**Fundamentals of Acoustics**
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**Lecture – 42**
**Complex Power, Pressure & Velocity for a Spherical Source**

Hello, welcome to fundamentals of acoustics. Today is the last day of this week which is the 7th week of this course. And today what we will do is, we will develop relations for power complex power and velocity, for point source or not necessarily point source, but a source with finite radius r naught.

But here, what we planned to do is that, we will calculate the relations for pressure and velocity, in terms of boundary conditions of spherical source which is emitting sound.

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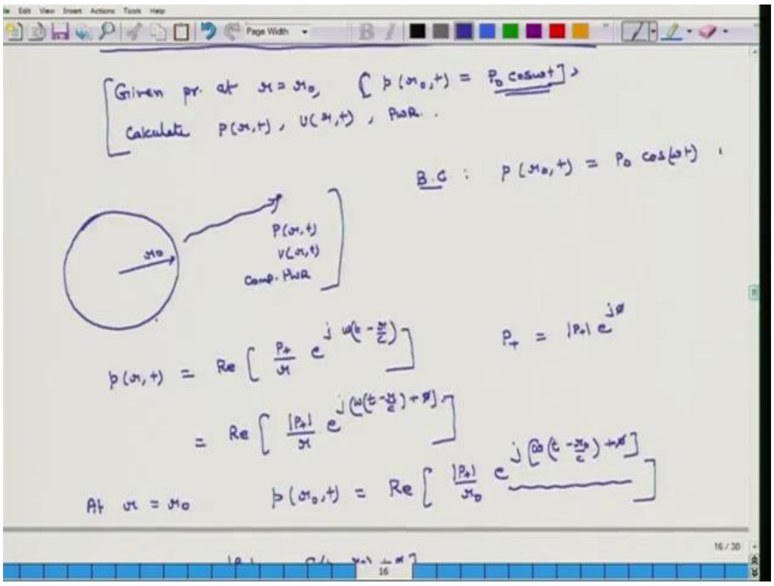

So, our aim is Complex Power, Pressure Velocity. So this is not a spherical source, but for a spherical source. It is not a point source but a spherical source of final width.

So, it is Spherical Source. So, here we are assuming that the spherical source is such that it is anything sound waves uniformly at all points, in all the directions, so it is uniformly expanding and contracting at all points.

So, problem a statement is that given pressure at r is equal to r naught, and what is the pressure? P at r naught t, equals some constant P naught, cosine omega t.

Calculate complex pressure, complex velocity and power, so this is what we have to calculate a complex power. So, to give a prospective, so suppose there is a sphere and it has a radius of r naught and the sphere is uniformly explained and a contracting so it is producing sound. As a consequence the sound is propagating in all the directions uniformly, it is propagating in all the directions uniformly. And at this, at a location close to the surface, of the sphere, the pressure is P naught, cosine omega t. So, we have to calculate at some point p, P complex pressure, complex velocity and power. This is also complex power; this is what we plan to do.

So, this is our boundary condition, our boundary condition, what is a boundary condition that pressure, at r naught t equals P naught cosine omega t.

Let us start this. First you will develop an expression write the expression, so pressure is a spherically symmetric system is, and if there are no reflected waves and we have developed this expression is, a real of some complex number P plus divided by r, e to the power of j, omega t, minus r over C. And now, P plus is a complex number, so it will have some magnitude and it will also have some phase, so let us call that phase phi.

If i plug this l, i get P plus divided by r, e to the power of j, omega t minus r over C plus phi. Now at r equals r naught, pressure, so what we are going to do is we are computing the boundary condition at r naught, so pressure at r naught is equal to real of P plus divided by r naught, e j omega t minus r naught by C plus phi, this entire thing.

So, pressure at location r naught is equal to. So, magnitude of P plus is a positive number, r naught is positive number so I can it out by r naught, and then this expression it is real component is cosine omega t minus r naught over C plus phi. So, this is expression 1.

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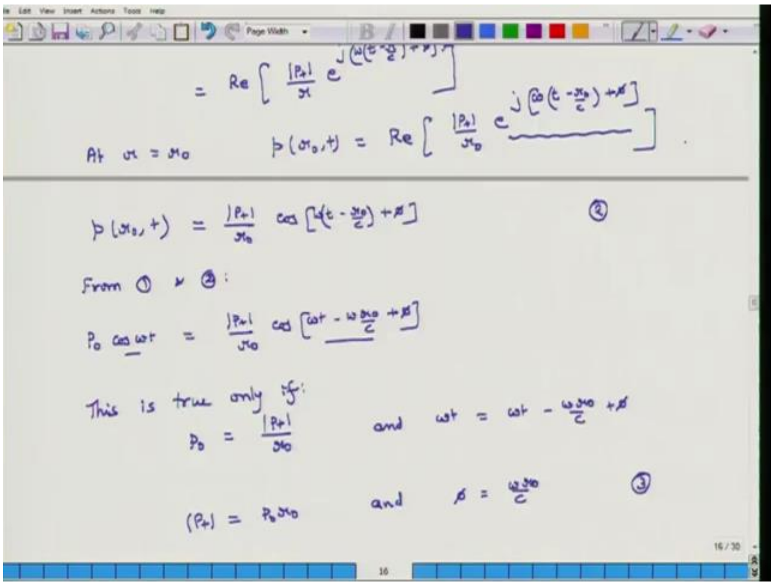


This is expression 2. Now we do not know P plus, but if we compare 1 and 2, we can calculate P plus, P plus from the boundary condition. From 1 and 2, what do will get from 1 and 2? We get P naught cosine omega t is equal to P plus by r naught, cosine omega t, minus omega r naught over C plus phi.

Now, this can be equal, right hand side, when left hand side, can be equal if, and only if. This will be true only if, P naught equals P plus over r naught, and omega t equals omega t minus omega r naught, over C plus phi.

So, what do I get? I get P plus equals P naught r naught, and I get omega t cancels out phi equals, omega r naught over C. So, this is equation 3 and if I substitute 3 in my expression for pressure, which is this equation let us call this equation A, then what I get is that the expression for pressure is p of r and t is equal to real of, P naught, r naught, over r P j omega r naught over C, e minus j, omega r over C, e j omega t.

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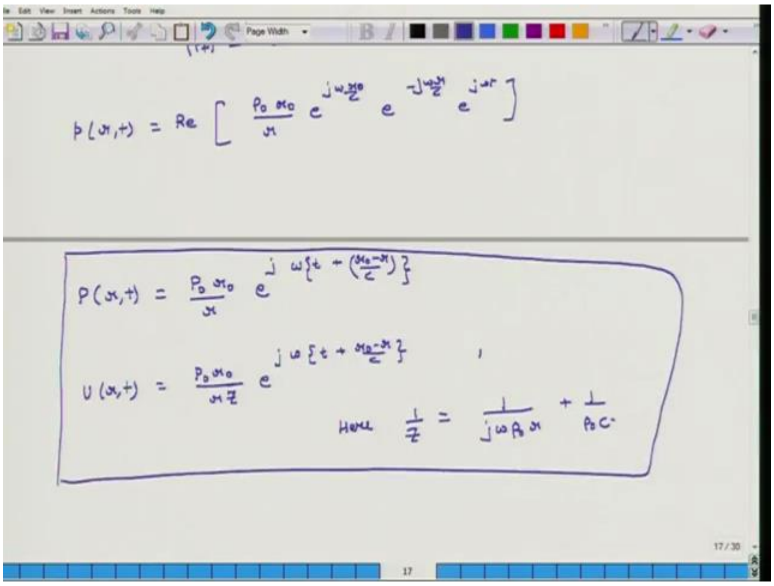

And if we go back and look at our earlier lectures, I can write that complex pressure is equal to so, I am going to just reframe this is equal to P naught, r naught, divided by r, e to the power of. So, it is e to the power of j, times omega t plus r naught minus r over C.

So, that is my complex pressure and my complex velocity is complex pressure, divided by r and also divided by Z times exponent, j omega, t minus, t plus, r naught minus r, divided by C, that is there. And Z it changes with r and we had calculated that, actually I will write the expression for 1 over Z, so Z. So, here 1 over Z, equals 1 over j omega rho naught r, plus 1 over rho naught C. So, this is my general expression for complex power, complex pressure and complex velocity.

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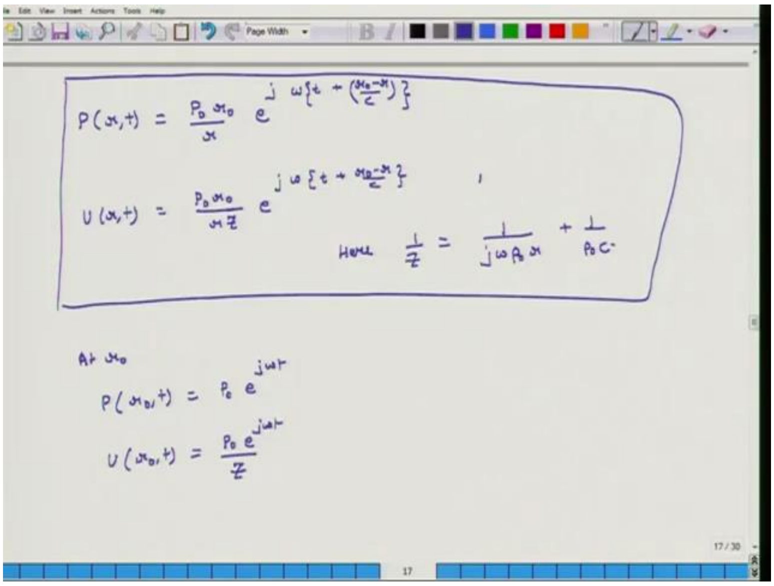


Now, we will calculate at r naught, at r naught, complex power no pressure, pressure complex pressure at r naught t is equal to P naught exponent j omega t. I can just plugin, r is equal to r naught here and I will get the same expression. Also U, r naught t equals, P naught e j omega t by Z by Z.

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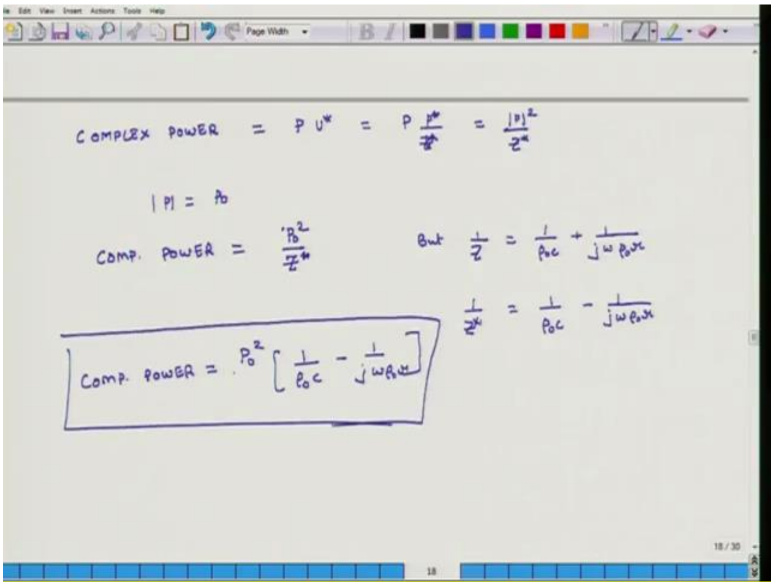


And lastly we will develop our relation for Complex Power, that is defined as, P times U star right. If I take it is real value, then I will get the real power dissipated; complex power is real plus whatever is the cyclic power. So, this is equal to a new star is P, times

P star divided by Z star, and P p star is modulus of P whole square divided by Z star. What is modulus of P? Now modulus of P is equal to P naught. So, complex power is equal to P naught square by Z, and Z star sorry Z star. But 1 over Z equals 1 over rho naught C, plus 1 over j omega rho naught r. So, 1 over Z star is equal to 1 over rho naught C, minus 1 over j omega rho naught r. So, complex power is equal to, so this is P naught square by Z star. So, I am just multiplying by 1 over rho naught C minus 1 over j omega rho naught r, and that is my expression for complex power.

What this tells us is that, when I have a sound source which is being emitted in a spherically symmetric way, this portion of the power, it gets dissipated, so it just travels out and never comes back.

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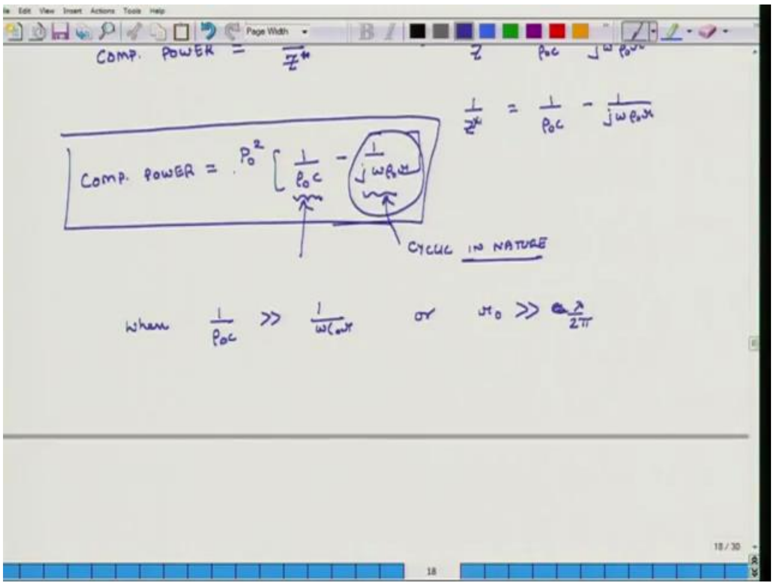


But this portion of the power is cyclic in nature. It is cyclic in nature. So, in part of the cycle it gets emitted by the system, and in other cycle half of the cycle it gets back in to the system. For instance like, if this is something similar to a capacitance or an inductor, in half of the cycle power is getting out of the source and then other half of the cycle power comes back in to the source. So, in spherically symmetric ways which are propagating outside, some power gets dissipated which relates to 1 over rho C.

But there is some other power, which gets those bia can 4th in to the system. Now, when 1 over rho naught C is extremely large compare to 1 over j omega rho naught r. So, this j should not be there. In that case or what this means is when r naught is extremely large compare to, lambda over 2 pi. Then this portion of the power is negligible and on the power which is there is getting dissipated out in the system. So that means is that if our sphere is very large compare to 1 6th of the wavelength, which means r naught is very large compare to the wavelength.

Then all the power, which is being consumed by the system it just gets dissipated and nothing comes back. So, that pretty much completes our discussion for today and we will meet once again tomorrow.

Thank you and have a great day bye.