Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

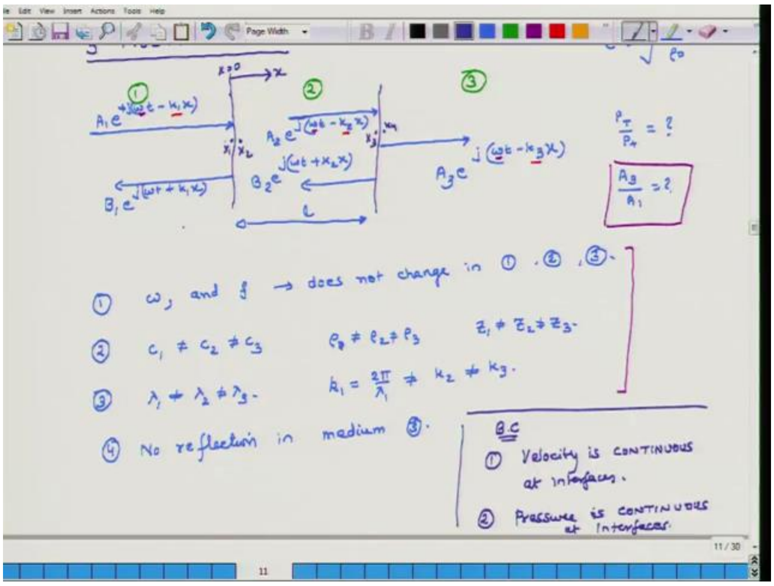
Lecture – 46
3 Media Problem

Hello. Welcome to Fundamentals of Acoustics, today is the fourth day of the current week. And today as well as in during the remaining portion of this week we will develop a mathematical formulation which we will use in the subsequent week; to figure out what type of frequencies and to what extent particular frequencies that can be handled by devices such like mufflers or silencers.

And specifically, we will be looking at the performance of reactive mufflers where the sound as it passes through the muffler it does not get absorbed, but rather in this it gets reflected back into the sound source because there are 2 types of mufflers. One category is reactive muffler where sound pressure as it enters the muffler it gets reflected, and in the other type of mufflers which are known as resistive mufflers and the sound as it travels through the length of the muffler it slowly gets absorbed and gets out of the muffler at much reduced sound pressure level. So starting today we will lay down the foundation, the mathematical foundation for reactive mufflers.

So, to solve the problem of a muffler or to develop a mathematical model of a reactive muffler we have to solve a particular type of problem known as 3 Media Problem.

(Refer Slide Time: 01:48)



Once we learn how to solve this 3 media problem, we can relatively easily use this knowledge to design a theory muffler of our choice. So what is the 3 media problem? A 3 media problem is, so first thing is that it deals with one dimensional planar wave. So, what we are trying to understand is that suppose there are 3 media; so medium is singular and media is plural. So this is media medium one first medium, this is second medium, third medium. Examples of these three mediums should be; here I may have air, in region 2 I may have carbon dioxide and region 3 I may have nitrogen whatever. And these 3 media are separated by sharp boundaries 1, 2 and 3, so these are the boundaries.

So, our aim is that if there is an incident wave in the first medium and the complex pressure due to the incident wave alone is P plus e minus j, omega t over this plus thing minus k 1 x. So, k 1 is the wave number for the first medium, so suppose the incident wave is of this nature then this sound, some of this sound will get reflected some of it will get transmitted into 2 and finally some of the wave will get transmitted into 3 and it will move out.

So, let us say the transmitted wave is P T e j omega t minus k 3 x. So, what we are interested in understanding is that what is this ratio P T over P plus, that is what is the main issue; so the problem is that if we know P plus and we also know the properties of medium 1, medium 2, medium 3 and I also if we know the length of the intervening medium which is medium 2 then can we figure out the ratio of P T and P plus that is our fundamental problem, so this is the 3 media problem because there are 3 media involved.

Now, in context of this particular problem I will not use terms P plus and P T because there will be several P plus as and P T and P minus is rather in medium 1; I will use the term. So, the incident wave amplitude is A 1 and then as this wave hits the boundary, part of it gets reflected and the complex pressure amplitude for the or complex pressure for the reflected wave is A 2 e j omega t minus k 2 x- sorry it is k 1 x, so it will be plus k 1 x.

Now, so some of it will be reflected and then some of the sound will get transmitted into medium 2. In medium 2, the incident wave hitting at the next boundary is A 2, e j omega t minus k 2 x and the reflected wave is.

Student: (Refer Time: 06:09).

So, I am sorry this should be B 2 actually it should be B 1 and the reflected wave is B 2; e j omega t plus k 2 x and finally, the wave which is getting transmitted out of medium 3, it is complex magnitude constant is not P T, so that rather it we will call it A 3. So, what we are looking at is the ratio of A 3 and A 1.

Now also some important things to note, the first things to note first thing as sound is propagating from travelling from medium 1 to medium 2 to medium 3, the frequency, so omega and frequency, it does not change in 1, 2 and 3 why they did not change. Suppose there are some fluctuations happening n number of times in medium 1 then the adjacent medium in the adjacent air particles or medium particles in medium 2, they also get oscillations at the same number of time right, their amplitude may go up or down, but the oscillations will remain the same because if I am hitting some particle hundred times then the other adjacent particle will also vibrate at the same frequency. So, frequency as I know from medium 1 to medium 2 to medium 3, it does not change.

Second these 3 media have different material properties, so the pressure ambient pressure in these three could be different or the density could be different. So, c 1 is not equal to c 2 is not equal to c 3, densities of these 3 media need not be same right because what is c, c is equal to p naught gamma over rho naught. If I have air nitrogen and carbon dioxide there is no reason to think that c has to be the same. Similarly rho 1 is not necessarily equal to rho 2, is not necessarily equal to rho 3 and because rho 1, c 1 is characteristic impedance for the first media is for the same reason Z 1 is not equal to Z 2 is not equal to Z 3.

Third, if frequency is same, but velocity of sound is different than lambda 1 is not necessarily same as lambda 2 is not necessarily same as lambda 3; and if lambda is not same then k 1 is which is equal to 2 pi over lambda 1 is not same as k 2 is not same as k 3. So, for that reason we have three different wave numbers in our expression and for the reasons discussed here, we have same angular frequency in our formulation, so this is important to understand. So, we have used these assumptions as we have written down expression for incident and reflected wave for each of these media terms.

The last assumption is that there is no reflection in medium, so there is no reflection in the third medium, so that is also something which we have incorporated into the model. Now our aim I will go back is to find out the ratio of A 3 and A 1, so how do I do that we once again we go back, so this is again a one dimensional problem all the waves are propagating in one direction planar waves. So, we once again go back and use transmission line equations, so we will get 2 transmission line equations corresponding to medium 1, 2 from 2 and 2 equations from the third medium. So, overall we will have six transmission line equations, but then how many unknowns we have; A 1, B 1, A 2, B 2, A 3, so 5 and then of course, k 1, k 2 all these things are also unknowns. So, we have more unknowns and less number of equations, so we need some additional conditions; so those additional conditions come from the interface.

Now, before we talk about the interface between medium 1 and medium 2 and medium 2 and medium 3, let us define our coordinate system. So, our coordinate system is such that this is the direction x and at this first interface x is equal to 0. So, let us look at the boundary conditions which I will give us the extra conditions. So, the first boundary condition will be that at x is equal to 0; consider a point particle of fluid here and another particle of fluid. So, let us consider this point as x 1 and this point as x 2 and x 1 and x 2 are extremely close to each other actually the distance between them is theoretically infinitely this will be a small actually it is 0 and what; that means, is that whatever is the velocity of the system at point x 1, it has to be the same at point x 2, so what; that means, is velocity is continuous at interfaces. Same thing is true for point side the second interface, so this is let us say x 3 and let us say this point is x 4, so the same thing is toward the second interface.

The second consideration is on pressure, now consider the situation where pressure at x 1 is different than pressure at x 2, if it was different what is the distance between these 2 points theoretically it is 0. So, you have non zero pressure difference at 2 points which are separated by 0 distance which means that the acceleration between point x 1 and x 2 because there is very less mass contained between 0 mass, if there is this 0 distance the mass contained in this is 0, but pressure is non zero; difference in pressure is non zero which means that the fluid will accelerate at infinite acceleration; m times a equals change in for difference in pressure times area difference in pressure is non zero, area is non zero, mass is 0; so, acceleration has to be infinite; that is not physically possible.

So, what that means is that pressure cannot be different, so essentially it means is that pressure is continuous at interfaces. So how many conditions we get; we get 2 conditions at the first interface which is v 1 equals v 2 and p 1 equals p 2 and 2 conditions at the second interface, v 3 equals v 4 and v 3 equals v 4, so total of four extra conditions.

So, this is what the overall problem is what we will do is, we will start solving this problem in our next class and that is tomorrow and also day after tomorrow and then we will draw some very important conclusions from such an analysis. So, with that I close for today and we will meet once again tomorrow.

Thank you.