**Fundamentals of Acoustics**
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**Lecture – 47**
**3 Media Problem**

Hello. Welcome to Fundamentals of Acoustics, today is the fifth day of the eighth week of this course. What we plan to do today is continue our discussion which we initiated in the last class which is yesterday and what we did yesterday was we had framed the 3 media problem which is shown here.

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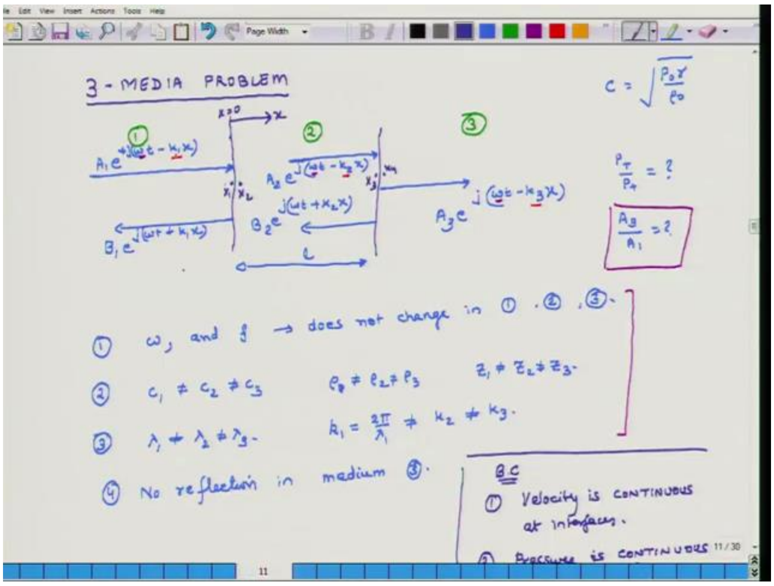


So, you have 3 media and in the first media there is a resident wave A 1 e plus j omega t minus k 1 x and in the third media median, the exiting on the transmitted wave is its complex pressure is a three e to the power of j omega t minus k 3 x. So, what we are interested in finding out A 3 over A 1 to find out this ratio, we have to develop number of equations and what we will do is we had also specified that there are velocity boundary conditions and pressure boundary conditions at each interface, so what we will do is we will list bound these boundary condition.

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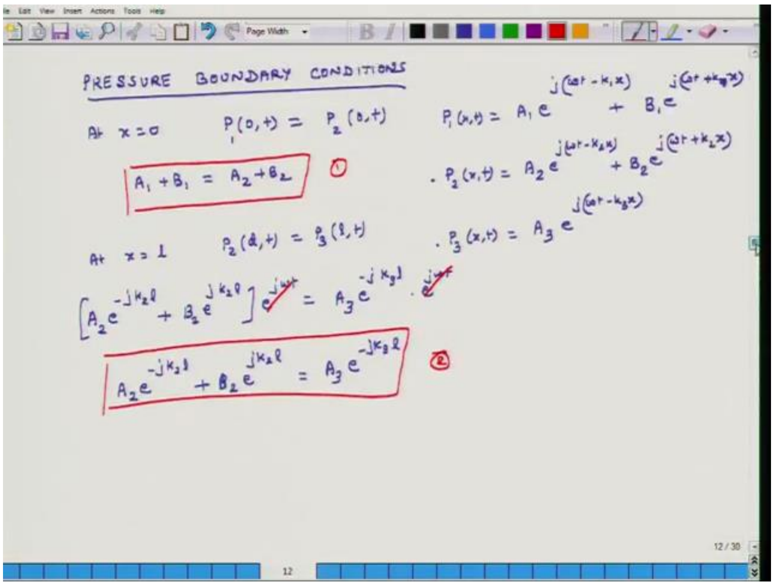


So, first we will list down the pressure boundary conditions, so the pressure boundary condition is that at x is equal to 0, P 0 t equals on in media 1 is same as P 0 t in media 2, now what is pressure; now P 1 is equal to A 1 e j omega t minus k 1 x plus A 2 e j omega t minus k 2, k 1 x. So, this is P 1 at location x and t and this constant should be d 1 and delta plus here.

Similarly P 2 x t, so where are we getting these equations, we are getting them from transmission line equations. So, P 2 is equal to, A 2 e j omega t minus k 2 x plus B 2 e j omega t plus k 2 x. So, if I put x is equal to 0 in this relation similarly I will for sake of completeness pressure in media 3 is P x t is equal to A 3 e j omega t minus k 3 x and here I do not have a reflected term, so no B 3 term. So, if I put x in equals 0 in this case I get the condition for this P 1 equals P 2 at x is equal to 0. So, essentially what I get is A 1 plus B 1 equals A 2 plus B 2, so let us call this equation 1.

The next boundary condition for pressure is at x is equal to l, what is l it is the length of the medium; second medium. So, here x is equal to l for x is equal to l P 2 equals P 3. So, I can write it as complex pressure; l t equals complex pressure P 3 evaluated at l in time 3. So, if I put in these two equations x is equal to l, I get A 2 c minus j k 2 l plus A 3; e j k excuse me this is B 2, B 2 e j k 2 l e to the power of j omega t equals A 3 e j k 3 l minus times e to the power of j omega t, so this and this cancels out.

So, what I am left with is A 2, e to the power of minus j k 2 l plus B 2 e to the power of j k 2 l is equal to A 3 e to the power of minus j k 3 l. So, I will box these important equations, so that is my second equation. So, these are the pressure boundary conditions, so I get two equations in A 1, B 1, A 2, B 2 and A 3 from here.

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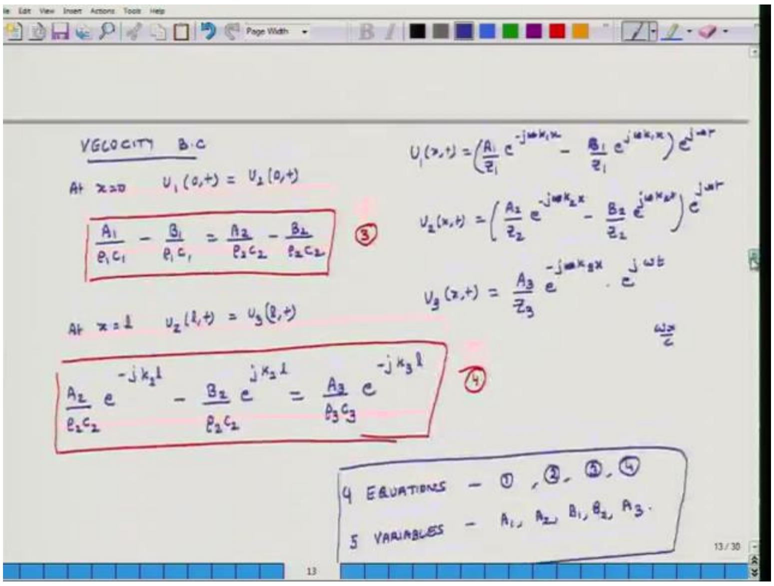


Next we will look at velocity boundary condition, so before we write expressions for these just write down the expression for velocity. So, complex velocity u in medium one equals A 1 by Z 1; Z 1 is rho naught, rho 1 is C 1 rho 1 is C 1 c minus j omega k 1 x plus actually this should be minus. So, this I am getting directly from transmission line equation minus B 1 over Z 1, e j omega k 1 x, e j omega t, u 2; x t is equal to A 2 over Z 2, e minus j omega k 2 x minus B 2 over Z 2 e j omega k 2 x, e j omega t and u 3, x t is equal to A 3 over Z 3, e j omega A 3 x times e j omega t.

So, the boundary condition for velocity at x is equal to 0 what is it, u 1 complex velocity u 1 at 0 equals u 2 at 0 t. So, I put x in the first and second variations and I equate them, so what I get is essentially A 1 over Z 1; I write it as C 1 minus B 1 over rho C 2 equals A 2 over rho 1; rho 2 c excuse me. So, this is rho 1, C 1 and this is equal to A 2 over Z 2 minus B 2 over rho 2, C 2, so this is the third important equation.

The fourth boundary condition corresponding to velocity is that x is equal to l, u 2 l t equals u 3; l t. If we do the substitution here, so I get A 2 over rho 2, C 2, times e minus j omega k 2 l minus. So, these are my expressions for complex pressures and in these expressions for velocities, I by error have included this term omega actually omega is one expression would have been omega x over c, but omega over c is k 1. So, I should not have included omega in these expressions associated with x, so I am going to scratch off omega at these places.

So, then two are velocity boundary condition what we have is A 2 over rho 2, C 2 exponent minus j k 2 l minus B 2 over rho 3, C 3 exponent j k 2 l. So, this should be rho 2, C 2 and this equals A 3 over Z 3 which is rho 3, C 3 exponent minus j k 3 l, so that is my fourth equation.

So, let us look at these all the four equations, so in these four equations I do not know A 1, B 1, A 2, B 2, A 3; I know k 1, k 2, k 3 because I know frequency and because I know frequency I know omega and because I know C 1, C 2, C 3; I also know k 1, k 2, k 3 because I know what gases are these. So, I know the type of gases and condition in the gas, so total four equation in red boxes and five variables. So, four equations 1, 2, 3, 4 and 5 variables, what are those five variables; A 1, A 2, B 1, B 2, A 3, so four equations and five variables. So with this situation, I can indeed compute the ratio of A 3 over A 1.

Now these are four equations and they are not straight forward equations, there are linear equations because e to the power of j k 2, l is a constant, k 2 is constant, l is a constant. So, all these equations are linear, but it will take time to solve these equations so that we can get this ratio. So what I will do is, I will write down the relations directly, but instead of finding the expression for A 3 over A 1, we will actually look at a slightly different entity.

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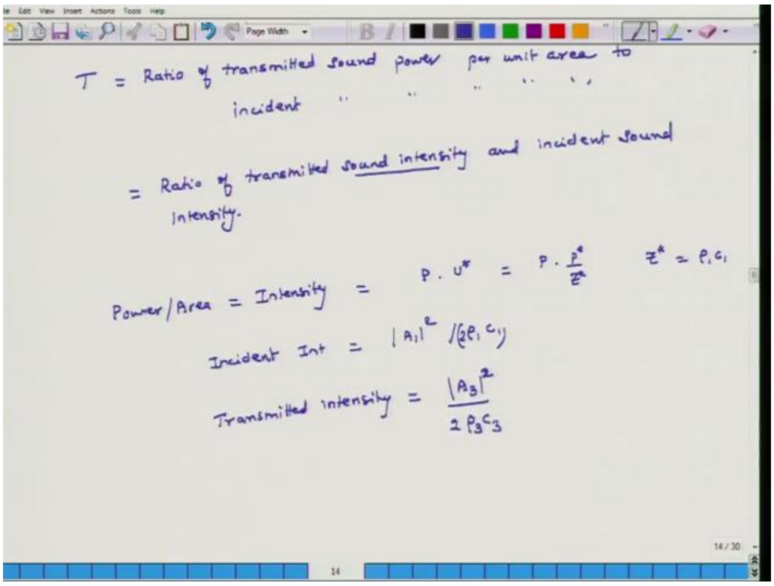


So, what we want to know is. So, we could find out this expression, but alternatively we would be interested in this parameter t and what is T, it is the ratio of transmitted sound energy per unit area. So, actually I will call it some power as two incident sound power per unit area or what is power per unit area it is intensity, so it is ratio of transmitted sound intensity and incident sound intensity.

Now, so this is sound intensity which means power per unit area and if you remember now power per unit area. This is intensity and the sound is travelling in the x direction we have said that is equal to pressure times u star right amplitude of pressure times u star and what is this, this is pressure and because this is wave travelling in forward direction no reflection is happening. So for the incident what is the incident energy, incident energy will be its magnitude pressure times u star and what is u star, it will be P star divided by Z star and for the incident wave alone; what is Z star, Z star will be rho 1, C 1. So, similar so incident intensity is equal to what, it will be and what is t it will be its magnitude is A 1 and P times P star is A 1 square divided by rho 1, C 1 and the last thing I wanted to say is because it is a sinusoidal function. So, I have to divide it by a factor of 2 right; RMS intensity is this divided by 2, if I do not divided it by 2 then that will be giving the (Refer Time: 18:17).

So, this is the incident intensity and transmitted intensity, so that equals using same logic A 3 square divided by 2 rho 3, C 3.

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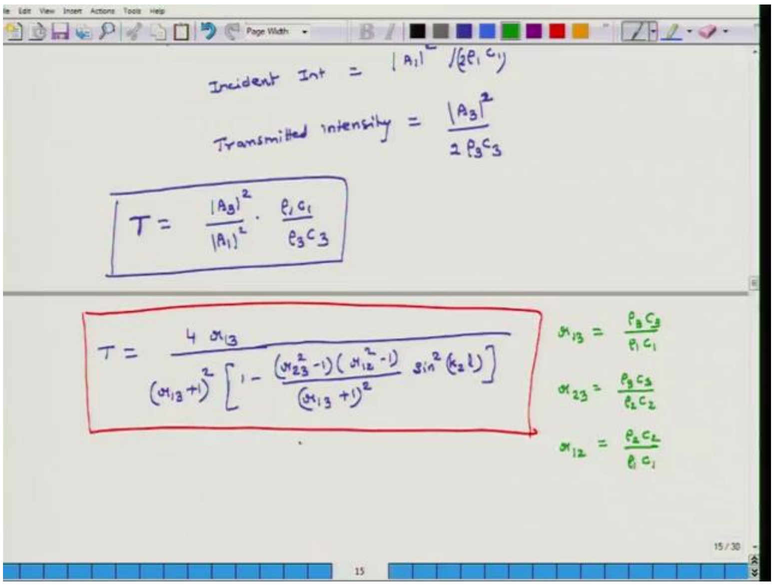


So, T equals A 3 square divided by A 1 square times rho 1, C 1 divided by rho 3, C 3. So, this is what we are interested in is how much energy we are putting into the system and how much energy is getting out of the system. So how do we calculate it from these four equations, first we calculate A 3 over A 1 and then we find out the magnitude of A 3 over A 1, square it and multiply it by rho 1 C 1 divided by rho 3, C 3 and we will get the answer, but still you know is that particular ratio.

So we can do all the math; we know that there are four equations five unknowns. So, in terms of one unknown I can express all other unknowns. So, I am going to just directly write the relation for t. So, t if I do all the math correctly it comes out as four times some constant r 13 divided by r 13 plus 1 whole square, 1 minus, r 2; 3 square plus 1 times, r 12 square minus 1 divided by r 13 plus 1 whole square, sin square k 2 l. So, that is the ratio t, so what are r 13, r 23, r 12. So, here r 13 equals rho 3, C 3 over rho 1 C 1 r 23 equals rho 3, C 3 over rho 2, C 2 and r 12 equals rho 2, C 2 divided by rho 1, C 1.

So, this is the relation. And what I planned to do tomorrow is discuss this relation for some specific cases and that will help us conclude make some important observation. And in the next class we will use the same relation for designing and actual muffler. So, with that we will close our discussion and we will meet once again tomorrow.

Thank you.