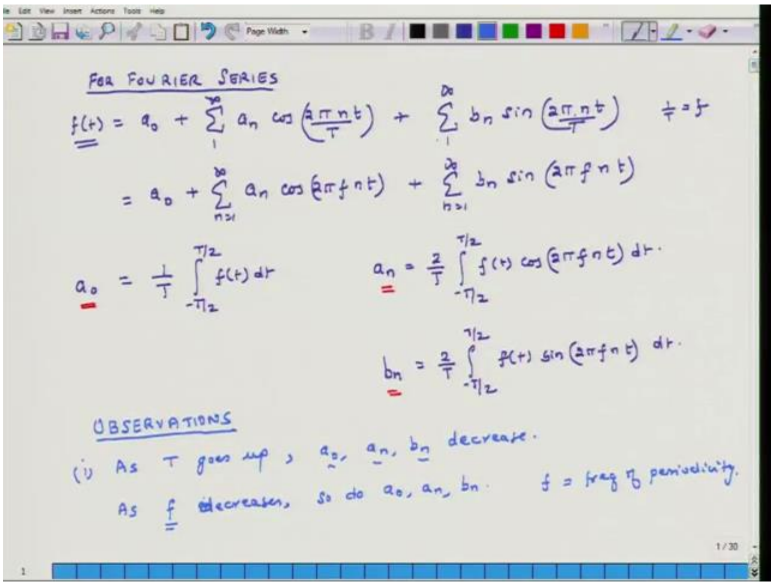
**Fundamentals of Acoustics**
**Prof. Nachiketa Tiwari**
**Department of Mechanical Engineering**
**Indian Institute of Technology, Kanpur**

**Lecture – 55**
**Fourier Transform**

Hello, welcome to fundamentals of acoustics. Today is the start of the 10th week of this course and over this week we will cover Fourier transform and fast Fourier transform for time series signals. So, what these techniques will help us to is convert data which is in time domain into frequency domain. So, last week we had seen that if there is a periodic function and the periodicity of that function is T seconds then we can represent that time series signal in the frequency domain and in the frequency domain we represent that in terms of 2 parameters, one is the amplitude of the signal corresponding to each frequency and the second one is phase of the signal, corresponding to each frequency

So, first what we will do is we will revisit those relations and see what they tell us as we change the time period of the signal.

(Refer Slide Time: 01:31)




So what we had said in the last week is that for Fourier series, if there is a signal which is periodic then its periodicity can be represented as a Fourier series such that, f t equals a naught plus sum of a n; cosine 2 pi, n t over T plus b n sin 2 pi n t over T, so here n is equal to varies from 1 to infinity and same thing here.

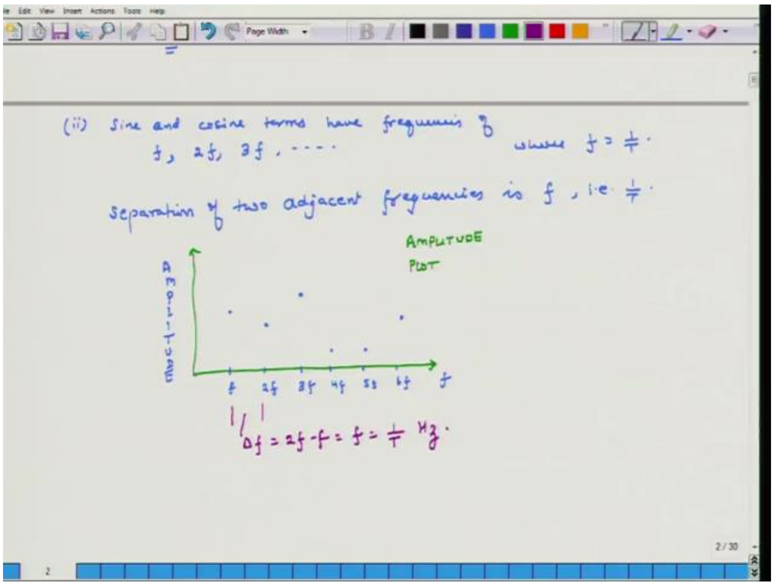
Now, we realize that 1 over T equals f, so another way of writing the same expression is a 0 plus summation of a n cosine 2 pi f; n t plus b n sin 2 pi f n t and n equals 1 to infinity and these coefficients we had defined them, so what are these values? So, if we know f t then a 0 equals 1 over t minus t over 2 to t over 2 f t d t and a n which are the coefficients associated with cosine terms, they are defined as 2 over t minus t over 2 to t over 2. So, I integrating over these limits and the integrated f t cosine 2 pi f n t, d t and b n is 2 over t integrated over the limits minus T to T over 2 f t sin 2 pi f n t d t. So, these are the expressions for a naught, a n and b n and these are constants which are there in the Fourier series.

So, let us make some observations these are important observations because they will be useful subsequently. So, the first observation is that suppose this f t does not change, but time period; its periodicity changes, if only periodicity is changing then, so the first thing is as T goes up; a 0; a n, b n decrease and if all things are same then as time period doubles a 0 a n b n they half and so on and so forth, so this is one important relation, important trend we should understand.

So, an equivalent statement would be as frequency, so what is frequency? Frequency is 1 over T. So, as 1 over t decreases, so do a 0, a n and b n. So what is this frequency? This frequency is frequency of periodicity, so what does that mean? That if the time period of a signal as it becomes larger and larger, these terms a 0, a n, b n they become less and less. So, theoretically if time period is extremely large and if it is theoretically infinity then these coefficients if f t is not changing they will approach 0, this is important thing.

The second thing to note is, so what is the second observation? So, let us look at f t how I have expressed it, I have expressed it as a 0 plus a 1 cosine 2 pi f t plus a 2 cosine 2 pi times 2 f t plus a 3 cosine 2 pi times 3 f t and so on and so forth; plus similarly we have a large number of terms in sins.

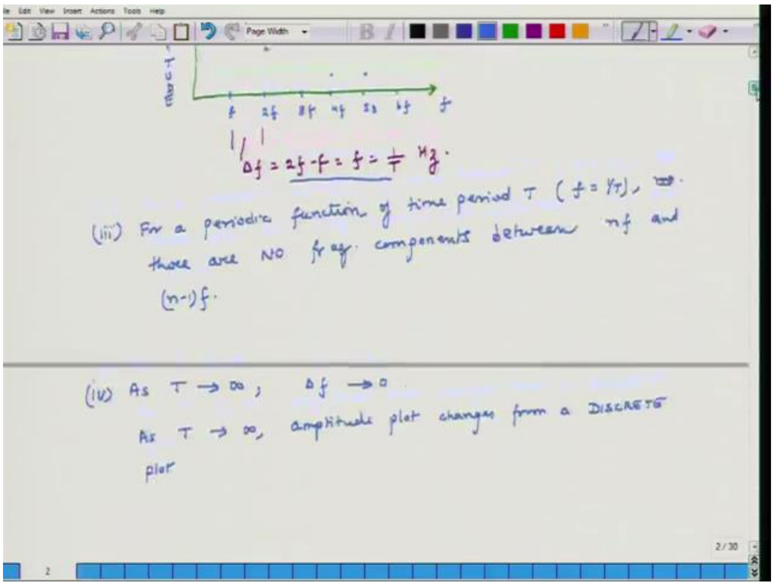
(Refer Slide Time: 08:05)




So, the sin and cosine terms have frequencies of f, 2 f, 3 f and so on and so forth, where f equals 1 over T. So, the fundamental frequency will be f which is 1 over T then the next Fourier component will be 2 f, third Fourier component will be 3 f and so on and so forth. So, what does that mean? So the separation of 2 adjacent frequencies is f that is 1 over T, so, what does that mean? So what graphically what that implies is that if I am doing the amplitude plot for a function which is periodic in nature then it will have of course, it will have an a 0 component which corresponds to 0 hertz, but then it will have its first frequency which will correspond to f, the second frequency will correspond to 2 f, third frequency will correspond to 3 f and corresponding t o each frequency they will be some amplitude and so on and so forth; 4 f, 5 f, 6 f, so this is my f and this is the amplitude.

So, it will be something like this and the separation between these 2 frequencies; 2 adjacent frequencies. So, this is delta f is what; is equal to 2 f minus f is equal to f is equal to 1 over T hertz, now think about it.

(Refer Slide Time: 11:00)

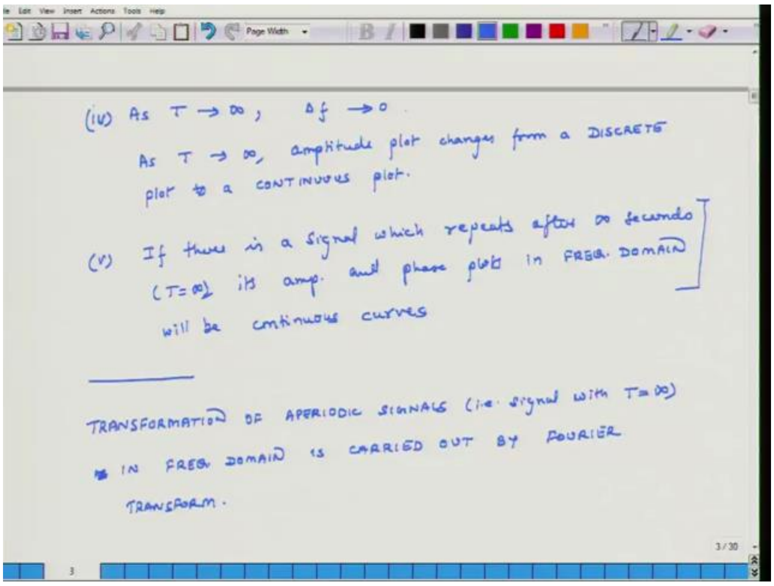



So this separation is 1 over T hertz, there are no frequency components between f and 2 f so, for our periodic function. So, based on this we make an important observation, for a periodic function of time period T and we will define f is equal to 1 over T, there are no frequency components between; let us say n f and n minus 1 f, there are no frequency components. So a periodic function may have infinity frequency components, but between 2 frequency components which are adjacent to each other which are separated by f there are no other intermediate frequency components. Forth observation we make is as time approaches infinity, what happens? This delta f approaches 0, as time approaches infinity delta f approaches 0. So, what that means is that as time approaches infinity this amplitude plot.

Student: (Refer Time: 13:03).

Delta f, so delta; so what is delta f; delta f is this thing, 2 f minus f is delta f, 3 f minus f is delta f. So, it is f and that equals 1 over T hertz, so as time period is approaching infinity; 1 over T is approaching 0. So, what; that means, is that these points become extremely closed and when time period becomes infinity then the separation between them is theoretically 0, so what does that mean? That as time approaches time period approaches infinity amplitude plot changes from a discrete plot. So, what is discrete plot see amplitude plot is right now if time period is non infinity; not infinity then these points are separated by finite distance right and there are no intermediate points.

(Refer Slide Time: 14:00)




But as T becomes infinite then this amplitude plot changes from a discrete plot to a continuous plot, I mean this is what continuous plot. So, this is important and this thing to understand right.

So, if there is a signal which; so what does that mean? So the last thing is if there is a signal which repeats after infinity seconds. So, what does that mean T is equal to infinity its amplitude and phase plots will be then its amplitude and phase plots in frequency domain will be continuous curves. So, why are we discussing all this, so till so far what we have done is that we have discussed that if I have a function which is periodic and its periodicity is T seconds then I can represent it as a Fourier series, but consider a function which is not periodic, which does not repeat itself then I can say that this function is going to; so in one set of words I say that this function is not periodic or I can also equivalently say that this function is going to repeat itself after infinity seconds.

So, what that means is that if I have a function which repeats itself after infinity seconds then I will not get its transformation in frequency domain as just dots, but it will be theoretically a continuous curve. So the transformation of aperiodic signals, so what is aperiodic signal? That is signal with T equals infinity is. So, transformation of aperiodic signals in frequency domain is carried out by Fourier transform. So, what is a Fourier transform do? You have a signal which is not necessarily periodic, you use a Fourier transform and you can find out its frequency components, and at what frequencies you will find out these components? You will find these its components at all possible frequencies. So, you will find it at 0 hertz, 0.1 hertz, 0.001 hertz, 0.0001 hertz and all possible frequency values you will find the, it will have some.

Student: (Refer Time: 18:05).

It will have some Fourier components, why? Because of this understanding that as the time period is between very large, the frequency separation between 2 points is shrinking to 0. So, this Fourier transform helps us achieve this thing. So this sets the background for more detailed discussion on Fourier transform and that is something we will do tomorrow, so till then please digest this basic understanding what is it that we are trying to do and then tomorrow onwards we will discuss Fourier transform in greater detail. So, till then have a great day and we will meet once again tomorrow.

Thank you.