

Fundamentals of Acoustics
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Lecture – 44
Noise Reduction by Mass Attenuation

Hello, welcome to Fundamentals of Acoustics. Today is the second day on the eight week of this course. What we will do today is continuation of our discussion which we were having yesterday, we had started discussing how using a wall which has a non zero mass and which is free to move, can be used to reduce the sound pressure level on the transmitted side.

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$u_B(x,t)$ z_0 no extra cond.
 st B.C. Val. of air on side A @ $x=0$ equals
 Val. of air on side B @ $x=0$.
 $u_A(0,t) = \left(\frac{P_+ - P_-}{z_0}\right) e^{j\omega t} = u_B(0,t) = \frac{P_T}{z_0} e^{j\omega t}$
 $P_T = P_+ - P_-$ ②
 Putting ③ in ② we get:

$$\left. \begin{aligned} P_B(x,t) &= \left(\frac{P_+ - P_-}{z_0}\right) e^{-j\omega x} e^{j\omega t} \\ u_B(x,t) &= \left(\frac{P_+ - P_-}{z_0}\right) e^{-j\omega x} e^{j\omega t} \end{aligned} \right\} ④$$

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The slide contains a diagram and several equations. The diagram shows a vertical wall at $x=0$. Region A is to the left ($x < 0$) and Region B is to the right ($x > 0$). In Region A, an incident wave $P_+ e^{-j\frac{\omega}{c}x} e^{j\omega t}$ and a reflected wave $P_- e^{j\frac{\omega}{c}x} e^{j\omega t}$ are shown. In Region B, a transmitted wave $P_T e^{-j\frac{\omega}{c}x} e^{j\omega t}$ is shown. A box contains the question $\frac{P_T}{P_+} = ?$. To the right of the diagram are three numbered points:

- $d \ll \lambda$
We don't have to worry about change in phase over distance 'd' in wall.
- $M =$ mass per unit area of wall.
- $Z_A = Z_B = Z_0 = \rho_0 c$.

Below the diagram are two matrix equations for pressure and velocity:

$$\begin{cases} P_A(x,t) \\ U_A(x,t) \end{cases} = \begin{bmatrix} P_+ & P_- \\ \frac{P_+}{Z_0} & -\frac{P_-}{Z_0} \end{bmatrix} \begin{cases} e^{-j\frac{\omega}{c}x} \\ e^{j\frac{\omega}{c}x} \end{cases} e^{j\omega t}$$

$$\begin{cases} P_B(x,t) \\ U_B(x,t) \end{cases} = \begin{bmatrix} P_T & 0 \\ \frac{P_T}{Z_0} & 0 \end{bmatrix} \begin{cases} e^{-j\frac{\omega}{c}x} \\ e^{j\frac{\omega}{c}x} \end{cases} e^{j\omega t}$$

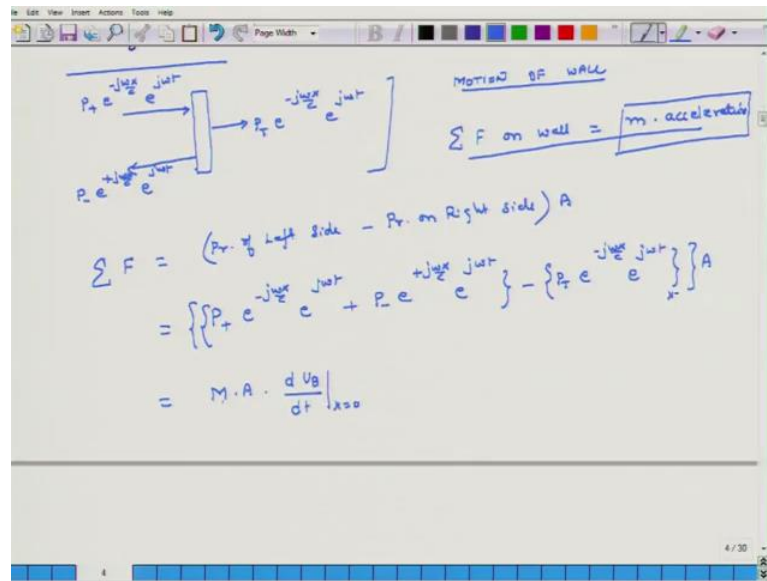
Notes on the slide:

- 7 unknowns, 4 eqns.
- To find $\frac{P_T}{P_+}$ we need two extra conditions.
- St. B.C: Vel. of air on side A @ $x=0$ equals vel. of air on side B @ $x=0$.

So, that is the mathematics which we are trying to develop and in that process yesterday what we had accomplished was we had developed a relation for P_B and U_B in terms of P_+ and P_- , where P_B and U_B are complex pressures, for pressure and velocity on the transmitted side and P_+ and P_- are magnitudes of complex incident pressure and complex reflected pressure, so that expression has been shown here and it is done by that is four.

Now, so once we have done this we have eliminated P_T and to get to about ratio of P_+ and P_T and P_+ we have to apply one more condition. So, and once we applied one additional condition then we will be able to eliminate P_- also and we will be able to get the relation between P_+ and P_T . So, that is what we planned to do in today's lecture.

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One condition is what we have already applied, but then there is another condition and the condition is that this is the wall it is being incident, it is being struck. So, the incident complex pressure is $P_+ e^{-j\omega x/c} + P_- e^{+j\omega x/c}$ and reflected pressure wave in complex terms is $P_- e^{+j\omega x/c}$ and transmitted pressure is $P_T e^{-j\omega x/c}$ and this term should be plus.

Now when this wall has been acted by pressure on in the transmitted side and pressure on the incident side because if the pressures are different then this wall will move because it is free to move, so the motion of the wall how do we figure out, what is the nature of this motion it is equal to total force on the wall and that is equal to mass of the wall times acceleration.

So, this is the equation we will develop total force on the wall is $\sum F$ and that equals pressure on left side, minus pressure on right side times area. So, the pressure on left side is going to push the wall in the positive x direction, pressure on right side is going to push the wall on the negative x direction. So, that is why I have a negative thing and times area of the wall.

So, what is pressure on the left side it is equal to incident pressure plus reflected pressure, so it is $P_+ e^{-j\omega x/c} + P_- e^{+j\omega x/c}$. So, this is on the left side incident component plus $P_- e^{-j\omega x/c}$. So, this is on the left side of the wall minus $P_T e^{-j\omega x/c}$ times a . So, this

is the sum of forces and so this is the complex pressure. So, when I add this up it will be complex force and this equals. So, this is sum of forces and then what is the right side. So, right side of this equation is mass times acceleration, so this has to equal mass of the wall and what is mass of the wall, it is m times a because m represents mass per unit area.

So, there is one more bracket needed and then I multiply it by a . So, on the right side it is mass times acceleration, so mass is m times a here; a is the area of the wall, times dU/dt and this has to be evaluated at x is equal to 0 and also right side also has to be evaluated, x is equal to 0.

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The image shows a whiteboard with handwritten mathematical derivations. The equations are as follows:

$$\Rightarrow \cancel{a} (P_+ + P_- - P_T) e^{-j\omega t} = \frac{m a}{Z_0} \cdot P_T j \omega e^{j\omega t}$$

$$P_+ + P_- - P_T = \frac{m}{Z_0} j \omega P_T \Rightarrow P_T \left(1 + \frac{m}{Z_0} \right) = P_+ + P_-$$

$$P_T = (P_+ + P_-) \cdot \frac{Z_0}{(Z_0 + m j \omega)}$$

$$P_T = P_+ - P_-$$

So what do I get; I get when I put x is equal to 0 in this entire relation I get P_+ plus plus, P_- minus minus P_T , $e^{-j\omega t}$ to the power of $j\omega t$ times a and that equals mass times mass per unit area times area and what is U/B ; U/B you use this relation U/B is P_T divided by Z_0 in to $e^{-j\omega x}$ times $e^{j\omega t}$. So, it is $e^{-j\omega x}$ over c , $e^{j\omega t}$, but then I have to differentiate it and for differentiating I did not leave a lot of space. So, I am going to re write it, so $MA \frac{d}{dt}$ this entire thing and this also I have to evaluate it, x is equal to 0.

Student: (Refer Time: 07:57).

And there is of course, P_T , so what I get is; so, P_+ plus plus, P_- minus minus P_T , $e^{j\omega t}$ equals $M A$ over Z naught times P_T and when I differentiate the term in the bracket only this thing gets effected. So, I get $j\omega$; $e^{j\omega t}$ and this term becomes 1 when x is equal to 0, so I am just omitting that out.

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And then of course, here is an A.

Student: (Refer Time: 08:55).

I am sorry, so there is also an A here, so A and A cancel out, so the $e^{j\omega t}$. So, I get P_+ plus plus, P_- minus minus P_T is equal to m over Z naught, $j\omega P_T$ this gives me now. So, P_T into $1 + m j\omega$ over Z naught is equal to P_+ plus plus P_- . So, this is another expression or I can say that P_T equals P_+ plus plus, P_- minus times Z naught divided by Z naught plus $m j\omega$.

So, this is another expression for P_T , so we had developed one expression for P_T which was from 3 P_T equals P_+ minus P_- . So, we will rewrite that equation again, so the other expression for P_T was P_T equals P_+ plus minus P_- .

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The image shows a digital whiteboard with handwritten mathematical equations. At the top, there is a small equation: $P_+ + P_- - P_T = \frac{Z_0}{Z_0 + Mj\omega}$. Below this, there are several boxed equations:

- $P_T = (P_+ + P_-) \cdot \frac{Z_0}{Z_0 + Mj\omega}$
- $P_T = P_+ - P_-$
- $P_+ - P_- = (P_+ + P_-) \left[\frac{Z_0}{Z_0 + Mj\omega} \right]$
- Thus $\frac{P_-}{P_+} = \frac{j\omega M}{2Z_0 + j\omega M}$ or $P_- = \frac{j\omega M}{2Z_0 + j\omega M} \cdot P_+$

At the bottom, the final equation is: $P_T = P_+ - \frac{j\omega M}{2Z_0 + j\omega M} P_+$

So, if I equate these two, so if I equate the rhs of both of these equations then what do I get, I get $P_{-} + P_{-}$ is equal to $P_{+} + P_{-}$ divided by $Z_0 + j\omega m$.

So, now this equation has only P_{-} and P_{+} , so if I do the math for this equation I can get the ratio of P_{-} over P_{+} . So, if I do the math what I get is P_{-} over P_{+} is equal to $j\omega m$ divided by $2Z_0 + j\omega m$, so this is what we get, P_{-} over P_{+} is this thing or P_{-} equals $j\omega m$ divided by $2Z_0 + j\omega m$ times P_{+} , but our goal is to find the ratio of P_T and P_{+} . So, what I do is I put this back in this relation, so what do I get.

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The image shows a whiteboard with the following handwritten equations:

$$P_T = P_+ - \frac{j\omega m}{2Z_0 + j\omega m} P_+$$

THUS

$$\frac{P_T}{P_+} = 1 - \frac{j\omega m}{2Z_0 + j\omega m} = \frac{2Z_0}{2Z_0 + j\omega m}$$

The final result is boxed:

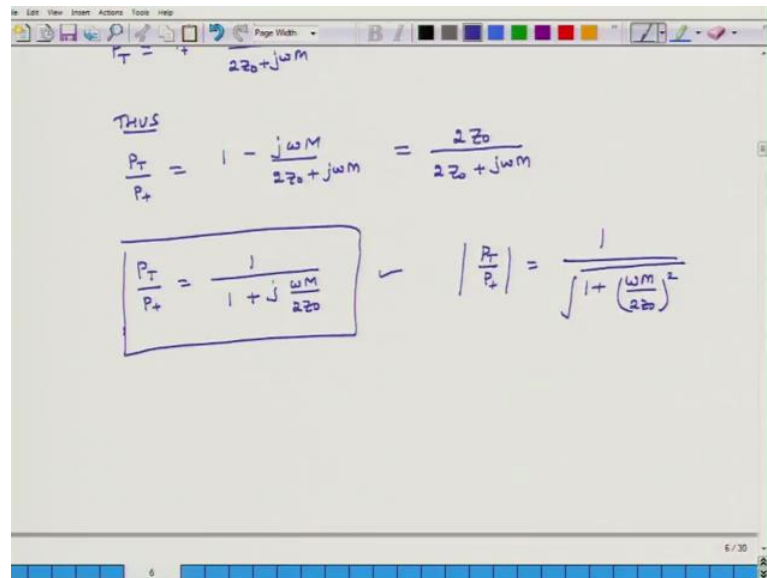
$$\frac{P_T}{P_+} = \frac{1}{1 + j \frac{\omega m}{2Z_0}}$$

So, what I get is P_T equals P_{+} minus P_{-} and P_{-} is $j\omega m$ divided by $2Z_0 + j\omega m$ times P_{+} . So, ultimately what I get is P_T equals. So, I will write down the ratio.

So, thus P_T over P_{+} equals $1 - j\omega m$ divided by $2Z_0 + j\omega m$ and that equals $2Z_0$ divided by $2Z_0 + j\omega m$ or finally, we can write P_T over P_{+} is equal to 1 divided by $1 + j$ times ωm divided by $2Z_0$. So, that is my ratio, so what does this say what does says is what this equation tells us is that as ω . So, consider the fact that there is a wall with the fixed value of m and if ω is extremely low.

Suppose omega is theoretically let us say it is 0 hertz, if omega is 0 then no matter how heavy the wall is and it can be extremely heavy, but the ratio of P_T and P₊ will be 1 which means that if frequencies are extremely small then even heavy walls will not be able to stop sound from propagating from one side to the other side and as omega goes up this P_T over P₊ it becomes less than 0.

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$$P_T = \frac{2Z_0}{2Z_0 + j\omega m}$$

THUS

$$\frac{P_T}{P_+} = \frac{1 - \frac{j\omega m}{2Z_0 + j\omega m}}{2Z_0 + j\omega m} = \frac{2Z_0}{2Z_0 + j\omega m}$$

$$\boxed{\frac{P_T}{P_+} = \frac{1}{1 + j\frac{\omega m}{2Z_0}}}$$

$$\left| \frac{P_T}{P_+} \right| = \frac{1}{\sqrt{1 + \left(\frac{\omega m}{2Z_0}\right)^2}}$$

Why because the negative of P_T over P₊ is equal to 1 over 1 plus omega m by 2 Z₀ naught; square this thing. So as omega becomes higher, this ratio becomes less than 1 and the higher I go up in frequency, the more efficient a wall becomes in terms of stopping the sound, it becomes less efficient.

So, that is the discussion I wanted to have today. And what we will do is tomorrow is continue this discussion, and we will also develop bode plots for transmission of sound associated with this type of a situation.

Thank you and we will meet once again tomorrow. Bye.