Fundamentals of Acoustics
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Lecture – 57
Fourier Transform

Hello. Welcome to Fundamentals of Acoustics. Today is the third day of the 10th week of this course. And we will continue our discussion of what of the topic we were discussing yesterday that is Fourier Transforms. And what we will do today is we will do couple of more examples so that we get a form grounding in Fourier transforms, and then we will move to another topic known as a Discrete Fourier transforms.

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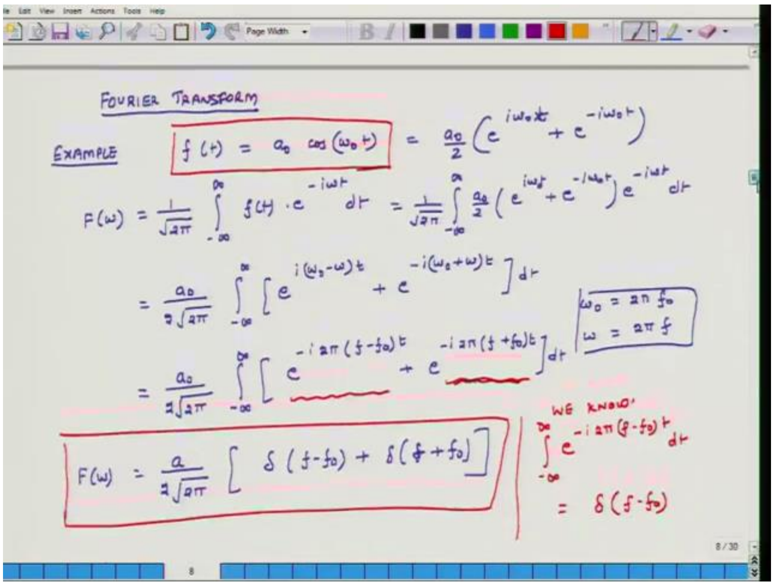

So, here today we will do one more example. And let us say that there is a function in time and that equals a naught cosine omega naught t. So, my aim is to develop its Fourier transform, I want to depict this in Fourier domain. So, this can be written as; so I will convert this into more form which is more convenient to handle. So, I can express it as e to the power of i omega naught t plus e to the power of i omega naught t. So, I am expressing it in terms of cosine in terms of it complex exponential form. So now, with this after this transformation I try to figure out its Fourier transform F of omega. So, what is F of omega? It is 1 over a square root of 2 pi integral from minus infinity to plus infinity f t times e to the power of minus i omega t dt; that is there. So, this equals 1 over square root of 2 pi minus infinity to infinity a naught over 2 e to the power of i omega t actually its omega naught plus e to the power of minus i omega naught t times e to the power of minus i omega t dt.

So, I bring a naught into out, so it is a naught divided by 2 is square root of 2 pi minus infinity to infinity integral of; and I have two terms e to the power of i omega naught minus omega t plus e to the power of minus i omega naught plus omega t dt. Now I do one small mathematical change, I say that omega naught equals 2 pi some number f naught and omega is equal to 2 pi f. So, what I get is a naught divided by 2 a square root of 2 pi minus infinity to infinity and here I get, minus i and then 2 pi f minus f naught t. So, here it was omega naught minus omega, but I wanted f minus f naught so I have taken a minus sign outside. And this is e to the power of minus i 2 pi f plus f naught t dt.

Now, at this stage we will say that we know, so we will use an existing relation and what do we know that integral of e to the power of minus i 2 pi f minus f naught t dt. If I integrate it from minus infinity to positive infinity this is equal to direct delta function evaluated at f naught. So, similarly the integral of the second function is direct delta function evaluated at positive f naught. So, what I get is a divided by 2 square root of 2 pi and then a direct delta function f minus f naught plus direct delta function; f plus f naught.

Now, let us look at how this function looks like. So, our original function is f t equals a naught cosine omega t. So, it has just one single frequency which is at omega naught, and let us plot this direct delta function let us plot this direct delta function. So, what is a direct delta function? A direct delta function is a function which has 0 for all values of t except at 0. Now this argument becomes 0 at f is equal to f naught, and this argument becomes 0 at f is equal to minus f naught.

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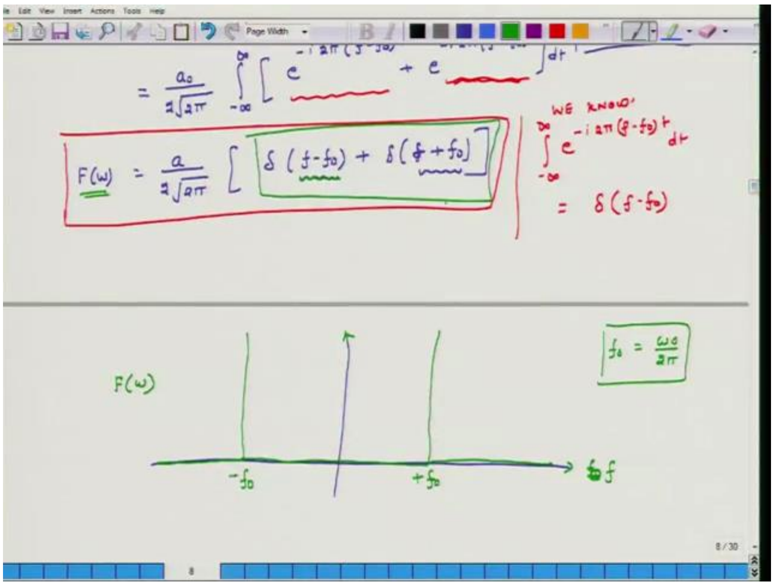

So, if I am going to plot this entire function, this here the argument become 0 at f naught. And in the second function the argument becomes 0 at minus f naught. So, if I plot this entire function and on the x axis suppose I plot f frequency and here I plot some value the function will look like this. And this is minus f naught and this is f naught.

So, what that means is that the relation for transform f is such that the Fourier components or the you know of in the frequency domain this signal a cosine omega t has only components at f naught and minus f naught. Where, f naught is equal to omega naught divided by 2 pi. So, f naught we all understand why will it have a component f naught, because this is cosine omega naught t. But what about minus f naught; again if I put omega naught instead of omega naught I put negative of that I still get t he same relation. So, this is the plot for F of omega.

Once again this plot has no re imaginary component. Why does it not have any imaginary component? So it does not have any imaginary component, because our original function f t is a pure cosine function which is an even function. So, because it is an even function we should not see any imaginary components in the expression for f omega.

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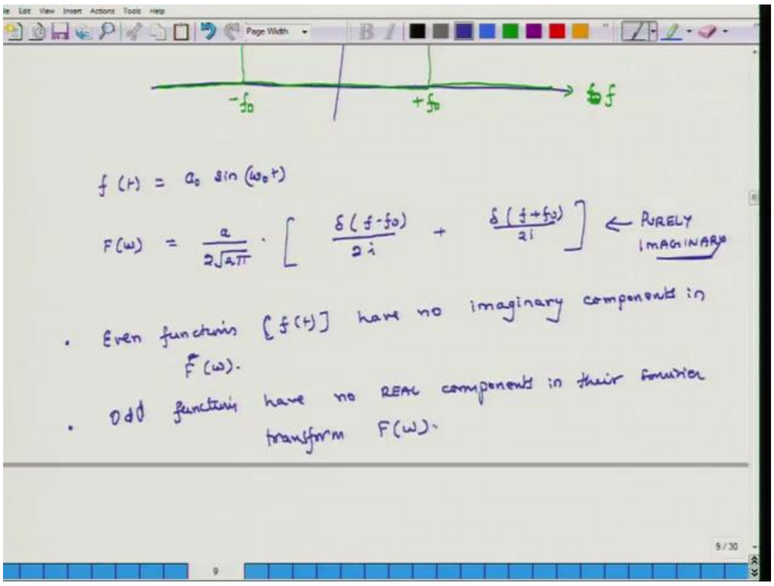

Another example would be an we will not do the mathematics for this in detail would be that if f t equals a naught sin omega naught t; if f t is equal to a naught sin omega naught t then we can show that its Fourier transform f omega is equal to what it is equal to a divided 2 under root 2 pi times delta f minus f naught, but that divided by 2 I plus another direct delta function f plus f naught divided by 2 i.

So, for a sin function pure sinusoidal function I have a Fourier transform which is purely imaginary; it is purely imaginary function. So, we will make two important observations. Even functions, and by even functions I mean f t have no imaginary components in there Fourier transform. And odd functions have no real components in there Fourier transform F of omega. It is important take away.

So, we will do one more example today and then we will close the discussion for today and then tomorrow onwards we will start discussing DFT’s.

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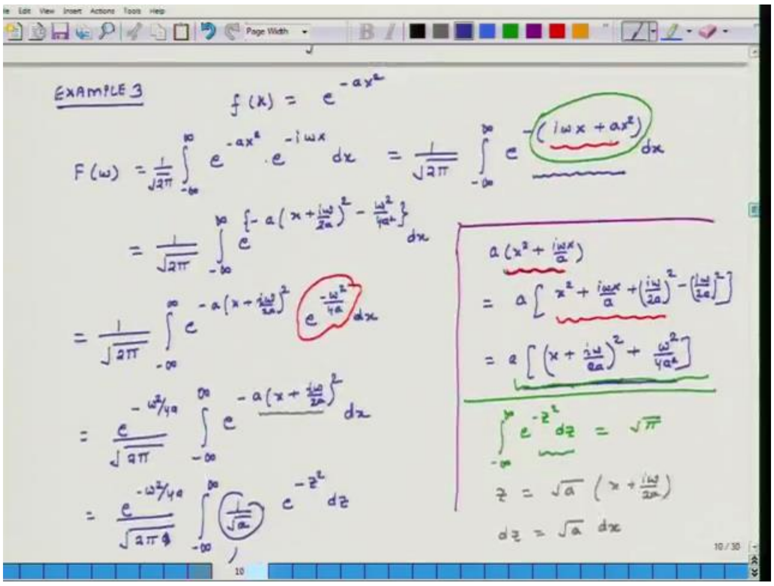

So we will do another example; example 3. So, let say f x equals e to the power of minus a x square. So, let say that f x is e to the power of minus a x square. Now is this an odd function or an even function or it is none of these. So this is an even function, so when we take its Fourier transform we should except that it should not have any imaginary components. Now we let us calculate its Fourier transform. So, F of omega equals 1 over square root of 2 pi integral minus infinity to positive infinity e to the power of minus a x square times e to the power of minus i omega t dt – oh, actually I am sorry here we are using not t but x, so e omega x d x.

So, this becomes 1 over square root of 2 pi minus infinity to infinity e to the power of minus and I take i omega x plus a x square dx. Or I can write it as square root of 2 pi integral minus infinity to infinity. So, what we want is we want to put this entire function because of this x square thing it is not easy to integrate. So, we want to change it into something which is easy to integrate or for which the results are known. So, what do we do? So we know that a x square plus i omega x over a see this thing is same as this term in parenthesis; this term is same as this term. So, this equals a and let us write it x square plus i omega x by a and then I add the term and subtract a term. So, I add this term i omega over 2 a whole square and I subtract the term i omega over 2 a whole square.

So, this entire thing can be expressed as a perfect square. So, this is equal to a x plus i omega over 2 a whole square and this term is plus omega square by 4 a square right. So, what we do is we take this entire thing. So, back and substitute it here. So, this is equivalent to this entire thing mathematically you can write. So, this is equal to e to the power of minus a times x plus i omega over 2 a whole square minus omega square by 4 a square and then I am integrating this entire thing with respect to d x

So, 1 over square root of 2 pi minus infinity to positive infinity e to the power of minus a x plus i omega over 2 a whole square times e to the power of minus omega square by 4 a square dx; actually once second. So, when I multiply this by a this square things goes away this is square thing goes away where is plus here.

So, this is a constant because I am integrating with respect to x so it will come out. So, this equals e to the power of minus omega square over 4 a divided by square root of 2 pi integral minus infinity to infinity e to the power of minus a x plus i omega over 2 a whole square; integrated with respect to x. Now there is a standard result in mathematics if you remember error functions and what is that; it says that integral of minus infinity to infinity of e to the power of minus z square dz, this is equal to a square root of pi. So, this function is called an error function, and the error function is e to the power of minus z square and if I multiply this function with respect to dz and integrate it between minus infinity and plus infinity I get square root of pi.

So this kind of looks like error function, because I have x plus i omega over 2 a whole square so there is something. So what I do, I do a transformation; so I say that z equals square root of a x plus i omega over 2 a, then dz is equal to square root of a times dx. So, I make this transformation here and what I get is e to the power of minus omega square over 4 a divided by square root of 2 pi integral minus infinity to infinity, and dx is what? dz divided by square root of a; so 1 over a square root of a times e to the power of minus z square dz.

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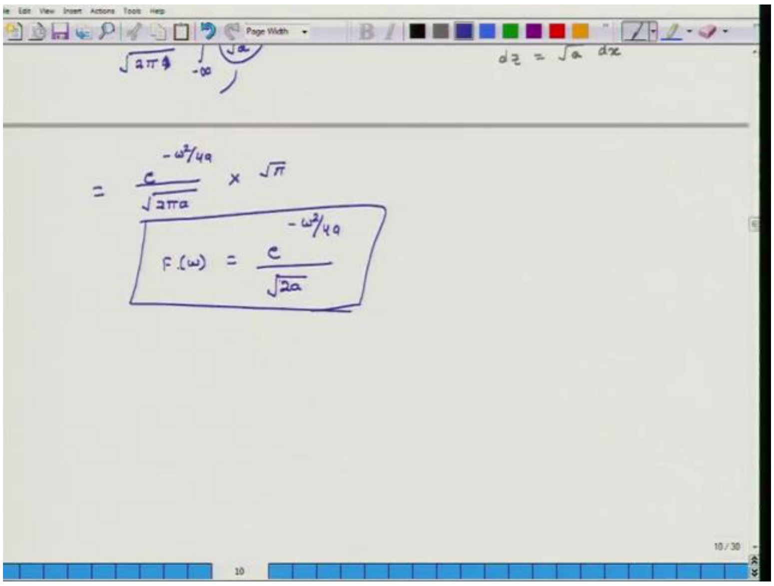

So, what I get is e to the power of minus omega square over 4 a divided by square root of 2 pi a because I am taking this term out and then I am left with just a pure error function integral so an its integral is square root of pi. So, my F of omega is e to the power of minus omega square over 4 a divided by 2 a.

Once again our original function of x was an even function, and when I develop its frequency domain representation I get a function which has no imaginary components. So, the point is that if you have a signal and when you visually look at it and you think that it looks pretty even, then while you are doing its Fourier transform. And if you get some imaginary components then you should be very careful and you should doubt the validity of your results and vice versa. If you think, if you have a feeling that your signal is odd then you should not expect any real component of the Fourier transform.

So, that concludes our discussion for today. And, tomorrow onwards we will extend this discussion for digital data and we will start discussing something known as discrete Fourier transform. So, with that I thank all of you and we will meet once again tomorrow. Bye.