Fundamentals of Acoustics

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Lecture – 52
Time and Frequency Domain Representation of a Signal

Hello, welcome to Fundamentals of Acoustics; today is the fourth day of the ninth week of this course. In the first half of this course we have discussed the mathematics underlying mufflers both of reactive as well as the resistive natures and starting today we will be discussing new area, this is again related to the application of fundamentals of acoustics and what we will learn in the remaining portion of this week as well as in some part of the coming week is how do we represent a time signal into frequency domain that and how do we convert frequency domain signal back to time domain.

So, you will be wondering what is time domain and what is frequency domain, so let us first understand this at a very fundamental level.

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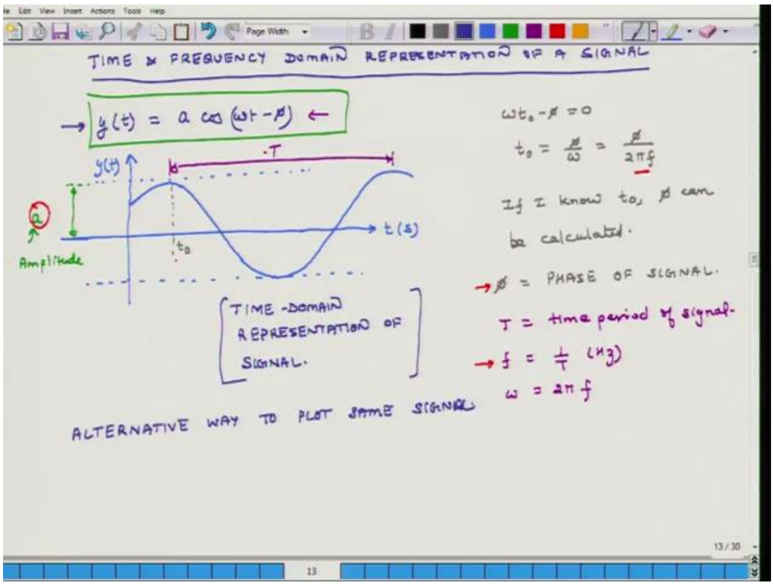

So time and frequency domain representation of a signal, so let us understand what this means and we will start with some very simple signal. Let us say that there is a signal y, which is a function of time and that is equal to a cosine omega t minus phi. So, let us plot this signal, so at time t equals 0 this signal will have a magnitude of a cosine phi at time t is equal to 0. So, it will be somewhere here let us say this is the maximum value, the maximum value of this signal will be a.

So, the signal and as time increases slowly omega t will approach phi and when omega t equal c it will be at its maximum. So, the signal will look something like this. So, on the x axis we are plotting time in seconds and on the y axis we are plotting the magnitude of the signal. Now this, its peak is a and that represents the amplitude of the signal, so we are discussing the very simple sinusoidal or cosine signal. The other thing to notice that this signal is not maximum at time t is equal to 0 rather it is maximum at some instant of time and let us say this time is t naught. So, at what time does this signal get its maximum position, it becomes maximum when omega t naught minus phi equals 0 or when t naught equals what phi over omega and omega is 2 phi f, so it is phi over 2 pi f.

So at p naught it becomes maximum, so if I know. So, the point is that if I know t naught phi can be calculated, so if I know t naught when in this relation I can calculate the value of phi, so that is the second thing and phi is called the phase of the signal, phi is the phase of the signal. The third thing is from this wave form, we know that this wave form repeats itself after t seconds. So, this distance the distance between 2 peaks is T, so T equals what, time period of the signal and f which is the frequency is equal to 1 over T and it is in written in hertz and omega is equal 2 pi f. So, from the plot of this simple wave form which is the harmonic wave form, I can deduce the value of phi, deduce the value of t deduce amplitude and if I have the plot of this function y t is equal to a cosine omega t minus phi.

Now, because I am plotting this signal with x axis we will seconds that is time and y axis is the amplitude this is called such a plot, so this is our function. Such a plot is known as time domain representation, why the time domain because I am plotting y as a function of time. So, if I plot y it does not have to be as sinusoidal signal any signal if I plot representation function of time, I say that it is time domain representation or the signal or time series signal, but I can plot the same information. So y t I can plot this same signal in another way, I can plot it in another way but before we plot it in different way, we should know what are important parameters of this signal the important parameters are frequency, so what are the important parameters.

So, I know everything about this signal if I know the frequency then I can calculate omega, I can calculate t, so this is an important parameter. The other important parameter of the signal is amplitude right and then the third important parameter is this phase. So, if I know that there is a harmonic signal and that harmonic signal and if I know for that harmonic signal its amplitude, its phase and its frequency; I know everything about this signal. So here I have in time domain representation of the signal, I use one graph to depict all these information. So, this is the depiction of T which is inverse of f; t naught will help us calculate phase and amplitude is their explicitly in the graph.

From t naught I can calculate phase using this relation, phi divided by 2 pi f is t naught, so if I know t naught then I can calculate the phase from here. Now I can plot the sole signal in another way and that is the frequency domains representation. So, let us look at, so there is an alternative signal, so what is this alternative way. So, in the alternative way instead of one graph I plot 2 graphs, this is very important it is not that one graph will give us all the information.

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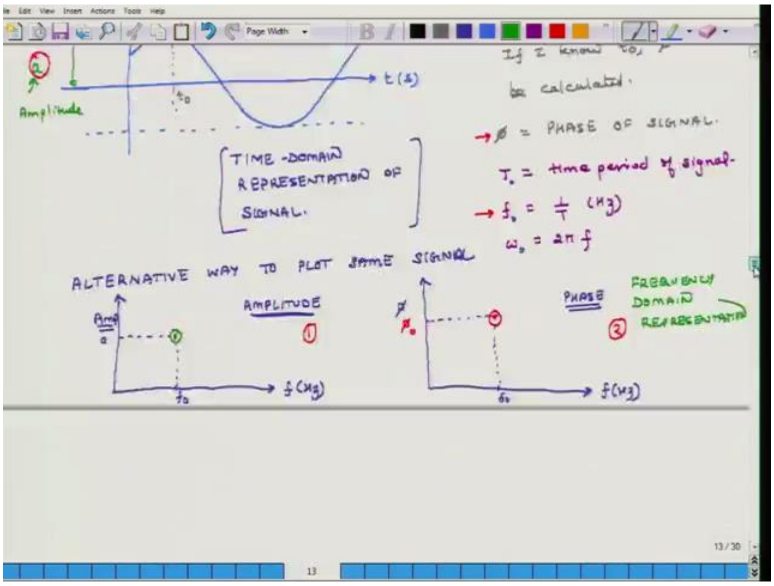

So in the alternative graph we have 2 graphs, the first graph is the plot for amplitude. Now what we do is this one, on the x axis we plot frequency in hertz and on the y axis we plot amplitude. So, this is the graph for the amplitude of the signal and what is the amplitude of the signal of y, what is the amplitude of the signal?

Student: a.

It is a and it is at what frequency? F, so it is at a frequency f next call this to be explicit, we will call it omega naught and we will call it f naught and we will call it t naught. So, add frequency f naught, the amplitude is a, so in the amplitude graph for this signal what do I have; I have just one point, where I have just one point.

And then I have another graph and in the other graph what I do is, I depict the phase of the signal, so it is a phase plot. So, on the x axis it is frequency and on the y axis I plot phase and let us say that the value of this phase is again phi naught, so I am going to plot this phi naught. So, at frequency which frequency phase is; what is the frequency f naught and at f naught the phase of the signal is phi naught. So, from these 2 graphs, so this is graph 1, this is graph 2; I get the same information about this signal y t because the movement I know amplitude and the movement I know phase, I can write down the mathematical expression that y t equals a because a is there on the graph times cosine omega t minus phi.

So, in the first way of represented the signal my x axis was time, so it is all time domain representation. In the alterative way my x axis is on in hertz, so this method this way of representation is called frequency domain representation and we have seen that v is frequency and time domain representations for this harmonic signal they are equivalent right because from each of these signals I can get this equation, y t equals a cosine omega t minus phi. So, let us look at another example, so here we had talked about just one particular frequency where it was omega naught. Let us consider that there is a signal, so we will do another example; example 2.

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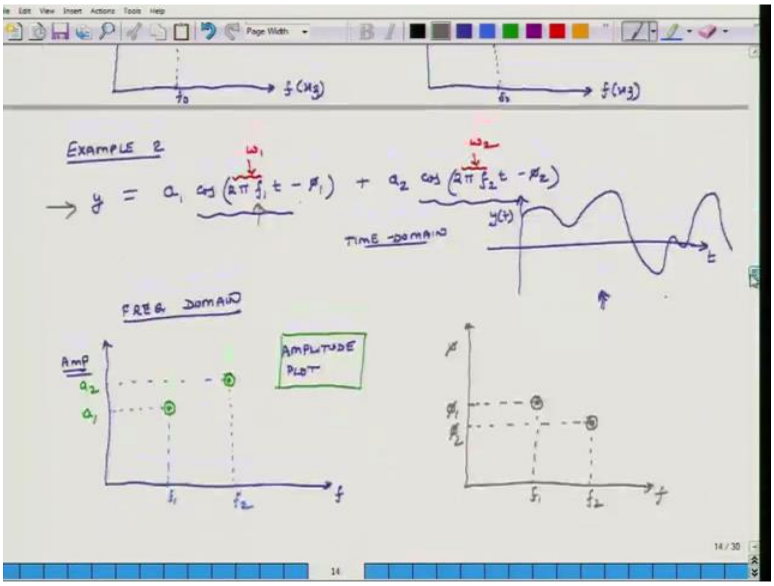

Let us consider y equals a 1 cosine, so instead of omega I am now going to write 2 pi f 1, t minus phi 1 plus a 2; cosine 2 pi, f 2 t minus phi 2. Now this 2 pi f 1 is omega 1; this 2 pi f 2 is omega 2. So, if I have to plot the time series representation or time domain representation then I do not know i mean the graph will look something which will not be explicitly harmonic right, but it is wave the puff to harmonic waves, harmonic signals. So, here I am getting time and in the y axis I am getting y is a function of t right; y is a function of t. From this graph, it is not straight forward to find the time period of each of these frequencies and the phase of each of these frequencies and will if I do a lot of mathematics I can still get from here.

But it is not very clear that how we can quickly get that information, but let us draw it in frequency domain. So in the frequency domain we have how many plots, every signal it has to correspond to 2 plots. So, the first plot will tell us about the amplitude, so this is the amplitude plot. Now in this case what is the amplitude associated with frequency f 1, what is the amplitude of the signal associated with frequency f 1, a 1. So, let us say this is f 1 and the amplitude associated with this frequency is f 1; f 1 is a 1 and what is the amplitude associated with frequency f 2, it is a 2.

So, let us say this is f 2, so frequency amplitude associated with frequency f 2 is a 2 and this is a point. So, this is the amplitude plot of the signal; one question I have is should I connect these 2 points or should I leave just these 2 points as is, why should I not connect these 2 points. So, I should not connect these 2 points because the moment I connect these 2 point, it means physically that at all other frequencies between f 1 and f 2 there are other components cosine signals, but this y t has only 2 components; one component associated with frequency f 1, another component associated with frequency f 2.

So, this type of a curve will have discrete number of points, it will not be a continuous curve. These 2 points can be little is very close or very far, dependent on the distance between f 1 and f 2, but the space between those 2 will not be connected by a line. Similarly, so now we had the amplitude plot and then the other plot in the frequency domain is the phase plot. So, on the y axis, x axis I plot frequency and on the y axis I plot phase and the phase is phi 1, at frequency f 1 and let us say phase 2; at frequency f 2 is phi 2, so this is the frequency domain representation.

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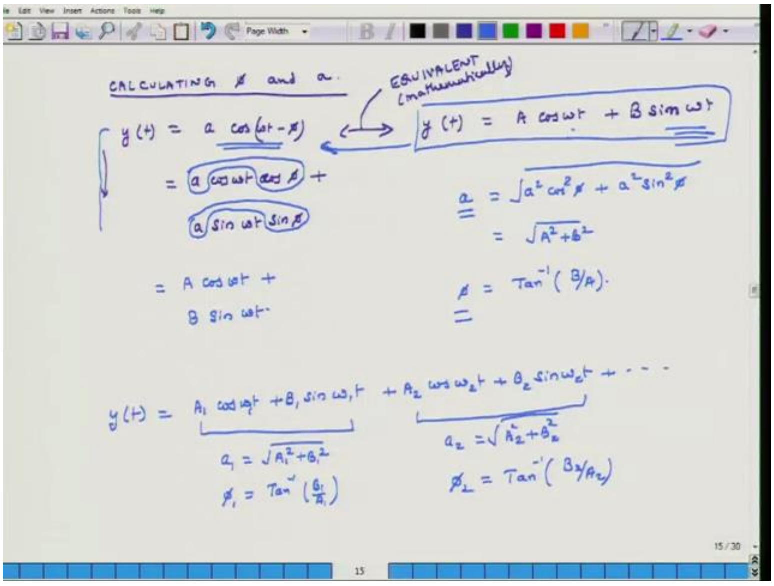

So, with this background I hope that I will be able to explain that one signal in time can be expressed either as a cosine as just a free time domain signal or we can also representing frequency domain. Calculating phi and a, this is the other part. So a lot of times; several times we write y t as; suppose there is one signal which has just one single frequency. So, I write it as a cosine omega t minus phi, lot of times we do not write the signal like this rather we write in an in some other way and we write y t is equal to a cosine omega t plus b sin sin omega t.

Both these representations are equivalent and it is important to understand why is that, so they are mathematically equivalent. So, they are mathematically equivalent; let us see why is the case, so what I do is I develop this further. So, this equals a cosine omega t times cosine of phi plus a, sin omega t sin of phi now I can, so phi is a constant. So, cosine phi times a is a constant, so will be a times sin phi is a constant. So, this I can write it as A cosine omega t plus B sin omega t. So essentially what I have done is, I have transformed this expression into the alternative expression, and what is the relation between capital A and small a? So, from here we see that a is equal to a cosine square phi plus a sin square phi a square the whole thing under route and this is equal to a square plus b square.

So, that is the relation between capital A and capital B, and phi is equal to what? Tan inverse of B over A. So, the point is that if you run into this type of an expression then from this you should be able to calculate and transform this relation into this type and from this type you should be able to develop a representation of signal y t in the frequency domain because from here you can calculate a, you can calculate phi and then from that you should be able to compute the frequency domain representation of the signal. So, there are there is signal y t equals A 1, cosine omega t plus B 1 sin omega 1 t plus A 2 cosine omega 2 t plus B 2 sin omega 2 t plus and so and so forth.

Then for this guy amplitude of the first signal is what a 1 square plus b 1 square and its phase is Tan inverse b 1 over a 1, for this guy amplitude is a 2 plus b 2 both squares under root and phi 2, so this should be phi 1, phi 2 equals Tan inverse b 2 over a 2 and so and so forth. So, all these understanding is very important because when you go into deeper into just time and frequency domain you should understand where we are coming from, what is it that we are trying to do.

So, I think what we will do is we will stop at these point of time and starting tomorrow we will develop this idea further and then tomorrow what we will discuss is; for a series representations of time domain signals. So, with that we close for today and we will meet once again tomorrow bye.