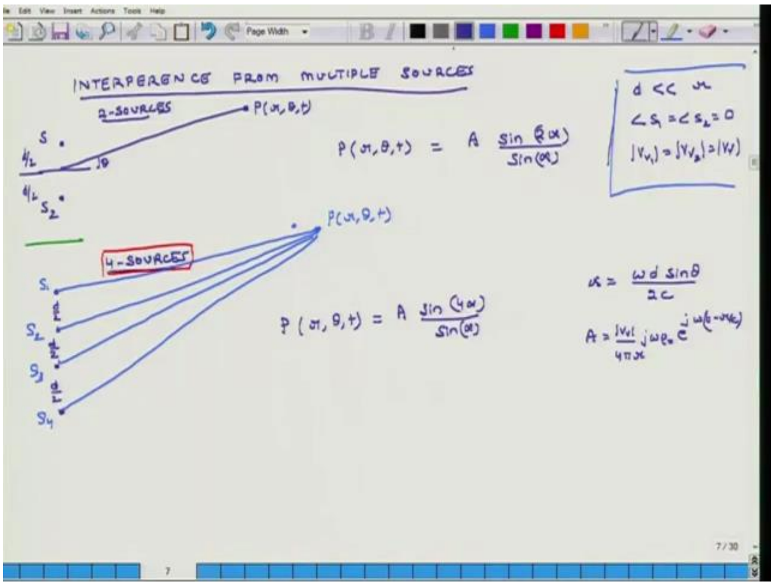
Fundamentals of Acoustics

Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 40
Interference of Sound Sources - Part IV

Hello, welcome to Fundamentals of Acoustics. Today is the 4th day of the 7th week of this course on acoustics and in last 3 lectures we have been discussing how sound waves interfere with each other. And specifically speaking in last two lectures we have developed expressions for interference of waves. First in presence of two sound sources separated by distance d; and actually in last class we discussed how sound sources interact when there are 4 sources.

(Refer Slide Time: 00:50)

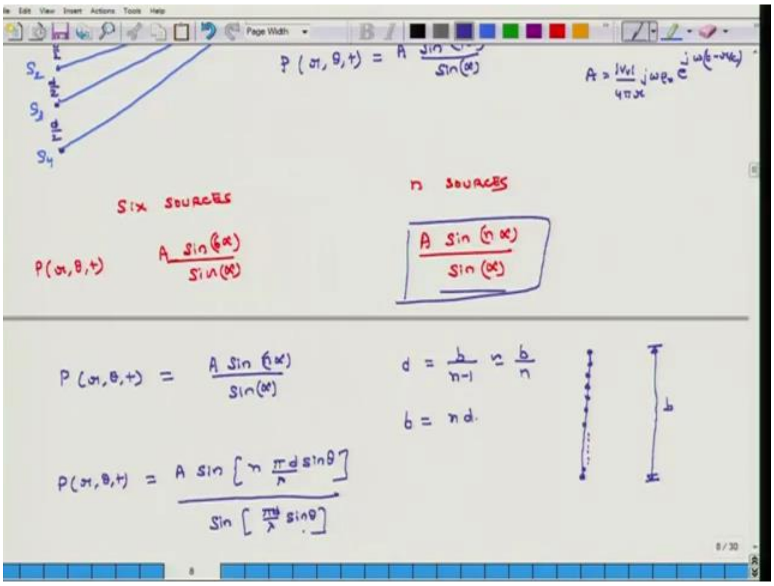


So, let us very quickly recap these results. So, the theme is interference from multiple sources. So, that are theme multiple sources, what happens when there are multiple sources. So, if there were two sources S 1 and S 2 and we were interested in pressure at r theta and t location and this angle was theta, then the result which we had developed was pressure at far field is equal to term A times sin 2 alpha divided by sin alpha. Of course, this result was is correct as long as some important assumptions hold, what are those assumptions that d is small or actually very small compare to r, d by 2 is this distance and d by 2 is the distance from midline to the second source. The second thing was that phase of S 1 is same as phase of S 2 and that is equal to 0 degrees and the third assumption which we had made was at the volume velocity magnitudes were same.

So, these 3 assumptions hold then this is the expression. So, that was for 2 sources then we developed an expression for 4 sources. So, this was for 2 sources and then we developed an expression when there are 4 sources. So, in this case we are still the point is a still far away let us say this is the point P r theta t and sound is coming from all the 4 sources and these separation distance between each of these sources as d over 2 and the sources are designated as S 1, S 2, S 3, S 4 and the assumptions which we had made earlier, similar assumptions are made in this case also. So, in such a case, P of r theta t equals A sin 4 alpha divided by sin alpha. And alpha we had defined as omega d sin of theta divided by 2 c.

And A was defined as so it was a there it will be largest expression. So, it is V v divided by 4 pi r, j omega rho naught, e to the power of j omega t minus r over c. So, that is what A was defined as. I can continue adding up sources and what I will find is that I will find a very specific and clear trend that for 6 sources.

(Refer Slide Time: 05:01)



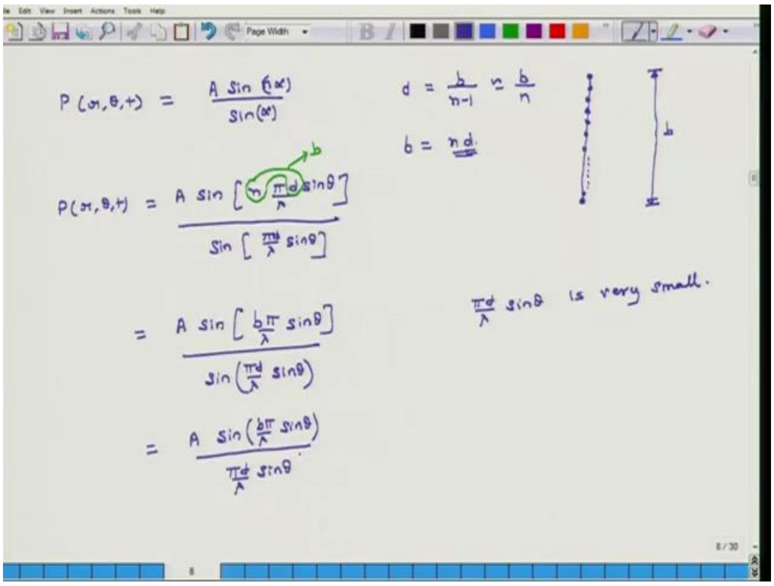
So, if I have 6 sources, my P r theta it will be A sin 6 alpha divided by sin alpha and so on and so forth. So, if there are n sources and n is a even number, I can generalize this strength through principle of mathematical induction. So, I can say it is equal to A sin n alpha divided by sin of alpha.

Now, n extremely large number it could be infinite and example of n sources could be that suppose you have a string, for instance of a guitar or something and if all points in the string are moving in and out by the same amount because volume velocity we are saying is same and they moving in face to each other, then that kind of a source could be considered as an array with n sources. So, now what we want to develop is an expression for what happens when there are n sources; because mathematically if n is larger, theoretically if n is infinite, then we cannot do this computation because I cannot use infinity to do calculations. So, I have to take it mathematically in some different form.

But if there are sources number of sources is very large, then P r theta t equals A sin n alpha divided by sin alpha. Now consider an array, so there lots and lots of sources and each of these sources are separated by a small distance d. And as suppose the overall length of the array is b and b is a finite number may be the string is 20 centimeters long so it is a finite number and if n is extremely large then d is equal to b over n minus 1 where 1 is the number of sources and if n is extremely large, then this can also be approximated as b over n. So, d is b over n so I can write as b equals n times d.

So, complex pressure which depends on r theta and t, is equal to A sin and n times alpha and alpha is pi d sin theta divided by lambda, divided by sin of alpha. So, where alpha is again pi d over lambda sin of theta. Now once again in this expression I have to use the term n which is extremely large and I have to use this expression d and d is also extremely small. So, on one case I am having an extremely large number theoretically infinite in the numerator and theoretically zero number in the denominator. So, I cannot do this mathematical operation. So, we have to somehow transform this equation into something usable. So, that is what our goal is. So, we are seen that n times d equal b.

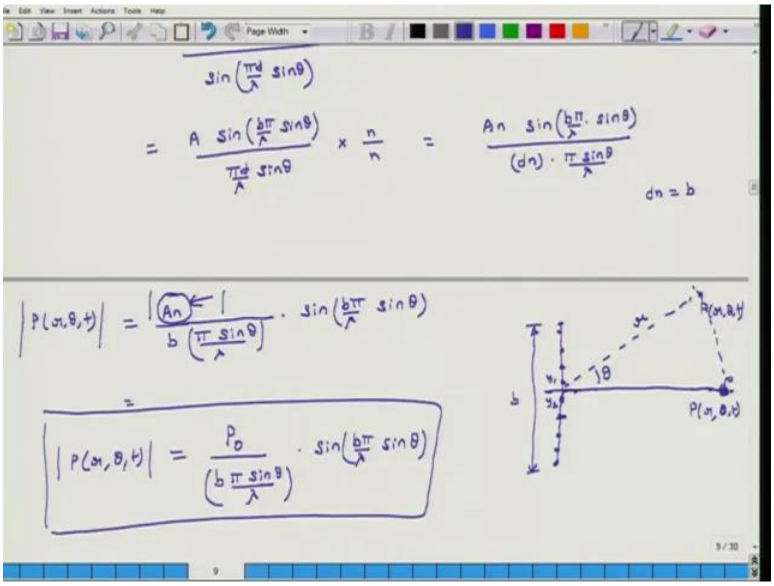
(Refer Slide Time: 09:13)



So what we do is, n times d is b and b is a finite number. So, I can write it as A sin b pi over lambda sin of theta, divided by sin pi d over lambda sin of theta. So, I have got in rid of n and d in the numerator, now I want to do something similar for the denominator also. So what we do is, we know that pi d over lambda times sin theta is very small, why is it very small because this finite string which is having n sources and n in as a extremely large number and if n is extremely large then d has to be very small. If d is a small pi is finite, lambda is finite, sin theta cannot exceed 1. So, pi d over sin theta is also extremely small.

So, for that is case this relation can be expressed as A sin b pi over lambda sin theta divided by pi d over lambda sin theta because sin of a small angle is same as the angle itself.

(Refer Slide Time: 11:00)

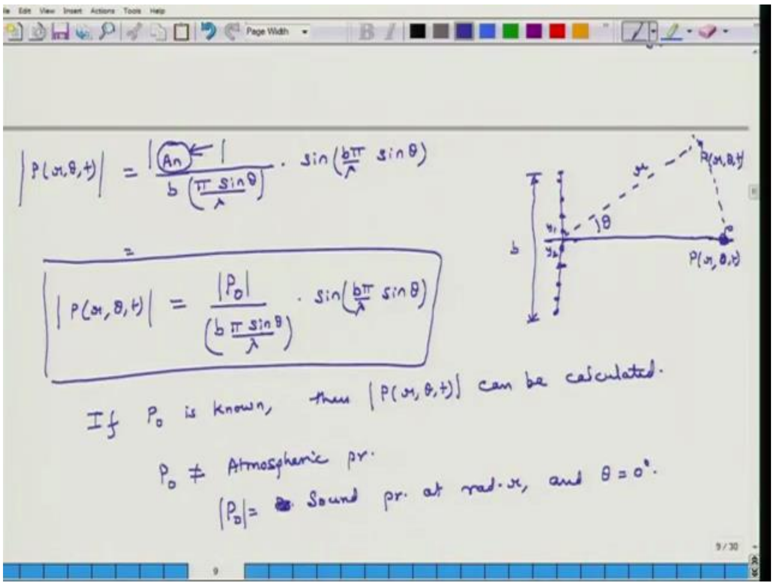


And now what I do is I multiply the numerator by n and divide the numerator by n. So, I get A n sin b pi over lambda times time theta divided by d n times pi sin theta over lambda. Now d n equals b, because b is the distance between sources this is the expression d n equals b. So, I can rewrite it as A n by b pi sin theta by lambda, times sin of b pi over lambda times sin of theta. So, that is my expression for P r theta t.

Now what we do is, so suppose this is the array with all these infinite or extremely large number of sources, and this array is b units long. Let us say this is my 0 degrees. So, if point p is located here, then this is point p r theta t and this angle is theta. So, what this expression tells is that at angle theta, what is going to be the complex pressure. Now for the same value of r, if theta is equal to 0 what will happen? The sound pressure is going to be maximum at theta equals 0, why will it be maximum? Because first thing is all the sources are moving in and out at the same time. So, at theta equals 0, the contribution from this source and the source which is just opposite of the median line will add up which are equidistant, because the distance travel by sound wave from this point to this. So, let us say this is P 1or not P 1; I will use a different term, let us say this is point y 1 and y 2, so at this location what is P? P is radiuses r theta is 0 and time is t.

So, the contribution of y 1 and y 2 will add up and they will not they will all come in phase. Similarly, y 3 and y 4 they will also add up. So, this the sound pressure level at p r 0 t it will be maximum, at other locations they may constructively or destructively interfere. So, what that means is, this term it represents the complex pressure at theta equals 0 that is what it means. So, if I take the magnitude of both the sides, then the magnitude of LHS is magnitude of the numerator, times this entire factor. So, the magnitude of sounds at position theta is equal to magnitude of sound at angle 0 which is P 0, divided by b pi sin theta by lambda times sin of b pi by lambda sin of theta. So, this is our final expression and what this expression tells us is that, at any angle if I have to calculate the value of sound pressure level, I can calculate in terms of theta and lambda and b as long as I know the value of pressure at theta equals 0.

(Refer Slide Time: 16:00)



So, the final comment if P 0 is known. So, this P 0 is then p r theta t can be calculated p r theta can be (Refer Time: 16:30) and here P 0 is not atmospheric pressure, this is just because we have run out of symbols rather P 0 equals sound pressure at radius r and theta equals 0 degrees. So, that (Refer Time: 16:59).

So, this concludes our discussion for today, and tomorrow we will like to such gears and we will introduce one term called directivity and the day after that we will talk about power in context of a spherically propagating waves. So, that is very much for it today and I look forward to seeing you all tomorrow again.

Thank you. Bye.