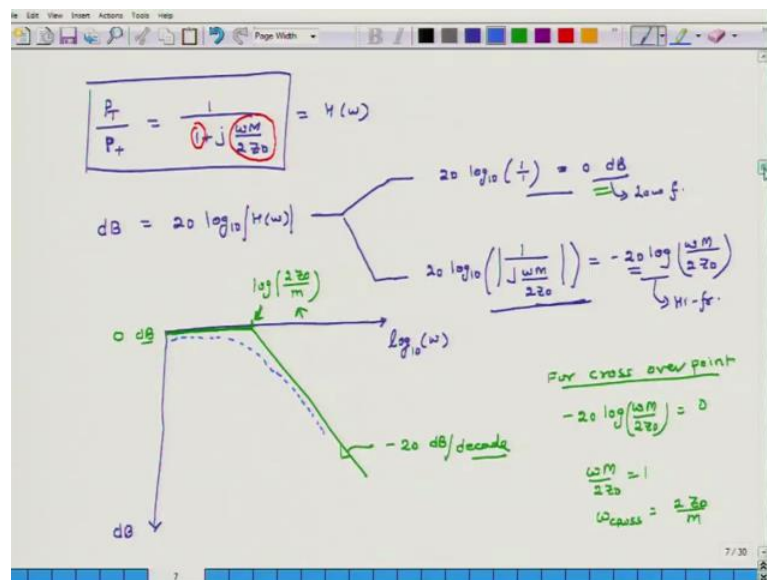


Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 45
Noise Reduction by Mass Attenuation

Hello. Welcome to Fundamentals of Acoustics; today is the third day of the eighth week of this course, and what we plan to do today is conclude our discussion for mass attenuation.

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So yesterday we had developed the expression for the ratio of magnitude of transmitted wave and magnitude of incident pressure wave. So, these are complex magnitudes and that was calculated to be 1 over 1 plus j omega m divided by 2 Z naught.

So these are ratios of pressures, and if I have to do it in decibels then in decibels when we are talking about pressures we have to take 20 log of this ratio. So, it is 20 log on base 10, so let us this call this ratio as H of omega. So, that is like a transfer function input is P plus, output is P T, so this is the function which defines whatever is being transmitted, so this is 20 log of H of omega and then I have to take its magnitude.

Now, what we will do is we will develop a bode plot for this transfer function, so what is bode plot it has two asymptotes; one asymptote corresponds to the condition as omega

goes to 0 and another asymptote corresponds to the situation when ω becomes very large. So, when ω approaches 0 what do I get, so I get, so as ω becomes very small as it approaches 0, I get $20 \log$ of 10 (Refer Time: 02:28) at half one over 1, so this is 0; 0 decibels and as ω becomes very large what happens; as ω becomes very large this term the term in red is very large compared to 1. So, this becomes $20 \log$ of 10 of 1 over $j \omega m$ divided by $2 Z_{naught}$ and I have to take its magnitude. So, this becomes, so magnitude when I take as this j goes away, so I get minus $20 \log \omega m$ over $2 Z_{naught}$. So, that is my low frequency, so this plot will give my low frequency curve and this plot will give me very high frequency curve.

Now, in the high frequency curve we notice one thing that suppose I; let us say initially ω is something such that the term in this bracket is 1, if the term in the bracket is 1 then this becomes 0 then let us say I multiply ω , I increase my frequency by a factor of 10 then this term in the bracket becomes 10. So, then this becomes minus 20 let us say I increase the frequency further and it becomes such that ω is hundred times the original ω then this thing becomes minus 40. So, what that means is that this is; if I plot this function on a log scale then it is a negatively sloped straight line and it has a slope of minus 20 decibels per decade and this is our flat horizontal straight line. So, if I have to plot these two asymptotes, so it will look something like this.

So, on the horizontal axis I am going to plot \log of ω , on the vertical axis I am going to plot decibels and my low frequency asymptote is going to be a straight line and this straight line corresponds to 0 decibels and then my high frequency asymptote is going to be a negatively sloped line and the slope of this line is going to be minus 20 dB per decade.

So, this is going to be my bode plot and the crossover point where the bode plot changes its slope, it will be when. So for crossover point it will occur when $20 \log \omega m$ over $2 Z_{naught}$ equals 0 decibels from here which means ωm over $2 Z_{naught}$ equals 1 or ω cross over point equals $2 Z_{naught}$ divided by m , so this point is going to be \log of $2 Z_{naught}$ divided by m .

So, that is how the bode plot looks like, so what does the bode plot say that in an asymptotic sense unless my frequency is more than this parameter angular frequency more than $2 Z_{naught}$ over m , my attenuation or sound reduction will be pretty much

negligible and only once I cross this important frequency then I start seeing reductions in my transmitted sound intensity and that reduction increases this time by 20 decibels this time by increased my frequency by a factor of 10 and the actual performance curve may look something like this, but it is important that if I want wall to be effected it is important that we design the wall in such a way that the parameter; $2 Z_{\text{naught}}$, the $2 Z_{\text{naught}}$ does not depend on wall it Z_{naught} is $\rho_{\text{naught}} c$ which is the speed of sound times density of air.

So, that depends on air, but the ratio of $2 Z_{\text{naught}}$ divided by m where m is the area density of wall should be such that it is below our expected frequency range, so that is important to consider.

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The image shows a whiteboard with handwritten mathematical derivations. At the top, the displacement $U_B(x,t)$ is given as $\frac{(P_2 - P_1)}{Z} e^{-j\omega t} e^{j\omega t}$. Below this, a diagram shows a vertical wall with an incident wave $P_+ e^{-j\omega t} e^{j\omega t}$ moving right and a reflected wave $P_- e^{+j\omega t} e^{j\omega t}$ moving left. The transmitted wave is $P_T e^{-j\omega t} e^{j\omega t}$. To the right, it says "MOTION OF WALL" and $\sum F \text{ on wall} = m \cdot \text{acceleration}$. The force calculation is shown as $\sum F = (P_+ \text{ on Left side} - P_- \text{ on Right side}) A$, which simplifies to $\sum F = \left\{ P_+ e^{-j\omega t} e^{j\omega t} + P_- e^{+j\omega t} e^{j\omega t} \right\}_{x=0} - \left\{ P_T e^{-j\omega t} e^{j\omega t} \right\}_{x=0} A$. This is then equated to $M \cdot A \cdot \frac{dU_B}{dt} \Big|_{x=0}$. At the bottom, a final equation is shown: $\left[(P_+ + P_- - P_T) e^{j\omega t} \right] A = MA \frac{d}{dt} \left[\frac{e^{-j\omega t} e^{j\omega t}}{Z_0} \right]_{x=0}$.

Let us look at couple of other factors. So, we want to know that when the sound is striking the wall as a shown in this picture, the wall is going to vibrate. So, what is going to be the velocity of that vibration?

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VELOCITY OF WALL

$$u_B(0,t) = \left(\frac{P_+ - P_-}{Z_0} \right) e^{j\omega t} = \frac{P_+}{Z_0} \left[1 - \frac{j\omega m}{j\omega m + 2Z_0} \right] e^{j\omega t}$$

$$= \frac{P_+}{Z_0} \left(\frac{2Z_0}{2Z_0 + j\omega m} \right) e^{j\omega t}$$

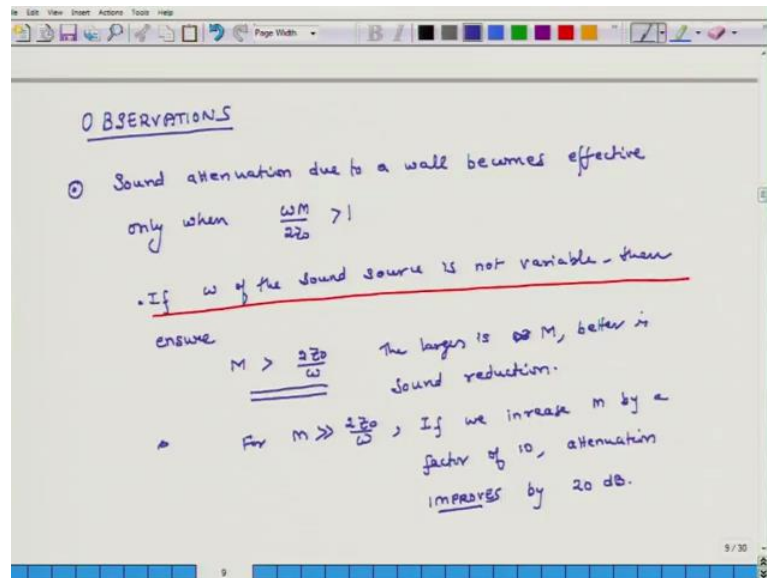
$$\boxed{|u_B(0,t)| = \frac{|P_+|}{Z_0} * \frac{2\rho_0 c}{\sqrt{4Z_0^2 + m^2\omega^2}}}$$

So, velocity of wall, so I can calculate velocity of wall from this relation U_B equals P plus minus P minus times e minus j omega x over c times e j omega t and here at the wall what is the value of x 0. So, if I put x is equal to 0 in this relation and I also put the value of P minus in terms of P plus which I had calculated then I will get my expression for velocity. So, $U_B(0,t)$ equals P plus minus, P minus divided by Z naught e j omega t and I know that P minus; I substitute the value of P minus in this equation. So I get P plus by Z naught, 1 minus j omega m divided by 2 j omega m plus Z naught, e j omega t . So, that equals e plus divided by Z naught times, 2 Z naught divided by 2 Z naught plus j m omega, e j omega t .

So, if I have to find the magnitude of this then it is U_B is equal to magnitude of P plus because P plus is a complex number divided by Z naught because Z naught is always real, so I do not have to worry about it. 2 Z naught is equal to into 2 rho naught c divided by 2 rho naught c plus, so I made a mistake here it is j omega m plus 2 z 0. So, this is going to be equal to 4 Z naught square plus m square, omega square and the magnitude of e j omega t is 1. So, that is my magnitude and what; that means, is that as I increase my either m or omega, the influence on U_B is similar because it is m square times omega square.

So, if I want to make my wall effective, if I double its velocity this m square times omega square becomes four times as much, so this is another important observation.

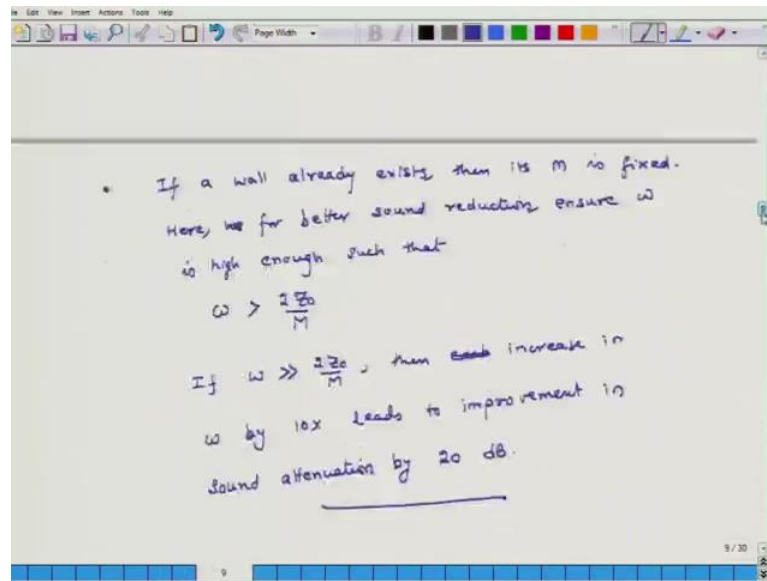
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Finally, we make some important observation, so observations first sound attenuation due to a wall becomes effective only when. So, we had developed that condition when this $j \omega m$ over $2 Z_0$ is in excess of 1. So, when ωm over $2 Z_0$ is large at least if not very large, but it should be larger than 1, now this can happen; so this can happen in two ways if ω of the sound source is not variable then ensure. So if ω , suppose there is a motor running at 1000 RPM and that is not variable, you cannot change it then ensure that m is more than $2 Z_0$ divided by ω , so this is important and in this case the larger is ω or not ω , m better is sound reduction. So, this there and then also in this case also for m very large compared to $2 Z_0$ divided by ω what happens when we are when m is extremely large compared to this ratio then what it means is that we are somewhere here.

So, then the response of the wall is pretty close to its asymptotic response. So, for m very large compared to $2 Z_0$ over ω , if we increase m by a factor of 10 attenuation improves. So, what does it improve means that less and gets transmitted, less sound, less transmitted improves by 20 decibels, so this is important to understand. So, this is the case when ω of the sound sources variable is fixed.

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So, if a wall already exists suppose there is already a wall in the system then its m is fixed. Here we for better sound reduction ensure ω is low enough such that; what is the condition, so the condition is this it should be higher, sorry. So it should be higher, now such that ω should be more than $2 Z$ naught over m and here the special condition is, if ω is very large compared to $2 Z$ naught over m then each, then increase in ω by 10 x, leads to improvement in sound attenuation by 20 decibels.

So, this is an important consideration, so these are two important conclusions based on this relation. Now later what we will see is that this formula works, so this curve theoretically it works up to a certain frequency and as you keep on increasing after a certain range, the efficiency of this formula is no longer guaranteed, but we will discuss that later, but whatever we have done over last three lectures is that we have learnt that how we can reduce sound in a particular area by just putting heavy wall and if we want to ensure that low frequency is do not get transmitted from one side to the other, we have to make the wall very heavy and in the and how have they have to be; they have to be heavy enough so that this condition is satisfied and if the mass per unit area of the wall is very large compared to $2 Z$ naught over ω then it will be effective for the cutoff frequency.

So, with this discussion we conclude our lecture for today, we will meet once again tomorrow and starting tomorrow we will start laying the ground work for solving the

problem of a muffler that is if you put a silencer or a muffler through contain or reduce the noise of a particular engine or some other source which has a particular frequency then how you actually mathematically develop the module for that. So, with that we will conclude for today and we will meet once again tomorrow.

Thank you. Bye.