

**Fundamentals of Acoustics**  
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**Lecture – 38**  
**Interference of Sound Sources – Part II**

Hello, welcome to Fundamentals of Acoustics. Today is the second day of the 7th week of this course and what we plan to discuss is continue our discussion on interference pattern as the emitting from multiple sources. So, yesterday we had discussed, how sound waves constructively and destructively interfere, when there are 2 sources separated by some distance  $D$ . What we plan to do today is we will revisit that discussion with a somewhat simplified assumption with an additional assumption, and we will again develop the relation for two sources and then may be today as well as tomorrow and day after tomorrow we will develop relations for 4, 6 and finally  $n$  sources. And when we have  $n$  sources then using that information we will be able to figure out that suppose you have a string, and it is emitting some sound source, and if the string is vibrating uniformly at all points then what kind of sound patterns we can expect. So, with that, we will start our discussion.

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INTERFERENCE FROM MULTIPLE SOURCES

Diagram showing two sources  $S_1$  and  $S_2$  separated by distance  $d$ . A point  $P(\theta, \theta, t)$  is shown at a distance  $r$  from the midpoint. Path lengths are  $r_1$  and  $r_2$ . The condition  $r \gg d$  is noted.

$$S_1 \equiv |V_{v_1}| = |V| \quad S_2 \equiv |V_{v_2}| = |V|$$

$$\angle v_{v_2} - \angle v_{v_1} = 0$$

$$r \gg d$$

$$P(\theta, \theta, t) = \frac{|V_v| j \omega \rho_0}{4\pi r} e^{j\omega(t - r/c)} \left[ e^{-j\frac{d \sin \theta}{2c} \omega} + e^{+j\frac{d \sin \theta}{2c} \omega} \right]$$

$$\frac{\omega}{2c} = \frac{\pi}{\lambda}$$

$$P(\theta, \theta, t) = A \left[ e^{-j\frac{d \sin \theta}{\lambda} \pi} + e^{+j\frac{d \sin \theta}{\lambda} \pi} \right] = A \left[ e^{-j\pi} + e^{+j\pi} \right]$$

$$= A \left[ e^{j\pi} + e^{-j\pi} \right] \times \frac{[e^{j\pi} - e^{-j\pi}]}{[e^{j\pi} - e^{-j\pi}]}$$

So, interference from multiple sources, so, suppose I have 2 sources; I will again start the discussion using 2 sources. So, that is  $r_1$  that is  $r_2$ . I am interested in finding

pressure at point P, which is distance  $r$  away. So, that is my median, this is my median line, the horizontal line, the distance from the median line, to point P is  $r$ , and this distance is  $D/2$ , and this distance  $D/2$ , the 2 sources are -  $S_1, S_2$  and the volume velocity for  $S_1$ , is  $V_{v1}$ , and its magnitude is such it is equal to  $V_v$ .

Similarly, for  $S_2$  we have  $V_{v2}$ , and its magnitude is equal to again  $V_v$ . But here we make one additional assumption, which makes things relatively simple, and that is the phase of  $V_{v2}$  minus that of  $V_{v1}$  is 0. So, earlier we had assumed that this  $V$ , but here it is 0. And the second thing is,  $r$  is still very large compared to the distance between 2 sources.

So, with that understanding, we will develop our expression one more time, and offer it in a somewhat different form it will be the same result or similar result. So, pressure at  $r$  theta  $t$  is equal to  $V_v$ , times  $j\omega\rho_0$  divided by  $4\pi r$ ,  $e^{j\omega t - r/c}$ , and in the brackets, in the brackets what we get is  $e^{-j d \sin \theta / 2c} + e^{j d \sin \theta / 2c}$  times  $\omega$ , plus  $e^{-j\phi} + e^{j\phi}$  in both these terms, but that has gone away.

Now, we say that  $\omega/2c$ , is equal to  $\pi/\lambda$ , and we call this entire expression as  $A$ . So, my pressure or complex pressure, equals  $A$  times exponent to the power of minus  $j$ , and  $\omega/2c$  is  $\pi/\lambda$ . So,  $j d \pi/\lambda \sin \theta$ , plus  $e^{j d \pi/\lambda \sin \theta}$ , and then of course, there is a  $j$  here and there is a  $j$  here.

Now, what we do is we define this thing as  $\alpha$ . So, this is equal to  $A$ ,  $e$  to the power of minus  $j\alpha$ , plus  $e$  to the power plus  $j\alpha$ . And the next thing is I take this expression, and in the parenthesis I have  $e^{j\alpha}$ , plus  $e^{-j\alpha}$ , and then I do a mathematical operation on this. So, what I do is I multiply it by  $e^{j\alpha}$ , minus  $e^{-j\alpha}$ , and I divided by the same thing  $e^{j\alpha}$ , minus  $e^{-j\alpha}$ .

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The image shows a handwritten derivation on a whiteboard. At the top, the pressure  $p(\alpha, \theta, t)$  is given as the sum of two waves:  $p(\alpha, \theta, t) = \frac{|V_r| j \omega P_0}{4\pi r} e^{-j\omega t + jkx} + \frac{|V_r| j \omega P_0}{4\pi r} e^{-j\omega t - jkx}$ . The amplitude  $A$  is identified as  $\frac{|V_r| j \omega P_0}{4\pi r}$ . The wave number  $k$  is related to wavelength  $\lambda$  by  $k = \frac{2\pi}{\lambda}$ . The expression is then written as  $p(\alpha, \theta, t) = A [e^{-j\omega t + j\frac{2\pi}{\lambda} x} + e^{-j\omega t - j\frac{2\pi}{\lambda} x}] = A [e^{-j\omega t} (e^{j\frac{2\pi}{\lambda} x} + e^{-j\frac{2\pi}{\lambda} x})]$ . This is further simplified using the identity  $e^{j\theta} + e^{-j\theta} = \frac{e^{j\theta} - e^{-j\theta}}{j} \cdot j$ . The numerator becomes  $(e^{2j\frac{2\pi}{\lambda} x} - e^{-2j\frac{2\pi}{\lambda} x}) + (1 - 1)$  and the denominator is  $2j \sin \alpha$ . The final result is  $p(\alpha, \theta, t) = \frac{2A \sin(2\alpha)}{\sin(\alpha)}$ .

So, what do I get? In the numerator, what I get is, so when I multiply  $e$  to the power of  $j$  alpha by  $e$  to the power of  $j$  alpha, I get  $e$  to the power of  $2j$  alpha, I also get  $e$  to the power of  $-2j$  alpha. So, this is one thing I get, and then when I multiply,  $e$  to the power of  $j$  alpha with this guy so I get 1, and when I multiply  $-e$  to the power of  $j$  alpha, with  $e$  to the power of  $j$  alpha I get  $-1$ . And in the denominator,  $e$  to the power of  $j$  alpha minus,  $e$  to the power of  $-j$  alpha is  $2j \sin \alpha$ . And the numerator becomes,  $2A \sin 2\alpha$ , and in the denominator I have  $2j \sin \alpha$ ; so  $p$  of  $r$  theta  $t$ . So, when I was doing this multiplication this thing should be a negative sign. Because, when I multiply this with this I get the first expression, and when I multiply this term with this term, I get the second expression and there is a negative sign here.

So,  $2j$   $2j$  cancel out, and essentially I am left with  $A \sin$  of  $2\alpha$  divided by  $\sin$  of  $\alpha$ . So, if I have 2 sources my pressure at a far field point is a times  $\sin$  of  $2\alpha$  divided by  $\sin$  of  $\alpha$ .

Thank you and have a great day, bye.