Fundamentals of Acoustics
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Lecture – 48
3 Media Problem - Special Cases

Hello. Welcome to Fundamentals of Acoustics. Today is the last day of the current week which is the 8th week. Yesterday we had developed an expression for T for a 3 media problem and specifically we had defined T as the ratio of transmitted energy intensity through incident and intensity for the system. And the ratio which we had developed looked something like this.

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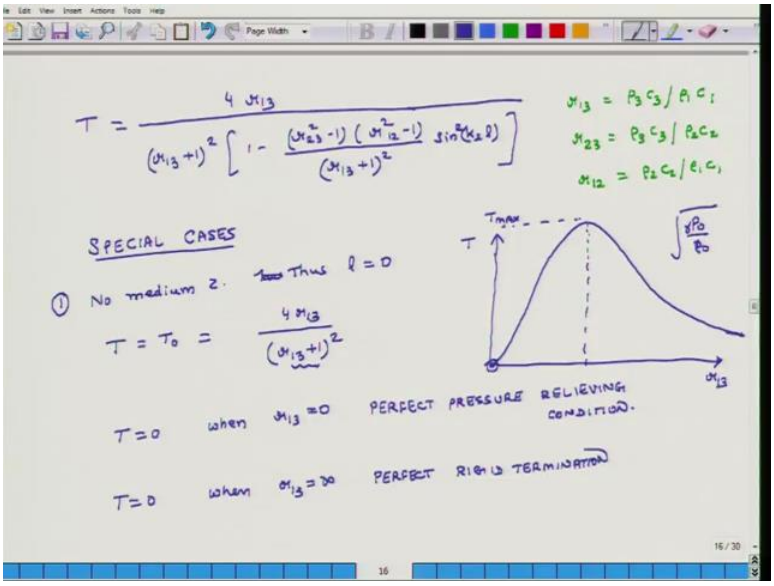

So, T equals 4 r 13 divided by r 13 plus 1 whole square times 1 minus r 23 minus 1 square times r 12 minus 1 square divided by r 13 plus 1 whole square sin square k 2 l. And r 13 equals rho 3 c 3 divided by rho 1 c 1, r 23 equals rho 3 c 3 divided by rho 2 c 2 and r 12 equals rho 2 c 2 divide by rho 1 c 1.

Now, we will look at some special cases of these 3 media problem. So, in a 3 media problem we have media 1, medium 2, medium 3, and the first case is. So, we look at two special cases for starters. So, in the first case we say that there is no medium 2. So, then it becomes a two medium problem if there is no medium 2 then the value of l becomes 0. So, here there is no medium 2. Thus, l equals 0. So, if l equals 0 then I say that when l is equal to 0 then T is equal to T 0 and this equals in the numerator I still have 4 r 13 divided by r 13 plus 1 whole square. Let us plot this.

From the x axis I am going to plot r 13 on the y axis I am going to plot T, the value of T. As at x or at when r 13 is equal to 0 T is 0, and when r 13 is extremely large then once again T becomes 0; when r 13 is very large this denominator is or approximately equal to r 13 square and that r 13 square cancels with r 13, so in I have still left with r 13 in the denominator. So, the overall plot it looks something like this and this is an asymptotic curve and it approaches the x axis at and it meets the x axis when r 13 is infinite. So, this is our T max.

So couple of things: T is equal to 0 when r 13 equals 0, what does this mean? What it means is when r 13 is equal to 0 either rho 3 0 or c 3 0 or rho 3 0 c 3 0 that is how we have defined r 13. So, either the air is very light or the speed of sound in this air is extremely small, so either the air is very light or the speed not speed of light the speed of sound in the air. So speed of sound is what, P naught over rho naught gamma. So, maybe the pressure in the air is extremely small then also, but density could be reasonable but a pressure is extremely small then also it could be the case.

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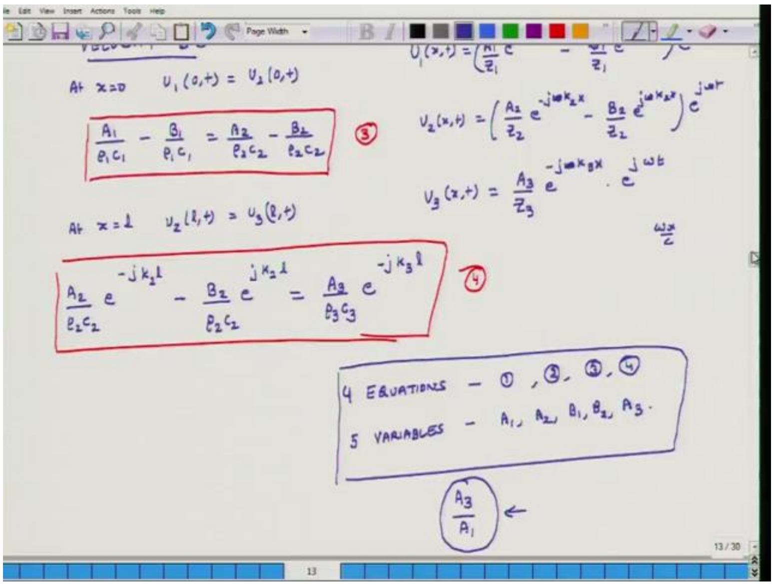

So, what happens in this case? So, what that means is physically what happens. So, when T is 0 it means that there is no energy entering the third medium, there is no entry energy entering the third medium because T is the ratio of exciting energy and input energy.

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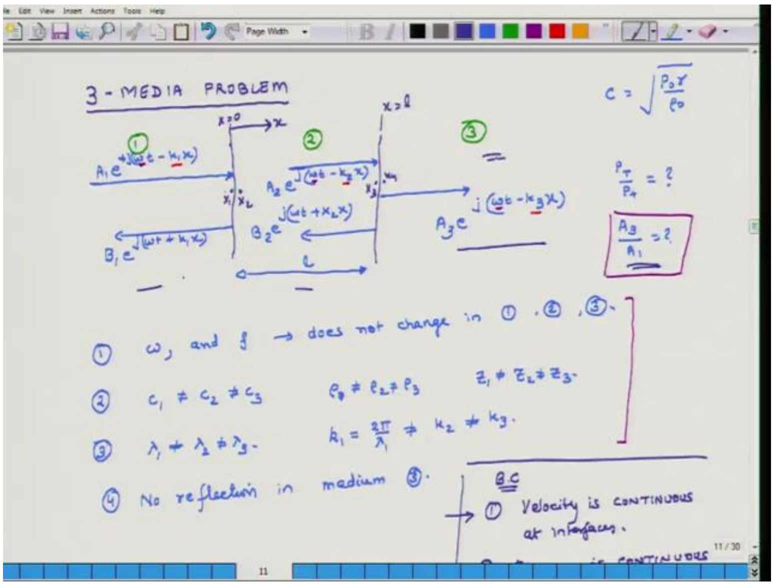

So, what that means is that when in this portion when either c 3 or rho 3 0 then whatever energy comes to this interface nothing gets out. One condition could be if pressure is 0 or pressure is extremely small then whatever fluctuations or pressure I have in media two at the interface they will become 0, because of this continuity of pressure condition right. So T 0 when r 13 is 0; so this type of a condition is known as Perfect Pressure Relieving Condition. What that means is that whatever pressure is there in the first medium at the interface it gets relieved and it becomes 0, so that is the first condition.

The other thing is that T is equal to 0 when r 13 equals infinite, r 13 becomes infinite when does it happen when either c 3 becomes infinite or rho 3 becomes infinite. So, either I have a very dense medium or c 3 is infinite which means the pressure in the system is extremely high. So what does that mean? That when some sound comes and it hits it is essentially hitting like a rigid to a rigid wall, so nothing gets transmitted to the second side. So, this kind of a situation is called Perfect Rigid Termination. So, either you have a perfect relieving condition where pressure is extremely small or you have a perfect rigid termination where P naught is extremely high; in both these cases the transmitted energy to the system is 0.

Now, when you go back we will look at some of the examples which we had done for open tube and a closed tube. What happens in a closed tube? You have perfect rigid termination. So, no pressure gets conveyed to the outside world, why because it is perfectly rigid there is no pressure going outside of the system. Similarly, in an open tube it is perfect pressure relieving condition, so no sound energy theoretically leads into the outside system, because there the overall p is 0. So, this is the first situation related to this graph that at when r 13 equals 0 and when r 13 is infinite the transmission factor it becomes 0.

Now, we look at the situation where what is the maximum value.

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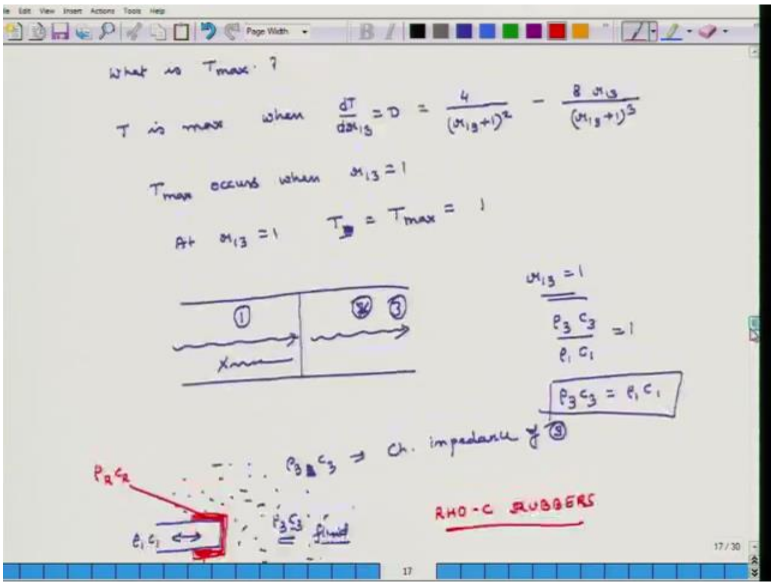

So, what is T max? Now theoretically T max cannot be what, more than 1 whatever energy is coming it cannot have more than energy getting transmitted back. So, T is max when dT over d r 13 equals 0. So, I differentiate this relation. And what does that give me? So that gives me it is 4 over r 13 plus 1 whole square minus 8 r 13 over r 13 plus 1 cube. So, when I do the math I find that T max occurs when r 13 equals 1, and at r 13 equals 1 T is equal to T max and its value is 1. Because, of this relation 4 r 13 when r 13 is 1 then it is numerator is 4, denominator is 4.

So, what does that mean? What that means is now this special case is a 2 media problem. So what does that mean? That if you have 2 media this is the boundary and r 13 equals 1 it means that whatever sound energy is coming in nothing is getting reflected back, all of that energy is getting transmitted. R 13 equals 1 means rho 3 c 3 over rho 1 c 1 equals 1 which means rho 3 c 3 is equal to rho 1 c 1. So, if there is medium 1 there is medium 2 and they can have their individually different densities and different values of velocities or sound, but if the product of these two entities for both media is same then r 13 will be equal to 1, and all the energy which is coming will get unified you know easily get it will get in easily transmitted.

So what is rho 3 c 3? Rho 3 c 3 is what its characteristic impedance of 3 right; and simply rho 1 c 1 is characteristic impedance of media 1. And in electrical engineering those of who have an electrical background you would recognize that when there is matching of impedance then all the incident energy gets easily transmitted to the other direction. If the impedance does not match then there is reflection and all the energy does not get transmitted. So, here the impedance of medium 1 and medium 2 they are matching. So, there is no problem in transmitting all the energy.

Medium 1 or medium let us call this medium 3 for consistency so that is the case now. Where would be like to use this kind of a situation. So, suppose you have a transducer and suppose there is this water or some fluid medium. So, this is fluid and this transducer is vibrating back and forth so it will emit some sound waves. Now all of the energy which is emitted by this sound wave will not get transmitted to the fluid, because this is like a two medium problem. This energy will get transmitted to the fluid 100 percent only if rho 1 c 1 for the transducer is same as rho 3 c 3 of the medium.

Now, a lot of times these transducers are made from quartz crystals and for sea water applications if I have to generate sound in sea then this is rho 3 c 3 corresponds to sea water. So, the quartz crystals impedance and the impedance of sea water it does not match, so what people do is, to make sure that they match they put some rubber and they design the rubber in such a way that its rho rubber times c rubber it matches that of sea water. So, in that case the impedance matches and all the energy gets easily transmitted to sea water. So, these make the transducer very efficient. These types of special rubbers are called Rho -c Rubbers, because their material properties are designed in such a way to meet that of sea water. So this is case 1, now let us look at case 2.

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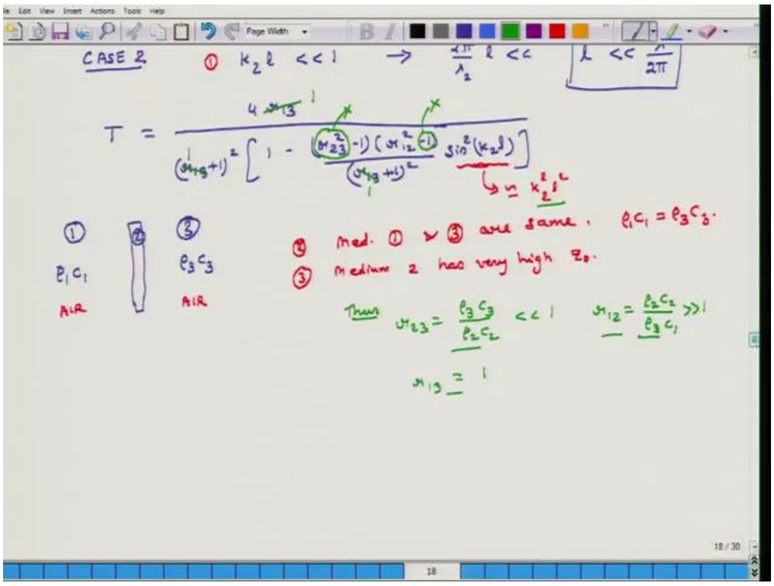

So what is case 2? In case 2 we will say that k 2 l is very small compared to 1. Where does k 2 l come? So first we will write down the expression for T one more time. So, T equals 4 r 13 divided by r 13 plus 1 whole square times 1 minus r 23 square minus 1 r 12 square minus 1 divided by r 13 plus 1 whole square sin square k 2 l.

So, k 2 l is coming in this term; when k 2 l is very small compared to 1 it means what is k 2, it is lambda 2 where excuse me. So, k 2 is 2 pi over lambda 2 times l is very small compared to l, which means that the length of the second medium is very small compared to one sixth of lambda. So, if this condition is satisfied then this term it does not approximate 0 when sin theta is very small if I make it 0, then everything becomes 0 if sin theta is small then it this approximates k 2, but this sin square so it becomes k 2 square times l square. So, this is one thing,

The second thing is that; so this is one assumption simplifying assumption, the second assumption we make; so this term goes away and the second assumption we make. So, before we make the assumption we let me define it. So, in the 3 media problem here what we have is that the width of this medium 2 is very small in the sense that its length is very small compared to the one-sixth of the wave length. The second thing is that rho 1 c 1 and rho 3 c 3 are same. So, what does that mean that medium 1 and medium 3 are same. So, that is the second assumption. So, medium 1 and 3 are same which means rho 1 c 1 equals rho 3 c 3. This is the second assumption. So, if that is the case then what happens to all those r 23 r 12 and so on and so forth? So, r 23 is equal to rho 3 c 3 divided by rho 2 c 2 rho.

And the third assumption we make is, so at this stage we also assume that medium 2; I am going to erase this right now because I will write it later so I am making three assumptions. So, medium 2 has very high characteristic impedance relative to medium 1 and medium 3. So, what could be an example? So, I could have an air here, I can have air here and in between it could be wood. So, in that case, so because of these three assumptions I can write; thus r 23 which is the ratio of rho 3 c 3 divided by rho 2 c 2. So, this is very small compared to 1 r 13. So, there is no r 13, r 12 is equal to what; rho 2 c 2 divided by rho 3 rho 1 c 1. So, this is extremely large compared to 1 and is there r 13.

Student: r 13.

R 13 so that is equal to 1. So, three assumptions in case 2; one is the middle portion is very thin, second thing is medium 1 is same as medium 2, third thing is medium 1 and medium 2 are same, but they are not only same medium 2 is very stiff compared to medium 1 and medium 2. So, because of these assumptions we get these approximations. Once we have done this r 13 equals 1. So, this is 1 r 13 equals 1 so I put 1 here, r 13 equals 1 so I put 1 here. Sin square k 2 l can be approximated as k 2 square l square. Now let us look at r 23 minus r 23 minus 1 r 23 is very small compared to 1. So, this term I can omit. So, I will be left with just minus 1.

And the other one is r 12 is very large compared to 1. So, I can omit this term.

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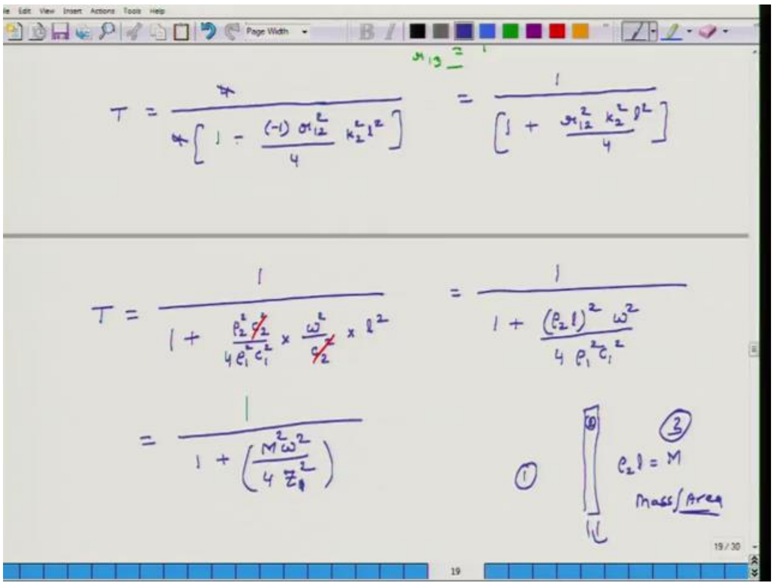

So, with these simplifications I get T equals 4 divided by 4, I get 4 from here times 1 minus, minus 1 r 12 square divided by 4 k 2 l square. So, 4 4 cancels out so I get T equals 1 over 1 and this minus 1 and minus 1 becomes plus, so plus r 12 square k 2 square l square by 4.

So, T equals 1 over 1 plus. So r 12 is what? Rho 2 c 2 over rho 1 c 1 and its r 12 square so squares. And then there is a 4 in the denominator times k 2, k 2 is angular frequency divided by c 2 either it you can write it as 2 pi over lambda or omega over c 2, and it is omega square over c square times l square so this c square gets cancelled out. So, what I am left with is equal to 1 over 1 plus rho 2 l square omega square divided by 4 rho 1 c 1 square.

Now, what is rho 2 l? Rho 2 is the second medium l is its width. So, what is this rho 2 l is equal to m which is mass per unit area. So, this is equal to 1 over 1 plus m omega square divided by 4. What is rho 1 c 1? So this is medium 1, this is medium 3. So, this is Z naught square m square; m square omega square by 4 Z 1 square.

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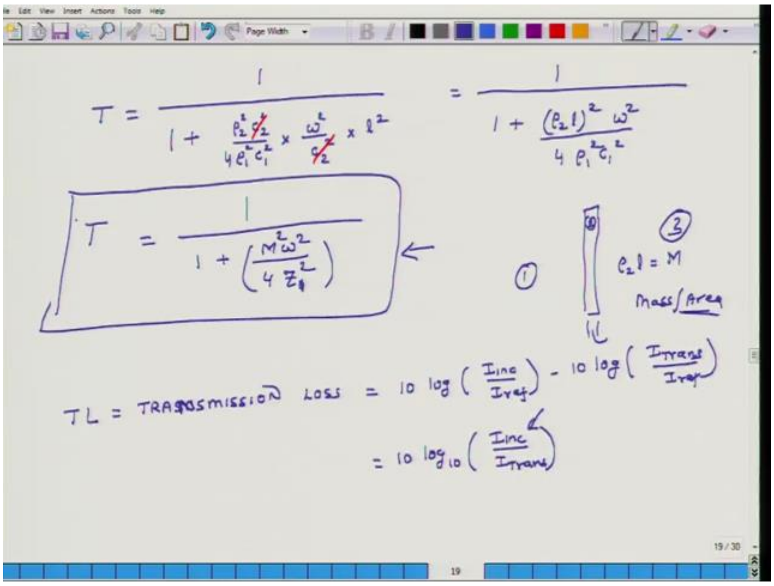

So, that is my this relation is exactly the same and the relation which we had developed for velocity if I square the velocity one of the air in the earlier problem which I had addressed in this week where you had a wall and we were interested in finding out the pressure as well as velocity at the transmitted end then the velocity if I square the relation for that and normalize it I will get this expression, you should go back and check that.

Now, I will define another parameter TL. So TL is transmission loss, because what I am really interested in is how much energy is getting lost so transmission loss and that is equal to 10 log of I incident divided by I reference minus 10 log of I transmitted by I reference so that equals log and both all these things are base 10 I incident divided by I transmitted.

Now this ratio is inverse of T, T means I trans divided by I incident.

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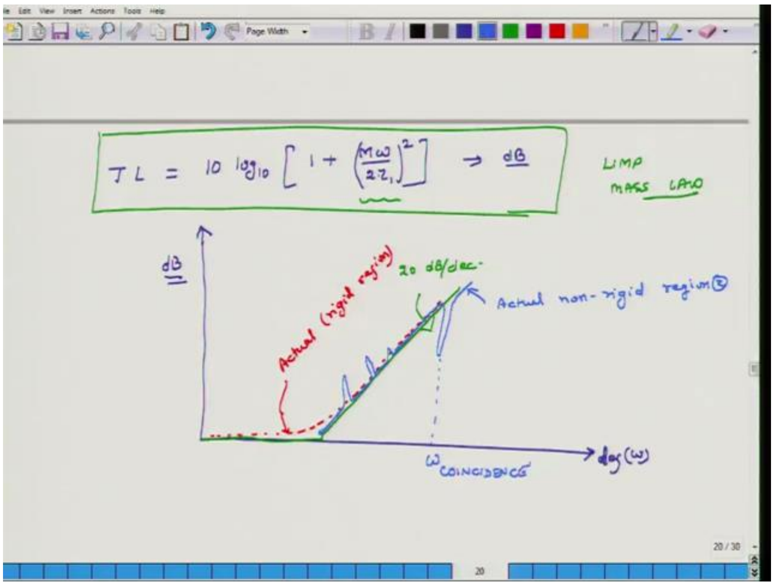

So, TL is equal to 10 log of 10. So, here the there is this denominator so when I take the log of the inverse this denominator will come up, so this becomes 1 plus m omega by 2 z 1 and whole square. And if I plot this curve on a bode plot. So, here I have log omega this is in decibels, so this is in here the units are dB. Now this has a low frequency asymptote, high frequency asymptote low frequency asymptote means when omega is going to 0 the transmission loss is 0, so it is like this. And high frequency asymptote is because of just this, so it a straight line like this and the slope of this is 20 decibels per decade.

Now, this equation for transmission loss is called Limp Mass Law. So, if your wall is rigid and it is not flexible so that it starts resonating then this law works and for limp; so this is the asymptotic response and the actual response will be something like this. But, in reality walls are not perfectly rigid. So, at very low frequency they behave as rigid masses, but as you increase the frequency they have their own resonances. So, when they have their own resonances as I am move on this positive slope line at resonance points what may happen is; so initially the actual curve may be like this, but these may be related to resonances. So, at resonance the transmission loss becomes higher and then it keeps on going and then you may see actually a dip. So, this is the actual non-rigid region 2. What is red? Actual for rigid region, so this is the real actual when it is non rigid.

So, at some frequency you will actually see a dip, and what this dip means is that at this frequency if the energy which is getting lost is less, transmission loss has gone down. This corresponds to angular frequency known as omega coincidence. We will not go into the mathematics of this, but what is omega coincidence?

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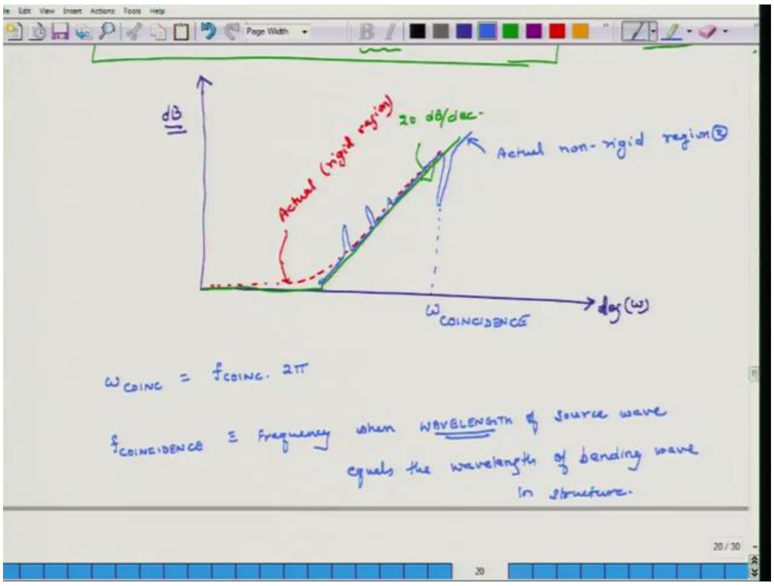

So, omega coincidence is equal to frequency coincidence times 2 pi, and what is frequency coincidence? This corresponds to frequency when wave length, so it is wave length of source wave. Or suppose you are exciting it by some frequency at this frequency there will be some wave length, this is the wave length we are talking about. So, wave length of source wave equals the wave length of bending wave in structure.

So, what does that mean? That when you are exciting it, structure will get excited and it may have some bending also, that bending will be associated with some bending wave and for 1 frequency the wave lengths will match each other, and when that happens when we will have a dip in the transmission loss. So, your region 2 will not work well for reducing sound at coincidence frequencies. At resonance frequency it will work very well at other frequency it will work moderately well, but at coincidence frequency it performs badly.

So, that concludes the discussion for today. And, we will meet next week, till then have a great weekend. Bye.