Fundamentals of Acoustics
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Lecture – 37
Interference of Sound Sources - Part I

Hello welcome to Fundamentals of Acoustics, this is the 7th week of this course which is 12 week long. So, starting today we are going to enter into the second half of this course and till so far what we have covered is a somewhat detailed treatment of 1-D waves which travel in Cartesian frame of reference that is planar waves, we have also introduced the concept of impedance and we have developed some basic relations for spherical waves which are traveling outwards, that is when these waves do not get reflected and come back to the source.

So, with that what we plan to do over this week is extend our discussion for 1-D waves moving outwards in an is spherical fashion and is specially what we plan to do is to figure out what happens when 2 or more sources of such waves produce sound and these sound patterns or sound waves interfere with each other and they are perceived and what is the nature of these waves at distance are away from these 2 or more sound sources.

So, another words we are going to discuss, how waves interfere with each other and that is going to be pretty much the theme of the discussion for this week.

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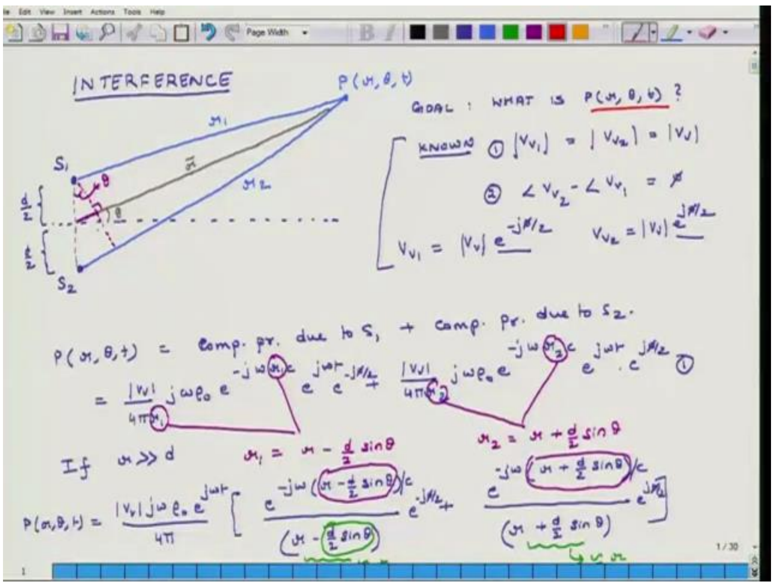

So, as I mentioned the theme of the discussion is going to be interference. So, we are going to discuss the interference of sound even (Refer Time: 02:17) from 2 is specific sources S 1 and S 2. So, these 2 sources are indicated here, this dotted line is the median line between these 2 sources and the distance of source S 1 from the median line is d over 2, and the distance of the second source from the median line is also d over 2; both these sources are emitting sound and what we are interested in is what happens. So, as sound is emitted it gets propagated and what we are interested in is what happens to the sound, which is perceived at point P and this point P is at a distance r at an angle theta away from the sources. So, what is r? We will draw that, so r is the vector as indicated here the distance between S 1 and point P is r 1 and the distance between S 2 and point P is r 2.

The other thing is that we want to define what is theta? So, theta is the angle of between vector r and the median line. So, with this understanding our aim is so our goal is what is the value of pressure at point p, which is r distance away, theta angle away and at any time; so what is p r t what is known? So, what we do know is that the volume velocity of sound source S 1 is same as the volume velocity of sound source S 2? And let us say we call it V v. So, the volume velocity is not same, but the magnitude of volume velocity of source 1 and source 2 are same. So, this is the first thing which is known; the second thing which is known is where the difference in phase of V v 1 minus V v 2. So, that phases phi actually it the other way round. So, V v 2 minus V v 1 is phi.

So, if these 2 parameters are known that the volume velocity is V v and the phase difference is phi, then we can say V v 1 volume velocity is equal to. So, V v 1 is the magnitude of the volume velocity is V v 1 magnitude V v, times e minus j phi over 2 and V v 2 equals V v its magnitude times e j phi over 2. So, with this knowledge we are interested in finding p r t; p r theta t which is the pressure at location r at an angle theta for time t. So, we will write down this expression. So, pressure at point p and this is capital P because its complex pressure. So, complex pressure at point p, is going to be complex pressure due to S1 plus complex pressure due to S 2. So, we will write down each of these expressions. So, what is a complex pressure due to S 1? So, it is V v over 4 pi r 1, j omega rho naught, e minus j omega r 1 c, e j omega t.

So, where did we get this expression from in last class we had developed the expression between volume velocity and with complex pressure. So, this is the complex pressure due to S 1 and similarly the complex pressure due to S 2 is V v divided by 4 pi r 2 because the second source is r 2 distance away from point p, times j omega rho naught, e minus j omega r 2 c, e j omega t. So, let us call this equation 1. Now what we have done here is that in equation 1 we have expressed complex pressure at point p, in terms of r 1 and r 2. So, now, what we will do is we will develop an expression between r 1 r 2 d and theta.

So, if r is extremely large compared to d, which means that this point is extremely far away then let us look at that. So, if I drop a normal this angle will be 90 degrees, this will be theta. So, if I am very far away the point p is extremely far away, then r vector r and r 1 they will be almost parallel to each other in such a case. So, if r is extremely large compared to d, then in such a case r 1 is equal to so this distance r 1 is equal to r minus d over 2 sin theta and r 2 is equal to r plus d over 2 sin theta.

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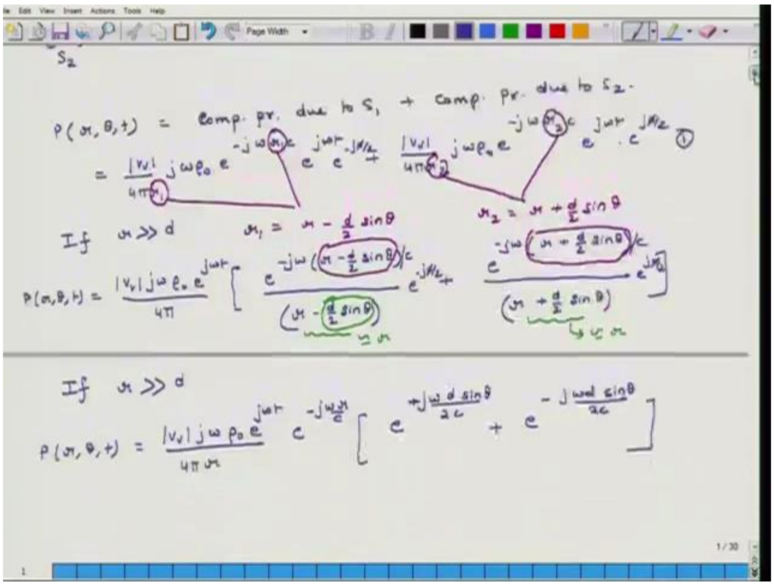

So with this understanding what we do is we put r 1 from here and we put r 2 from this relation. So, using this, what we get is P r theta t equals, now what I will do is I will take some of the components some of the portions of these this equation as common some terms. So, V v j omega rho naught e to the power of j omega t is common and also common is divided by 4 pi. So, in the brackets I get e to the power of minus j omega r minus d over 2 sin theta, divided by r minus d over 2 sin theta, plus e to the power of j omega there has to be negative sign, times r plus d over 2 sin theta and there is also c here, divided by r plus d over 2 sin theta.

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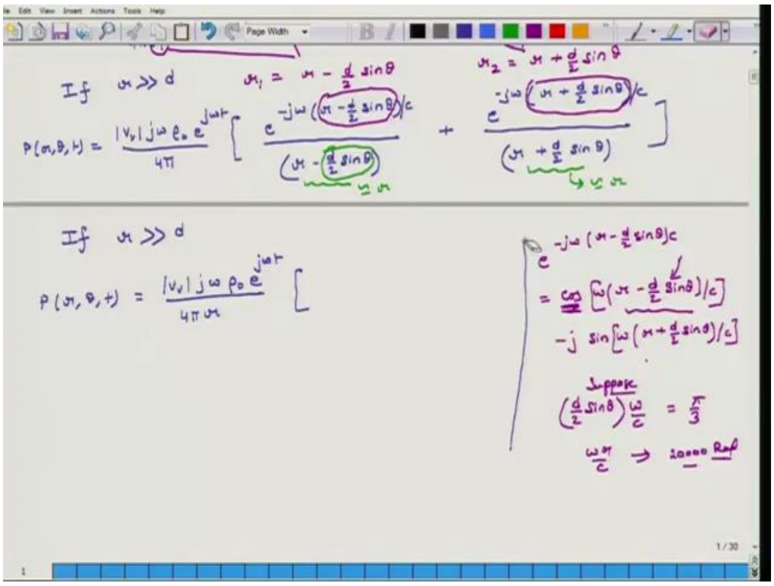

Now, if r is extremely large compared to d then in this term d over 2 sin theta sin theta maximum value of sin theta is 1. So, the product of d over 2 in sin theta can never exceed d over 2 and if r is extremely large compared to d, then in this thing this can be approximated as r, similarly this term in the denominator can be approximated as r. So, I am going to rewrite my expression P r theta t is equal to V v j omega rho naught e j omega t, divided by r. Now the other thing is that one may do a similar approximation in this term also, but then that will not be mathematically wise why I say that. So, just consider this, e to the power of minus j omega r minus d by 2 sin theta, c is equal to what? It is cosine of omega r minus d over 2 sin theta times c plus. So, actually there is a minus sign. So, it has to be minus j sin omega r plus d over 2 sin theta c.

So, actually there has a small error in this should be divided by c, now here I cannot drop the d over 2 sin theta term, why can I not drop it? I cannot drop it for the simple reason that suppose d over 2 sin theta times omega by c, suppose its value comes to be let us say pi over 3 radiance and omega r over c it comes to be let us say r is extremely large, it may be let us say r is 20000 radiance, but then an addition of pi over 3 will cause a significant change in the cosine of this angle, if the cosine of right if this number is pi over 2 then small changes in d over 2 sin theta times omega over c can cause significant changes in the cosine and sin of this angle. So, I cannot make the same approximation in this term which is encircled in the purple circle, I cannot do the same thing here. So, here I have to retain this otherwise I will be making a mistake. So with that understanding what we have to do is, you have to retain it and do the correct mathematics.

So, I am going to erase this because I need some space. So, I will continue writing this expression. So, so the other thing I can do this I can take e to the power of minus j omega r over c outside the bracket.

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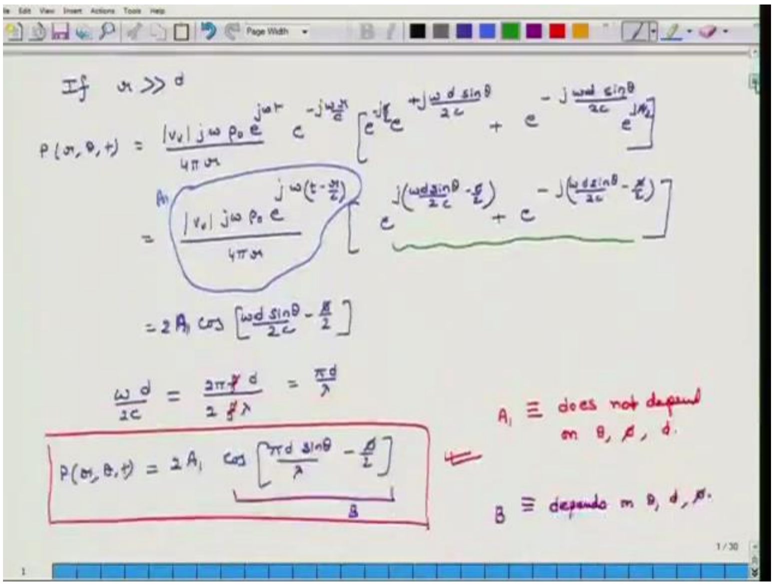

So, I am taken that outside the bracket and in the bracket I have e minus j omega d sin theta over 2 c and minus and minus becomes positive, plus e and here minus and positive became negative. So, minus j omega d sin theta divided by 2 c and then I think I made important unforgivable mistake. So, when I was doing this substitution for volume velocities, I should have also included the phase term which I did not. So, I will do that I will rectify that mistake here. So, here it as to be e minus j phi over 2 and here it has to be e j phi over 2, same thing here e minus j phi over 2 and e j phi over two. So, here it is minus j phi over 2 and this is e j phi over 2. So, volume velocity magnitude j omega rho naught e j omega t minus r over c, divided by 4pi r times exponent of j omega d sin theta over 2, minus phi over 2, plus e minus j omega d sin theta over 2 c minus phi over 2.

So, let us call this expression as A 1. So, this is A 1 and this expression in the parenthesis is what? So, this is something like e exponent j to the power of beta plus exponent minus j to the power of beta. So, this is A 1 times, this is equal to A 1 cosine omega d sin theta over 2 minus phi over 2. So, they should be a c here so that is there and there will be a factor of 2 here. Now omega d over 2 c equals. So, what is omega? Omega is 2 pi f d divided by 2 times f lambda. So, f f that is the frequency they get canceled out. So, this is equal to and 2 also get canceled out, so this is equal to pi d over lambda. So, my expression for complex pressure at point p is equal to A 1 times or actually it is 2 A 1 times cosine pi d sin theta divided by lambda minus phi over 2.

So, that is my expression. So, what does this expression tell us, that if there are 2 sound sources S 1 and S 2 separated by total distance d, then the sound experienced by at point t which is r distance away and at angle theta, the complex pressure is expressed by this relation; now let us look at this expression in detail, what we observe is that A 1. So, what is A 1? Its magnitude of volume velocity time's j omega rho naught e j omega t minus r over c divided by 4 pi r.

So, A 1 does not depend. So, it does not depend on what? It does not depend on theta, it does not depend on phase and it does not depend on the distance between the 2 sources, it only depends on r and the angular frequency. So, if I have to so and let us call this expression as this part term as B. So, B it depends on theta d and phi, if I have to figure out that how is sound going to change in this picture. So, in this picture as theta increases this p is going to move like this. So, as theta changes this p is going to move like this. So, for at a given distance r as I increase or decrease theta how will p change, that change will be decided by the behavior of B, because B is the parameter which depends on theta phi and d this is important to understand.

So, if I have to plot the variation of pressure with respect to theta, then that plot will have same shape as the plot for B as I change theta. So, we will actually make on such plot. So, to construct a plot we need to know the value of d, we need to know the value of theta and we need to know the value of phi, we will make some assumptions.

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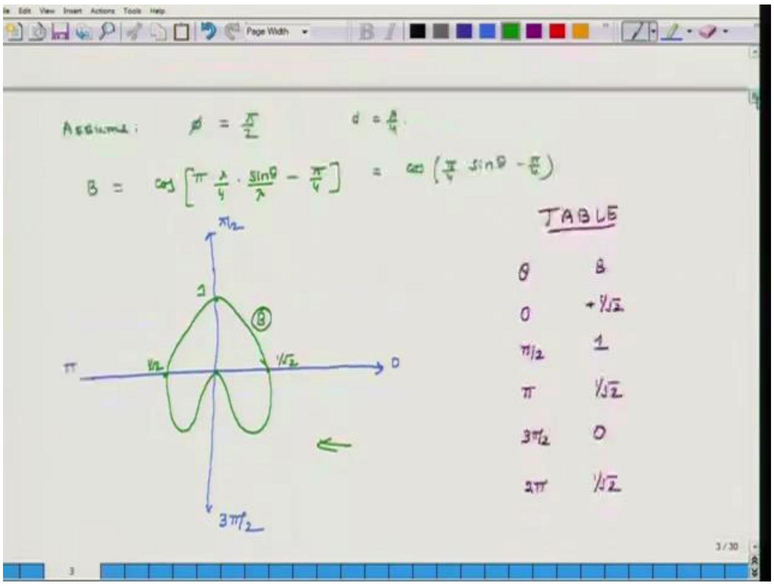

So, we assume that phi which is the phase of the volume velocity source is equal to pi over 2 and we pick d that is the distance between the sources such that d is equal to lambda over 4. So, then B equals cosine of pi time's lambda over 4, times sin theta divided by lambda minus pi over 4. So, this is equal to cosine of pi over 4, lambda cancels out times sin theta minus pi over 4. So, first let us construct a table. So, we will construct a table for B.

So, let us say theta. So, theta can have different value 0 pi over 2 radiance, pi radiance, 3 pi over 2 radiance and then of course 2 pi; then what is the value of B is. B is what? So, when theta equals 0 then it is cosine pi over 4 minus pi over 4 or when theta is equal to 0 then it is cosine pi over 4 minus pi over 4. So, that is 1 over is square root of 2, when theta equals pi over 2 then sin theta is 1 pi over 4 minus pi over 4 is 0. So, cosine was 0 is 1. When theta equals pi radiance then this is again 0 and the value of B comes to be one over square root of 2, when theta equals 270 degrees or 3 pi over 2 radiance, then this is sin theta is minus 1. So, minus pi over 4 minus pi over 4 becomes minus pi over 2. So, this value becomes 0 and at 2 pi it is one over this square root of 2.

So, with this these points we will draw polar plot. So, in the polar plot this is 0 degrees, this is pi over 2 radiance, this is pi radiance and this is three pi over 2 radiance and what we will do is this is my origin. So, the distance from the origin will reflect the value of d. So, at x theta equals 0 degrees, it is what is it? It is 1 over root 2, same thing here at pi also its 1 over root 2. So, the distance from the origin is same at pi over 2 it is1, and at 3 pi over 2 it is 0. So, my polar plot excuses me it is a little. So, it will look something like this; now this dip on the negative side is not that sharp, it will be a little softer dip, but the point is. So, this is the plot for B and what this says is that if point B is located at theta equals 90 degrees which is here, then it will hear the maximum sound pressure level because the value is 1 here. So, this is 1, this is 1 over root 2, this is one over root 2, and if our micro phone is located at the other angle which is minus 2017 or which is 270 degrees, then the sound perceived by the micro phone will be minimum and theoretically it will be 0 pascals.

So, this is the effect of interference and what we see is, that when 2 sources interfere even though 2 individually each source may be is emitting is spherically symmetric wave, but if there half interfere phase difference then they interfere with each other to produce some interesting polar patterns and these polar patterns can be calculated using the methodology, which we have discussed in this lecture.

So, with this I conclude the discussion for today and tomorrow will continue our discussion on interference, but with different situation.

Thank you and have a great day bye.