Fundamentals of Acoustics
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Lecture – 36
Comparison of Impedances for a Radially Propagating Wave and a Planar Wave

Hello. Welcome to Fundamentals of Acoustics. Today is the last day of the six week of this course. And what we will do today is essentially continuation of what we were doing yesterday that is discussion of Kundt's tube. And, the first part of this lecture will be essentially summarizing whatever we have discussed in terms of impedance for open tube, close tube, as well as the Kundt's tube. And then in the second part what we will do is we will also go back to spherically propagating waves and compare the radial the impedance of a radially propagating wave with that of a planar wave.

So, that will set the ball rolling for more detailed discussion on spherical propagating waves which we will restart in the next week; that is week 7.

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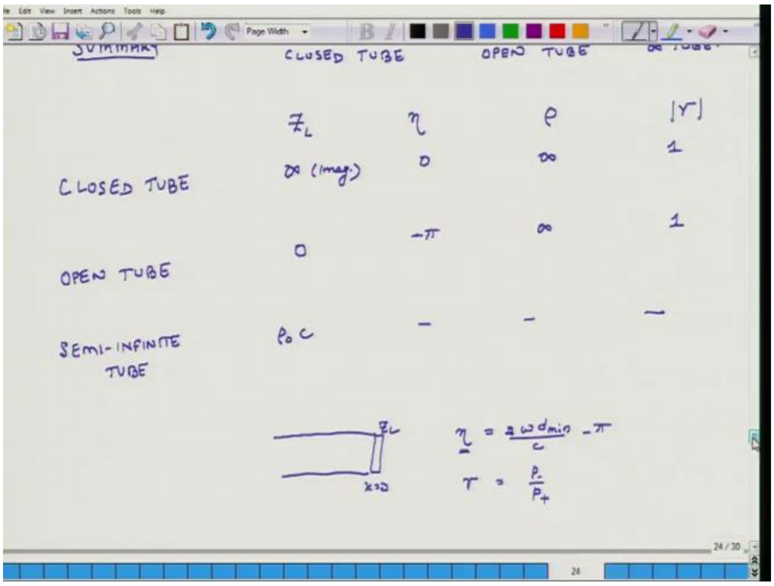

So, in the first part of this lecture we will develop a summary. So, we had discussed closed tube, and then we had discussed an open tube and then semi-infinite tube. So, we will summarize for these systems the value of Z eta, rho, and also the magnitude of gamma. So closed tube, then we have open tube, and then we have finally semi-infinite tube. So, what we will do is we will construct a table for Z which is specific acoustic impedance eta, rho, and rho is the ratio of maximum pressure amplitude and minimum pressure amplitude as I travel in the tube, and then finally gamma.

So for a closed tube this is infinity, Z is infinity why because, Z is defined. So, actually this is Z L. So, it is the tube, so x is equal to 0 and this is Z L. So, Z L is 0 is infinite because the complex amplitude of pressure is non zero and complex amplitude of velocity is 0, so this is infinite. But it is not a real number it is an imaginary number. So, if I plot it on the complex plane it will be positioned that at a infinite distance from origin, but on the y axis. Then we look at eta and this is equal to 2 omega d min divided by c minus pi.

Now, d min is the location where pressure is going to be minimum and if we do the math; in this case d min will correspond to half of the wave length of the system. So, if I put in d min as lambda over 2 in this equation. Then this entire term 2 omega d min divided by c comes to be pi, so pi minus pi is 0. And then rho is the ratio of maximum pressure amplitude divided by minimum pressure amplitude, so maximum pressure amplitude is non zero and minimum pressure amplitude is 0 which is at some specific location; so this is infinity. And then gamma, so gamma was defined as P minus over P plus. And for a closed tube this ratio P minus and P plus is 1; P minus is equal to P plus so it is 1.

For an open tube the value of Z L is 0, because pressure at the open end is 0. While the velocity is non zero, so 0 divided by non zero entity is 0. Eta is negative pi and its negative pi because the value of d min is 0, when d min is 0 then eta becomes negative pi. The ratio of rho ratio which is the ratio of maximum pressure amplitude divided by minimum pressure amplitude, maximum pressure amplitude is some number, but minimum it goes down to 0, so this is infinite. And then the value of gamma for an open tube it is P plus equals negative of P minus, but I am taking just the absolute value so it is a still 1. And then for a semi-infinite tube this is rho naught c, and all these terms they are not necessarily relevant in context of this table.

So, this is the discussion in context of impedance.

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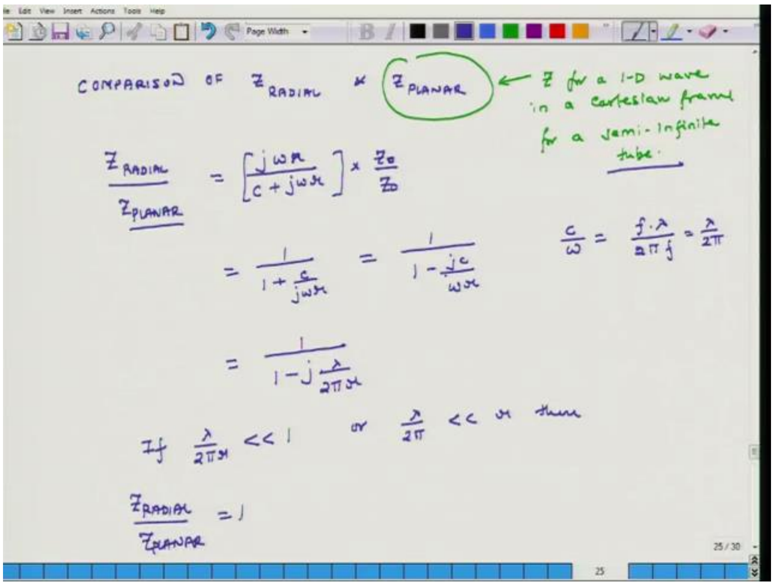

And the next thing which we will do today is we will compare of impedance for our radially propagating wave, so I will call it Z radial and planar wave which is a typically a wave which is traveling in a duct. And if the duct is infinitely long then it will be Z planar. So, Z radial divided by Z planar. This Z planar is Z for a 1-D wave in a Cartesian frame for a semi-infinite tube. So, this ratio is what. So, Z radial is j omega r divided by C plus j omega r into Z naught and Z planar is Z naught. So, this Z naught cancel out, so I am left with 1 over 1 plus c divided by j omega r. This is equal to if I take j on the numerator I get 1 minus j c divided by j; no excuse me divided by omega r.

Now c over omega equals, so c is frequency times wave length divided by 2 pi f; so this is equal to lambda over 2 pi. So, this becomes 1 over 1 minus j times lambda over 2 pi r. While we are doing this we must remind ourselves once again that Z planar we are taking as Z 0, because what we are comparing it with is it is the Z for a 1-D wave in a Cartesian frame for a semi-infinite tube; for a semi-infinite tube Z planar so those are planar waves. And the impedance is same as Z naught. We had developed that relation earlier in the in last couple of classes.

So, if it was a closed tube then the Z planar will change, but for a semi-infinite tube it is same as Z naught. So, it is a 0 you know, but I am just equating this Z planar as Z naught. And I have defined, explained that Z planar is in context of a semi-infinite tube, I have written it here. So, this is there. Then the ratio; so if lambda over 2 pi r is very small compare to 1 or if lambda over 2 pi is very small compared to r then Z radial divided by Z planar is equal to 1.

So what we will do is, we will actually do an example and get a feel of what this mean.

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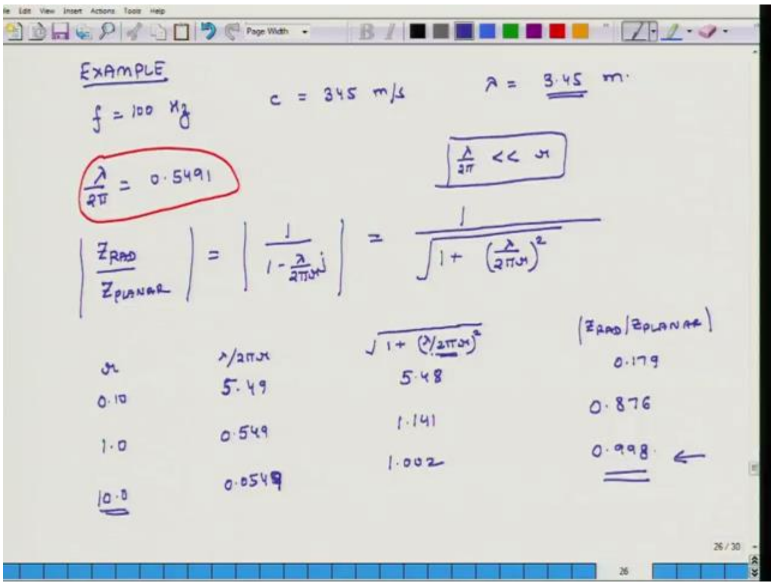

So, we do an example. We consider that frequency is 100 hertz and we know that velocity of sound is 345 meters per second. So, lambda comes to be 3.45 meters. So, lambda over 2 pi equals 0.5491. Now Z radial divided by Z planar, if I have to find its amplitude it is equal to 1 over 1 minus lambda over 2 pi r j. So I have to take the amplitude of this entire thing, and this is equal to 1 divided by 1 plus lambda over 2 pi r whole square, and the entire thing as to be put under a square root sign.

So, we know that lambda over 2 pi is 0.5491 is f is 100 hertz so we will compute, we will construct a table that if we are at a distance of 10 centimeters away we will calculate the value of lambda over 2 pi r. Then we will also calculate 1 plus lambda over 2 pi r whole square, and then we will calculate Z radial by Z planar its magnitude. So, when r is equal to 10 centimeters this term becomes 5.49, this is 5.48, and this works out to be 0.179. So, when we are 10 centimeters away from the source for 100 hertz wave the radial impedance and the planar impedance are not more or less; they are significantly different.

Let us see what happens at when we are 1 meter away. So, this becomes 0.549, this becomes 1.141, and the ratio of radial impedance and planar impedance it becomes 0.876, so it is reasonably close to 1 it is about 88 percent they are. But, if we are 10 meters away then this becomes 0.0549 and the number under the square root sign becomes 1.002 and the overall ratio becomes 0.998.

So lambda is 3.45 meters, this r is about 10 meters, so if roughly if we are 3 times away from the sources compare to lambda then the impedance for the radially propagating wave is pretty close to the to direct for a planar wave. So, the point is that this condition that lambda over 2 pi r should be very small compared to r; does not necessarily mean that this whole thing as to be extremely, this r as to be extremely large in an absolute sense. What matters is that this term lambda over 2 pi r, actually not just this actually its square should be very small compared to 1. So, even if it is moderately small number when I square it becomes even smaller and I get this kind of a result.

So, this concludes the discussion for this week. And in the next week which is the seventh week of this course we will start discussing radially propagating waves in far more detailed, and that is what I look forward. So, I hope that you have a wonderful weekend, please do review whatever we have been talking and discussing over these last 6 weeks. This is a week, this is course which is 12 weeks long, and so we are more or less half wave through the course.

So, this is an important milestone and I would specially encourage you to go back and review all the concepts and details which we have discussed over these lectures these last 6 weeks. And that kind of a review will be extremely helpful to what we will discuss in the second half of this course. So, with that I once again hope that you have a great week end and we will meet once again on the coming Monday.

Thank you and have a great day. Bye.