

**Fundamentals of Acoustics**  
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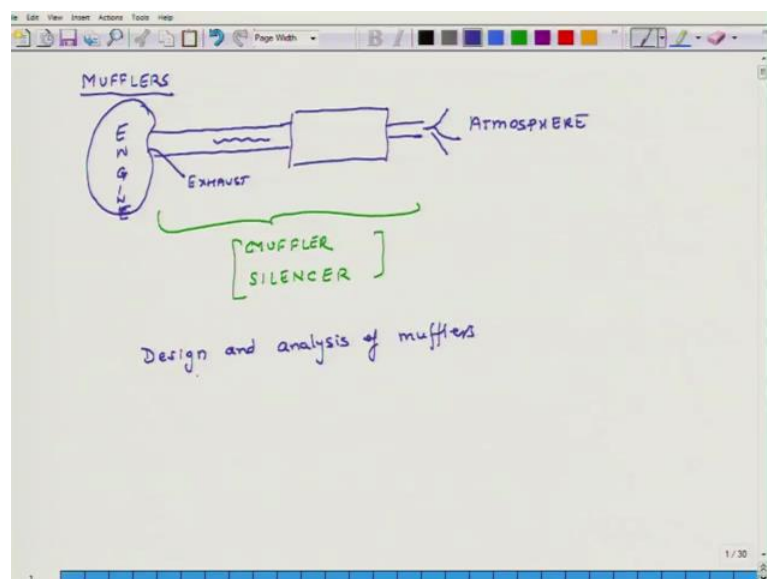
**Lecture – 49**  
**Mufflers**

Hello. Welcome to Fundamentals of Acoustics, this is the 9th week of this course. And this week till the end of the course what we will be focusing on will be a variety of applications of whatever knowledge we have learned till so far. And this process happened started happening last week itself.

So, last week we had discussed an actually week before that we had discussed how to measure impedance of materials which are not known, and then we also discussed about impedance for different types of tubes. And then last week we started discussing about how to model muffler. And in that particular context we discussed the 3 media problem, because once we understand how to solve the 3 media problem then we can use that knowledge to design and analyze a muffler.

So, all that development of understanding happened last week. So, we have developed the relation which helps as find out transmission loss in our fluid as sound propagates from media 1 to media 2 to the 3rd media. So, after doing that now we are ready to start talking about mufflers. So that is what we will do in the first part of this week.

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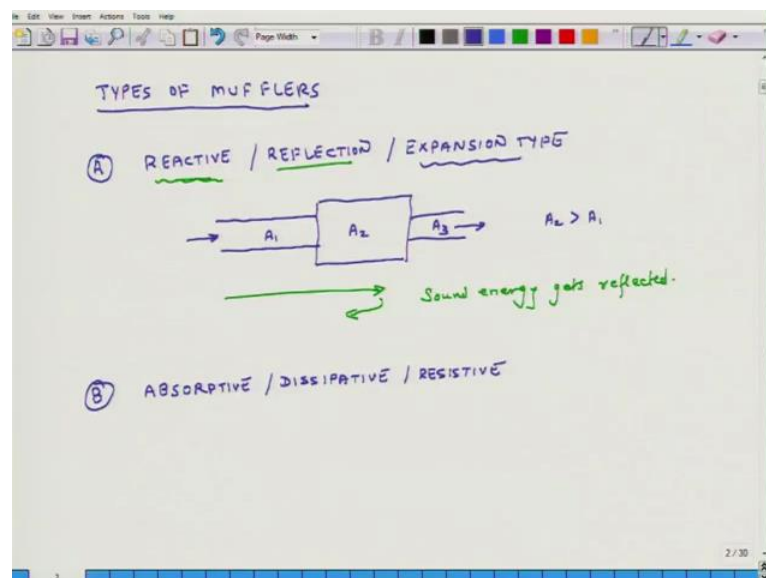


So, our focus will be Mufflers. Now what is the muffler? So, in a lot of cars or motor cycles you would have seen that from the engine there is a long tube comes out. So, let say this is your engine and let say this is the exhaust area. So, all exhaust gases they come out of a pipe and then you would have seen that there is another factor or wider pipe connected to it and then there is a small pipe. So, the gases flow out. So, this is your atmosphere. So, this entire contraction is called Muffler.

So why is it called muffler, because it muffles; so the sound which is coming out of engine is extremely loud because the pressures in the engine are extremely high. So, if those pressures get released into the environment then that can generate very significant levels of sound. So, what muffler does is it reduces the amplitude these pressure fluctuations such that the sound which you finally hear is not extremely loud, it may still be loud but it is not extremely loud so that it can hurt your ears or it can be significantly annoying.

So, that is why it is called muffler, because it muffles or suppresses the sound. Another name for this is specifically in our country is Silencer. So, what we will discuss today is design and analysis of mufflers. Now there are several types of mufflers.

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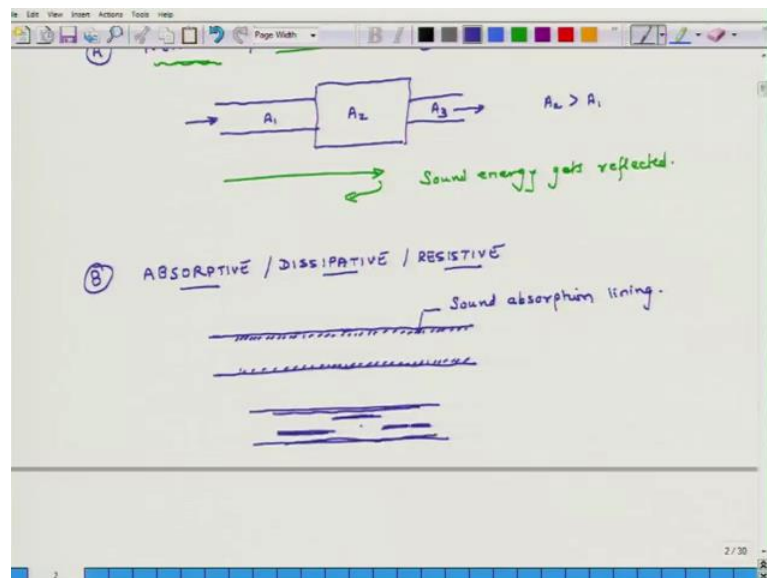


So, the first category of muffler they are known as Reactive Mufflers, this is the first category. Several times these reactive mufflers are also called Reflection Mufflers. And further in certain cases they are also known as Expansion Type Mufflers. So, how do

they look like? Typically the shape of these mufflers it looks like this. Here the gases enter from the engine and then you have a wider pipe or a cylinder whose cross sectional area is significantly larger than the input and output and then the gases exit out in the atmosphere. And because these cross sectional area  $A_2$ , let say this is  $A_1$  and this is  $A_3$ . So, because  $A_2$  is larger than  $A_1$  the gases expanded it so that is why it is known as Expansion Type muffler, because the gases can expand limit.

And then there are known as reactive mufflers or reflection mufflers, because the sound energy when it passes through ear through the muffler it does not get absorbed by the muffler, but it gets reflected. So, sound energy gets reflected. So, that is why these mufflers are also known as reflection type mufflers. And they are also reactive because, I mean either the sound energy has to get reflected or it has to get dissipated. So, it gets reflected if there is reaction occurring or reactants in the system. So, whenever you have reactance in the system using the electrical analogy or electrical world then all the energy does not get transmitted outwards. So, that is why they are known as reactive type of mufflers.

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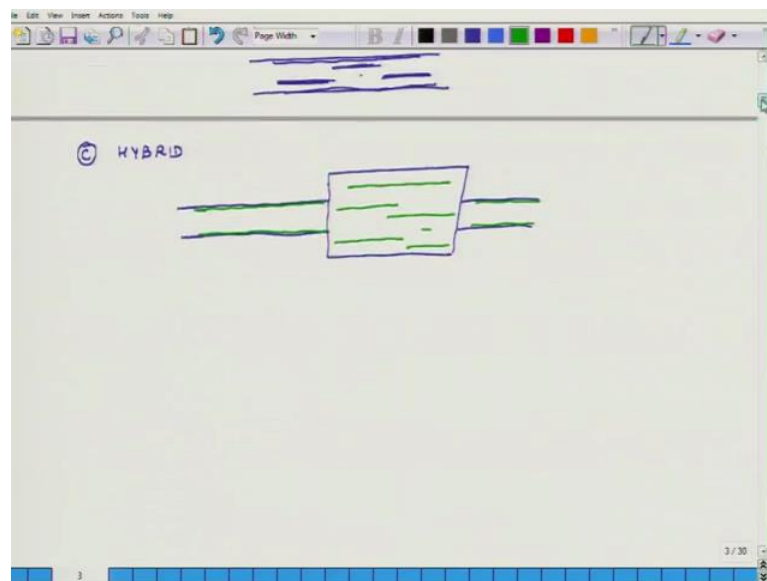


Now, then there are this another category of mufflers and they are known as Absorptive Mufflers or some people also call them Dissipative Mufflers or some people call them Resistive Mufflers.

So what happens in these types of mufflers? Well, you can have a very simple shape, but the inside lining or the inside surface of this pipe. So, this is the simplest topology for dissipative or absorptive or resistive type of muffler. So, the inside surface of this pipe has a sound absorption lining. So, as sound passes through this sound gets absorbed with by this lining and that is why it is known as either absorptive muffler or dissipative muffler, because the sound energy as it travels through pipe it gets dissipated, it gets converted into heat. And they are known as resistive, because the same way happens in resistor as using pass through resistor it gets converted into heat. So, that is why actually what is happening in these resistive types of mufflers.

Now this is a topology which have made which is looks extremely simple, but resistive mufflers may have several layers of resistive lining. So, each of these baffles may be lying with some sound absorbing material. So, as sound passes through it on several surfaces sound gets absorbed. So, these are Absorptive Mufflers.

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Then the third category would be Hybrid Mufflers. So, I just show you. So in hybrid mufflers we rely on both reactance and reflection of the sound energy as well as absorption. So, in these types of mufflers you can still have a topology like this, so this will call calls reflection. And then you can have sound absorption materials, you can have even like this. So, as sound travels through the entire structure it not only gets

reflected back, but whatever is getting out that also gets absorbed. So, these are hybrid muffler.

So, these are all the several types of mufflers. So, what we will do today is we will start the discussion on expansion type of mufflers, and then we will also briefly cover there is resistive type of mufflers in our discussion.

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**EXPANSION MUFFLERS**

Diagram: A chamber of length  $l$  is divided into three regions. Region 1 (left) has cross-sectional area  $A_1$  and density  $\rho_1$ . Region 2 (middle) has cross-sectional area  $A_2$  and density  $\rho_2$ . Region 3 (right) has cross-sectional area  $A_3$  and density  $\rho_3$ . Sound waves are shown entering from region 1, reflecting back, and transmitting into region 3.

$$T = \frac{4 \alpha_{13}}{(\alpha_{13} + 1)^2 \left[ 1 - \frac{(\alpha_{23} - 1)(\alpha_{12} - 1) \sin^2(k_2 l)}{(\alpha_{13} + 1)^2} \right]}$$

**ASSUMPTION:**  $A_1 = A_3$      $A_2/A_1 = m$

To simplify  $T \rightarrow$  we need to find  $\alpha_{13}, \alpha_{23}, \alpha_{12}$ .

**FROM LAW OF MASS CONSERVATION**

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$\rho A_1 = \rho_2 A_2$$

$$\rho_2 = \rho_1 (A_1/A_2) = \rho/m$$

$\rho_1 \rightarrow$  Total density.  
But  $v_1 = v_2$ .

So, we will start our discussion by considering expansion mufflers. And the typical geometry we will consider will be of this type. So, sound ear is coming in from this end it is getting out from here, let see this cross sectional is  $A_1$ , cross sectional area of the chamber so we call this chamber is  $A_2$ , sectional area here is  $A_3$ ; and let say that the length of this chamber is  $l$ . So, as air is passing through portion one it goes into portion two and then goes out from third portion it is essentially passing through three different media. So, the mathematics of house and sound travels to this type of a single chamber expansion muffler is dictated by the 3 media with solution for that 3 media problem. So, let say in this portion, so let us call these portions region 1, this is region 2 and this is region 3. So in region 1 we have density as  $\rho_1 c_1$ ; in region 2 we have density and velocity sound as  $\rho_2 c_2$ ; and region three it is  $\rho_3 c_3$ .

Now, for purposes of analysis we will make some simplifications, but the basic equation for  $T$  which is the transmission coefficient that is equal to  $4 \alpha_{13}$  divided by  $(\alpha_{13} + 1)^2$  times  $1 - \frac{(\alpha_{23} - 1)(\alpha_{12} - 1) \sin^2(k_2 l)}{(\alpha_{13} + 1)^2}$ , so this is  $\alpha_{23}^2 + 1$  actually  $\times$  is there it

is  $r_{23}^2 \sin^2 k_2 l - 1$ . And the other one is  $r_{12}^2 \sin^2 k_2 l$  divided by  $r_{13}^2 + 1$  whole square. So, that is the transmission coefficient for this kind of a muffler. And if we know the parameters  $\rho_1 c_1$ ,  $\rho_2 c_2$ ,  $\rho_3 c_3$  then we can easily compute the transmission ratio for this kind of a muffler.

Now, for purposes of our today's analysis we will make some assumptions which make the relation for T simpler. So, we assume; so what do we assume? We assume that  $A_1$  equals  $A_2$ . So, that is one assumption we make. And the other assumption we make is that the ratio of  $A_2$  over  $A_1$  is equal to  $m$ , some number  $m$ . So, the cross sectional area  $A_2$  divided by cross sectional area of the first section  $A_1$  is  $m$ . And the first assumption was that  $A_1$  equals  $A_3$ . So, with this these two sets of assumptions we will try to compute  $\rho_2 c_2$ ,  $\rho_3 c_3$  in terms of density and speed of sound for the first section.

So, to simplify relation for T we need to find  $r_{13}$ ,  $r_{23}$ , and  $r_{12}$ . Unless we do not know these we cannot compute the value of T. So, that is what we do. So, what we know from mass conservation that as air moves from section 1 to 2 and it goes from section 2 to 3 the total amount of mass which is entering in section getting out of section 1 is equal to the flow in section 2 also. So,  $\rho_1 A_1 V_1$  velocity 1 is equal to  $\rho_2 A_2$  times velocity 2.

Now, please realize that here  $\rho_1$  is total density. You are typically an acoustical system we just worry about the change in the density, but here  $\rho$  represents the total density of  $\rho$  only then this relation will be correct. And  $V$  is the velocity. Now, at the interface of 1 and 2 velocity in section 1 because of continuity has to be same as velocity 2. So we know that, but  $V_1$  equals  $V_2$ . So, that is the case then  $\rho_1 A_1$  equals  $\rho_2 A_2$  or  $\rho_2$  equals  $\rho_1 A_1$  by  $A_2$   $\rho_1 A_1$ .

So,  $\rho_2$  is equal to; so this gives me  $\rho_1$  by  $m$ . Well, I mean this velocity it is moving uniformly in first part then it is moving that is constant velocity, so what we do not worry in this case is that there may be some (Refer Time: 18:25) which are getting formed found here. So, we do not worry about those we think that the flow is happening uniformly, so this transformation is. So, with that understanding; so the density in medium 1 and density in medium 2 is of by this factor. So, similarly we can say  $\rho_3$  equals  $\rho_2$  times  $m$ . So, let us call these relations 2A, this is 2B.

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To simplify T  $\rightarrow$  we need to find  $v_1, v_2, v_3$ .

From LAW OF MASS CONSERVATION

$$\rho_1 A_1 v_1 = \rho_2 A_2 v_2$$

$$\rho A_1 = \rho_2 A_2$$

$$\rho_2 = \rho_1 (A_1/A_2) = \rho/m \quad T \text{ (A)}$$

$\rho_1 \rightarrow$  Total density.  
But  $v_1 = v_2$

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Similarly,  $\rho_3 = \rho_2 m \quad T \text{ (B)}$

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$$\rho_1 v_1 = \rho_2 v_2 \rightarrow \rho_2 = \frac{v_1}{v_2} \rho_1 = \rho_1 \frac{A_1}{A_2} = \frac{\rho_1}{m}$$

Similarly  $\rho_3 = \rho_2 m$

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SUMMARY:  $\rho_3 = \rho_2 m \quad \rho_2 = \frac{\rho_1}{m} \quad \rho_3 = \rho_2 m \quad \rho_2 = \frac{\rho_1}{m}$

Now the other thing is that as fluid is flowing, so there are two phenomena happening; one is that the fluid is flowing from section 1 to section 2 to section 3. So, fluid is flowing like that.

The other phenomena are that while fluid is flowing like that in that fluid sound is also propagating. So while fluid is flowing like that in the fluid sound is also propagating, and there is also some DC flow of fluid and low frequency flow of fluid. Now as fluid flows through it go from part 1 or section 1 and then it expands into section 2, and then again it gets contracted into section 3. And this expansion happens it does not happen rapidly it happens at a slope phase and then again it also contracts as it goes into the third section.

So, in a overall since the fluid also expanding and contracting and it is happening slowly and because it is happening slowly the overall process in the system is isothermal, because when it expands slowly the temperature goes down and all the extra heat from outside comes in to compensate for the temperature. So, the expansion and contraction of the overall gas when it flows is an isothermal process, but in that on top of that process sound is travelling and the propagation of sound is an adiabatic process.

So, what we are trying to find out now is the overall pressure of the system in section 1 section 2 and section 3 and that will depend on the isothermal process. So for an isothermal process we can write  $P_1$  which is the total pressure in part 1 times volume is equal to  $P_2 V_2$ , right. This gives me  $P_2$  is equal to  $V_1$  by  $V_2$  times  $P_1$ . So, here

capital  $V$  is the volume. Now in one second the volume will be proportional to the area. So, this I can write it as  $P_1 \text{ times } A_1 \text{ over } A_2$  is equal to  $P_1 \text{ over } m$ . Similarly,  $P_3$  is equal to  $P_2 \text{ times } m$ .

So, we can write 4 important relations. Let us summarize; so what is the summary?  $P_3$  equals  $P_2 \text{ times } m$ , then  $P_2$  equals  $P_1 \text{ over } m$ , and  $\rho_3$  equals  $\rho_2 \text{ times } m$ , and  $\rho_2$  equals  $\rho_1 \text{ over } m$ . So, what we will do is going forward we will use these relations to compute these parameters  $\rho_1 c_1$ ,  $\rho_2 c_2$ ,  $\rho_3 c_3$  then from those parameters we will calculate  $r_{23}$ ,  $r_{12}$ , and  $r_{13}$ , and then we will find the value of transmission coefficient. So, that part of the thing we will do in our next class, and till then have a great day and we will meet once again tomorrow.

Thank you.