Fundamentals of Acoustics
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Lecture – 56
Fourier Transform

Hello, welcome to Fundamentals of Acoustics, today is the second day of the 10th week of this course and today we will continue our discussion on Fourier transform, which we initiated in the last class.

So, what we will do today is I will first write down the expression for Fourier transform and then explain what it implies.

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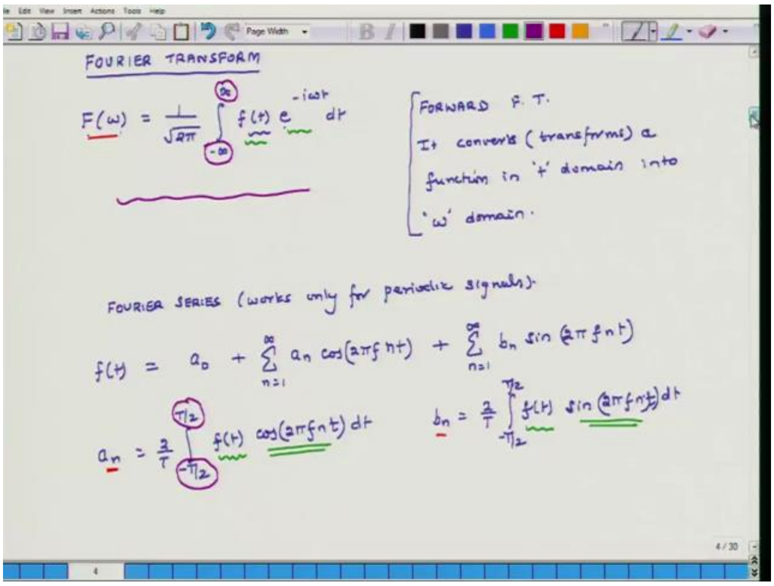

So, there are 2 types of Fourier transforms one in known is known as the forward Fourier transform and the other transform is known as the inverse transform. So, first we will let see what he forward transform expression looks like. So, capital F of omega equals 1 over square root of 2 pi minus infinity to positive infinity, f t e to the power of minus i omega t d t.

So, this is the expression for forward Fourier transform; what does this expression do? What this expression does is that if. So, what does it do? It converts or actually a better expression would be transforms a function in t domain, now this t could be anything it could be time or x or whatever because this is just a mathematical domain; so in t domain, into omega domain. In physical sense t could be time and omega could be 2 p times f. So, omega may represent frequency domain t may represent time, but this is just a mathematical. So, if there is something which can be measured in terms of t and if the same thing can also be measured in terms of omega and f t and omega are related in the sense that omega equals 1 over t times 2 pi, then this kind of transformation will help us convert or transform function in t domain into omega domain.

Now, here we are not going to prove this particular expression, but it will be important to compare this relation with the expression for Fourier series. So, Fourier series expression and this works only for periodic signals; if the signal is not periodic it does not work. The Fourier series signal what does it say that f t is equal to a naught plus a n cosine 2 pi f n t plus d n sin, 2 pi f n t and n equals 1 to infinity n equals 1 to infinity.

Now, just compare these 2 expressions, here I have this function and it is being multiplied by and what does this f omega represent? It represents kind of in some sense the value of this Fourier transform at omega and what is a n? A n equals 2 over t minus t over 2 to t over 2 f t cosine 2 pi f n t d t and b n equals 2 over t, minus t over 2 to t over 2 f t sin 2 pi f n t d t.

Now compare. So, these are kind of Fourier coefficients at different values of omega and a n and b n are also fourier coefficients for different values of for different frequencies, n is equal to one corresponds to frequency f, n is equal to 2 corresponds to frequency 2 f, n is equal to 3 corresponds so for. So, a n and b n corresponds to different frequencies, f is the coefficient corresponding to different omegas which is 2 pi f, which is similar.

The other thing is we have multiplied f t with a cosine term and here we have multiplied f t with a sin term, what have we done here? Here we have multiplied f t times e to the power of i omega t, now I can express e to the power of i omega t as a cosine plus I times sin. So, in a sense they are mathematically similar, they are not necessarily mathematically in the sense they are mathematically similar.

Third thing, you look at the limits of integration T over 2 to minus T over 2 and what does T represent? T represents the time period, here it is positive infinity to negative infinity and what is the time period of this signal it is infinite. So, in a sense this expression for fast Fourier for this forward transform it looks similar to this thing the third thing is of course, I have this integral here and the same integral is there also. So, in this sense this forward transform equation it looks similar. So, these are some of the important things.

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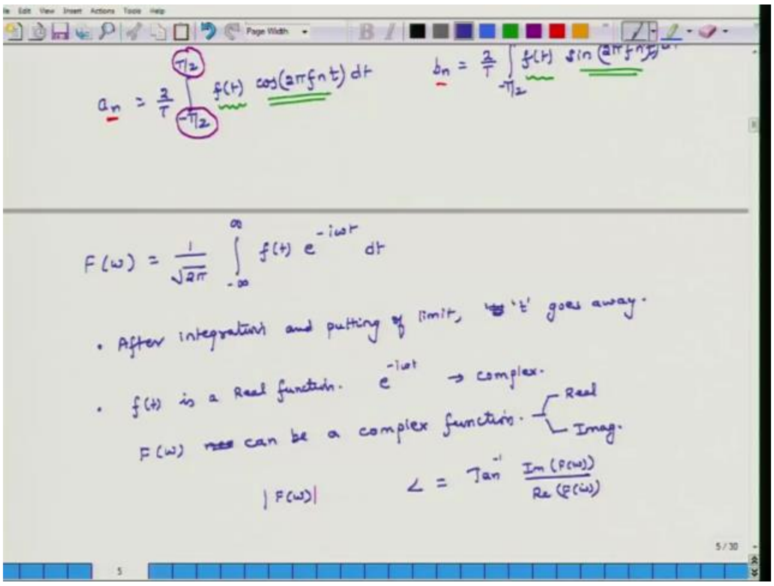

Now, couple of comments; So, we will first before we first comment on this f omega equals 1 over square of 2 pi minus infinity to infinity, f t e minus i omega t, dt that is our transformation. So, now, we make some comments after integration. So, this is integration on time and once we put the limits on the time, then after integration and putting off limits, parameter t goes away. So, we are left only with an expression with omega right. So, that is 1.

Second f t is a real function that is it involves only real quantities, but e to the power of i omega t is complex. So, f omega it can be a complex function. So, it will have a real portion and it will be having imaginary portion, f omega will have a real portion and an imaginary portion. So, for each omega I may get a real part and an imaginary part and then for corresponding to each omega, it will get its magnitude will be f of omega and its phase will be. So, this is magnitude and its phase will be tan inverse, imaginary of f of omega, divided by real of f of omega.

Essentially the imaginary component kind of maps to the sin term and the imaginary and the real thing maps to cosine thing. So, in this way F can be a complex function it can have a real part as an imaginary part, I am using those 2 different parts, I can find what is the absolute value; so I can again have an absolute a plot a continuous plot for absolute magnitude and I can also have a continuous plot for the phase of the representation of the signal in frequency domain.

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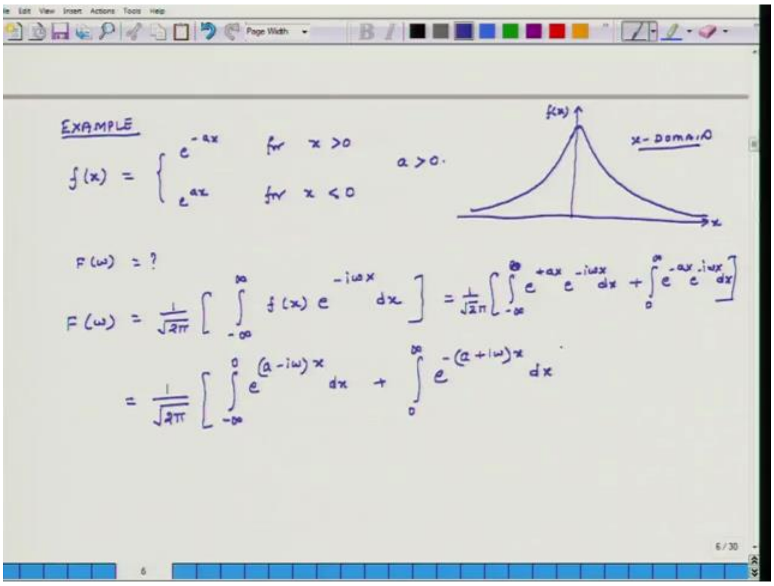

Let us look at an example, let us say that f x. So, here instead of time I am using this term x, but it is just a mathematical symbol. So, f x is equal to e the power of minus a x and e to the power of a x. So, it is e to the power of minus a x for x greater than 0 and it is e to the power of a x for x less than 0 and we say that a is a positive number, so a is larger than 0.

So, let us plot the signal, at x is equal to 0, the value is going to be 1 and as x increases on the positive side. So, this is x this is f of x. So, as x increases on the positive side this expression looks like this, as x increases or decreases on the negative side it looks like this right and this is a sharp. So, this is the slope at x is equal to 0 is a sharp point. So, this is the plot, this is in x domain.

Now, I want F its representation in omega domain I want its representation in omega domain and that is what we will calculate. So, F omega, this is a function we do not get in discreet values it is a function and that equals 1 over square root of 2 pi and then minus infinity to positive infinity f of x, e minus i omega x. So, instead of t I am just putting x here d x.

Now, I know that this function is not continuous over the entire domain, but it is piecewise continuous. So, it is continuous from minus infinity to infinity and it is an also. So, I will write it as 1 over square root of 2 pi integral, minus infinity to in; sorry, minus infinity to 0, e to the power of minus a x, e to the power of minus i omega x dx, plus and it is from infinity to 0, e to the power of minus a x e to the power of minus i omega x dx closing of brackets.

So, this equals 1 over square root of 2 pi, minus infinity to 0, e to the power of a minus i omega x dx, plus 0 to infinity, e to the power of minus a plus i omega x dx.

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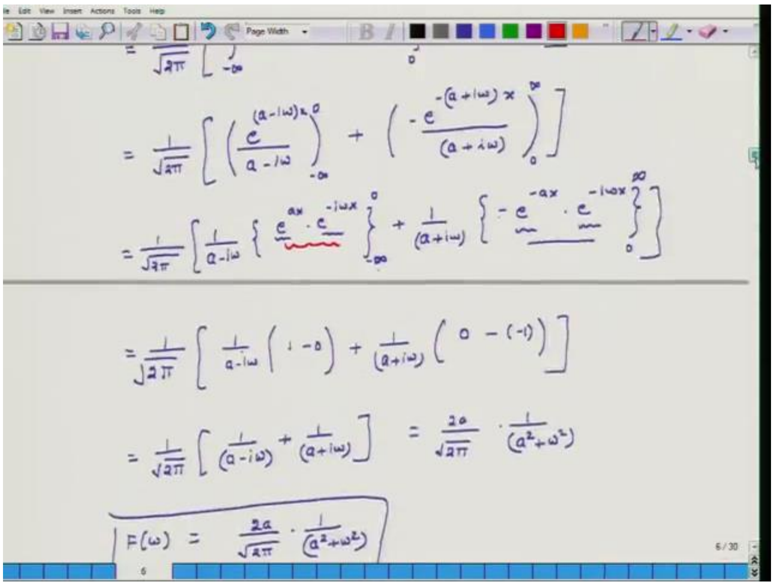

So, I compute it further and what I get is when I integrate the first term, I get e to the power of a minus i omega divided by I a minus i omega and this I am evaluating where from minus infinity to 0, plus e to the power of minus a plus i omega x and there is an x here also divided by a plus i omega and there is a negative sign here and this thing I am evaluating from 0 to infinity.

So, it is 1 over square root of 2 pi and what I will do is 1 over a minus i omega I am going to take out and the term in the parenthesis I will write it as, e to the power of a x times e to the power of minus i omega x, evaluated between 0 and no minus infinity and 0 plus 1 over a plus i omega in parenthesis, minus of e to the power of minus a x times, e to the power of minus i omega x that is what it is right.

So, first we will cal compute. So, this is 1 over a minus i omega and when I evaluate this function at x is equal to 0 what do I get? 1, when I evaluate e to the power of a x times e to the power of minus omega x at x equals 0 I get 1; what happens when I evaluate at minus infinity. So, e to the power of a x when x is equal to minus infinity is 0 and e to the power of minus i omega x as x becomes very large, it just fluctuates between plus 1 and minus 1 because there is an I here. So, this term may not necessarily become 0, but this term at x is equal to infinity this term it becomes 0. So, the multiple of these 2 is 0. So, this is 0, plus 1 over a plus i omega and then again. So, this I have to evaluate from 0 to infinity.

So, as x becomes infinity this term becomes 0 and this term fluctuates between positive and minus 1. So, the multiple of these 2 becomes 0 and then we evaluate this entire expression at x is equal to 0. So, this is minus because of this minus term, times minus 1 I get this. So, this is 1 over square root of 2 pi 1 over a minus i omega plus 1 over a plus I omega. So, what do we get out of here? So, what we get is if I sum them up, I get 2 a over root 2 pi times 1 over a square plus omega square. So, f of omega equals 2 a over root 2 pi times 1 over a square plus omega square.

So, that is our Fourier transform, this expression for this function. Now it happens that this Fourier transform has no imaginary component right, why does it not have an imaginary component something to wonder; this function affects is an even function every value for f of x and f of minus x the values are same. So, what does it mean? So, let us say this is a point x here and this is point minus x here the values are same. So, if f of x equals f of minus x then the function is even what does that mean? When I resolve this into sines and cosines sines by their very nature, they are not even functions they are odd functions and cosine function is an even function, why is it cosine function an even function? Because when I plot it, it is like this, so this is symmetric about x is equal to 0.

So, when I resolve it in its frequency component, it should have only cosine terms it should not have any sine term. So, that is why this function is having no imaginary component. So, any function if it is graphically if we see that it is symmetric about x axis, then it is an even function and then it will have only cosine terms. So, its Fourier transform will be a pure real function, it will not have any imaginary components. So, that closes the discussion for today and we will continue our discussion on Fourier transforms tomorrow as well. So, till then have a great day and bye.