuous function, but most of the times when we get data it is not continuous data it is just discrete points. So, I cannot use this relation not usable in experiments, I will not say all experiments but in most experiments I cannot use this formula. So, I cannot convert this frequency domain data which is made up of points into this time domain data which is made up of individual points into frequency domain. So, for that we have to learn a technique called Discrete Fourier Transform.

(Refer Slide Time: 08:47)



So, what we will do is we will not go around and try to prove the relation for discrete Fourier transform, but we will just state the results. I will first write down the relation and then explain it. So, the forward transform which converts digitize or discrete data in time domain into frequency domain, I can express it as capital X subscript k equals summation from n is equal to 0 to n minus 1, X n e to the power of minus i 2 pi k n divided by capital N, we will explain this formula. So, what is the X n? X n they correspond to n data points in time domain. So, what will their values be? Their values will be x 1, x 2, x 3 and so on so forth, but we do not start from x 1, because in this formula n starts from 0. So, the first point we call it as x 0 and if there are n points we go up to x to n minus 1, we start from 0 and we end up at n minus 1. So, X n are n data points in time domain. So, in the discrete Fourier transform and this is the forward transform relation.

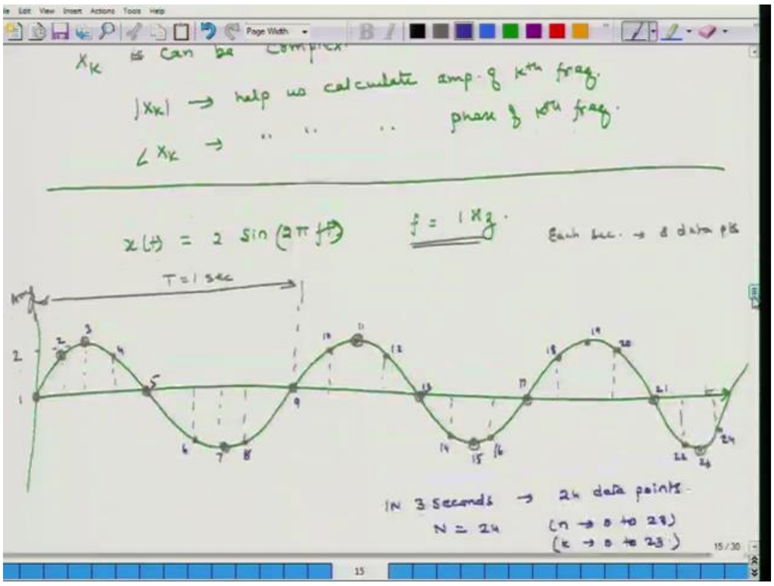
So, this converts time domain into frequency domain, we converts time domain data into frequency domain data. So, this is raw input data. So, if I know end points in time, I can use those end points to compute frequency domain data what is X k? So, there is an error here it should not be lower case n, but this is upper case N because total number points is N. So, this small n is just an index. So, before I talk about X k n is the index for time it is index for time. So, n is equal to 0 means t 0, n is equal to 1 means t 1, n is equal to 2 means t 2 and so on so forth.

So, finally, we get n equals n subscript N minus 1, it means t N minus 1. So, again there are end points, this is an index for time. Now let us X k. So, what is X k amplitude of signal at k th frequency and k is equal to 0 means first frequency, k is equal to 1 means second frequency and we go up to k is equals N minus 1 means nth frequency. So, k does not equal frequency, but it tells k is equal to 0 means first frequency, k is equal to it by itself a does not necessarily means frequency, but it tells that the amplitude of the first frequency is x 1, amplitude of the second frequency is actually amplitude of the first frequency is x 0, amplitude of second frequency is x 1, amplitude of the third frequency is x 2 and so on so forth.

So, k is the index on frequencies it is the index on frequencies. So, n is the index for time, k is index on frequencies. So, what does this relation show? So, if I have N data points in time, I will get N frequency components, this is important to understand.

If I have N points in time from that information I can extract n frequency component. Now all of them may not be unique some of them may be repeated, but we will get n different frequency components. So, what we do is. So, how do we use this formula? The way we do it is what we do is we first get all the values of x and then we multiply x 0 to e to the power of minus i 2 pi k times 0 divided by N, add it to x 1 times e to the power of minus 2 pi k times 1 divided by N and so on so forth. So, initially I said k to be 0 and I find the summation of this entire experience then I get x 0 then I change by k 2 1 and I do this entire operation again for n points. So, I some I make N summations so I get x 1; then I get I go to k is equal to 2, so initially I calculate the this co efficient x for 0 th frequency or for first frequency when k is equal to 0, then for second frequency k is equal to 1 and so on so forth. So, in this way step by step I calculate x 0, x 1, x 2, x 3, till x n minus 1 and I get for n different frequencies these co efficient x's and then from that.

(Refer Slide Time: 16:43)




So, last thing X k is it can be complex. So, magnitude of X k will help us. So, again X k is not same as magnitude, it will give us an idea of magnitude exact magnitude will tell what it means. X k its m it is modulus will help us calculate amplitude of k th frequency, and phase of X k will help us calculate phase of k th frequency. So, what we will do is we will now do an example. So, that thing becomes clear.

So, let us considered the relation x t 2 sin 2 pi ft and let us say f equals 1 hertz. So, if I plot it my signal will look like this and these are the points. So, my experimental set up is giving me some points. So, I am going to set up the problem today and then we will continue this discussion tomorrow. So, f is frequency is 1 hertz and what my x data acquisition system is doing is; that in each second it is acquiring 8 sample points. So, each second I get 8 data points. So, this is my time axis, this is my amplitude. So, this maximum values to and this is how much this duration T is equal to 1 second. So, in each second it is getting 8 data points, let us count. So, and it is finding data points after equal intervals of time. So, it is not that.

So, these are the data points. So, my data acquisition system is not giving me the green curve, it is just giving the data points and in 1 second it is giving me 8 data points. So, let us just count these numbers of points. So, this is point number 1 2 3 4 5 6 7 and 8, in 1 second I am getting 8 data point. So, I am not picking up the 9th point because at the 9th point the cycle is going to be complete.

So, I am getting 8 then in the next cycle I get 9 10 11 12 13 14 15 16. So, in 2 second I get 16 data points then what is it? 17 18 19 20 21 22 23 and 24, so in 3 seconds, I am let us my data acquisition system is acquiring data for 3 seconds. So in 3 seconds I get 24 data points, which means that n is equal to 24. So, I am having 24 data points and using these 24 data points, how many frequencies, frequency amplitude I will be able to compute?

Student: 24.

24; so, n is going from 0 to 23 and the index on frequency is also going from 0 to 23. So, in the next class we will learn how to use this relation which we have shown in particle way. So, that concludes our discussion for today and we will meet once again tomorrow.

Thank you.