Fundamentals of Acoustics
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Lecture – 35
Volume Velocity

Hello, welcome to Fundamentals of Acoustics. Today is the fifth day of this particular week, and today and tomorrow we will discuss propagation of one dimensional waves and spherical reference frame. So, we will go back to the topic which we are discussing earlier and specifically today we will introduce concept called volume velocity.

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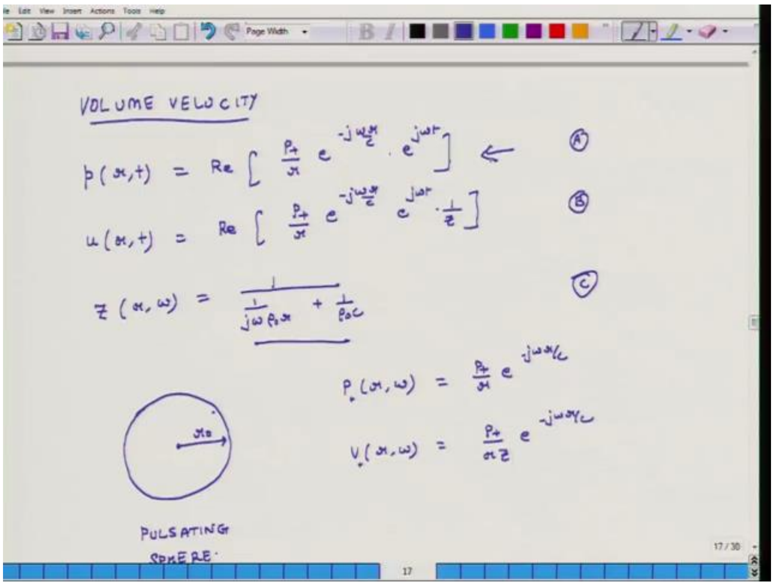

So, this particular concept becomes very handy and useful in are attempts to compute because typically what happens in suppose this pulse rating surface or for that sake loud speaker and if its emitting sound then we do not necessarily know how much pressure is being generated just at the (Refer Time: 01:15), but through laser measurements or some other techniques we can measure the motion of (Refer Time: 01:22) and we also another area of dia forming which is moving. So, we know that how much air it is displacing each second. So, that is essentially known as volume velocity and we will see ad we will develop it mathematically and that is the concept we are going to develop today.

So, last week we had developed equations for one dimensional a spherical waves and we had said that pressure was real portion of P plus over r e minus j omega r over c times e j omega t. And velocity for 1-D waves in the spherical frame of reference is real of P plus over r, e minus j omega r over c, e j omega t and then of course, divided by a specific acoustic impedance which is Z and we had actually calculated Z and we found that Z was a function of r and omega, and the relationship for Z was 1 divided by 1 over j omega rho naught r plus 1 over rho naught c. So, this is what we had developed last week and now we will develop it further and introduce this notion of volume velocity.

Consider as here and let us say that its sphere of a radius r naught and what this sphere is doing is that it is emitting sound and it is expanding and contracting and it is uniformly expanding and it is uniformly contracting. So, it is emitting sound and we are interested to find out its volume velocity. So, this is A this is B this is equation C. So, from A and B we know that the complex amplitude of pressure is P plus over r e minus j omega r over c and complex amplitude of velocity is P plus over r Z, e minus j omega r over c, the complex amplitude and because both these waves for pressure as well as velocity they do not have a reflected component there is no P negative, because in our assumptions we had said that there is no reflected wave.

So, this p r over r time p which is a function of r n omega complex pressure amplitude, it corresponds to amplitude only of the out outward moving wave, so I will also put up positive on these expression.

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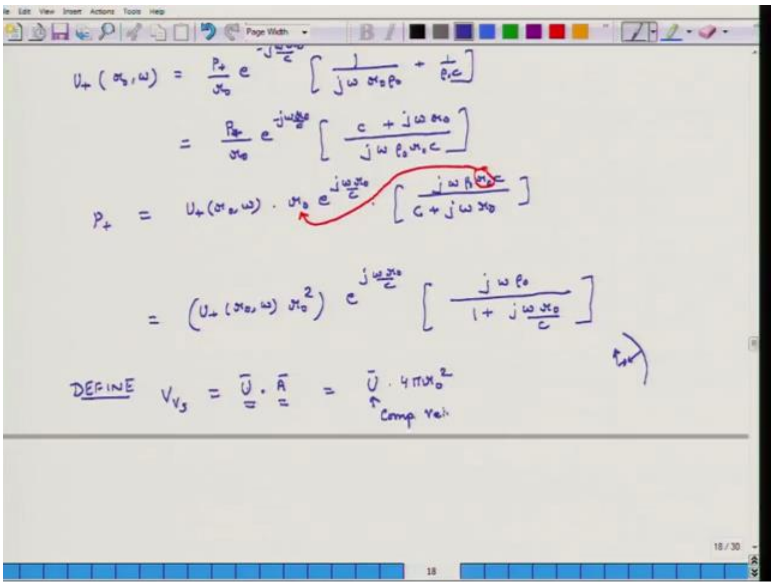

So, from here I can say that U plus r omega equals P plus divided by r p minus j omega r over c and times 1 over Z and 1 over and Z is 1 divided by this entire expression. So, 1 over Z is 1 over j omega r rho naught plus 1 over rho naught c. Now we go back to this sphere pulse rating sphere, if we are interested in pressure field and velocity field at the surface of this sphere, then the value of r at the surface of this sphere is r naught and the surface of this sphere right.

So, at the surface of his sphere r is equal to r naught, so U plus. So, the velocity on the surface of this sphere is U plus which is a function of r naught and omega and that equals P plus which is the constant divided by r naught, e minus j omega r naught divided by c times 1 over j omega r naught rho naught plus 1 over rho naught c. So, what we do here is now I can re write it as P plus divided by r naught, e minus j omega r naught over c times and I will at these terms j omega rho naught r naught c and what I get here is, c plus j omega r naught. So, from this I can express P plus as; so I am going to just cross multiply I will get U plus which is the complex velocity amplitude, times r naught e j omega r naught over c. So, exponent of minus j omega r naught over c when it goes to the positive the other side it becomes of positive thing, times j omega rho naught r naught c divided by c plus j omega r naught.

So, now what I do is I take this r naught out of the bracket and I multiply this with the other r naught. So, this is equal to U plus r naught omega which is the velocity of the die from of this sphere, times r naught square, exponent j omega r naught over c, times j omega rho naught c divide by c plus j omega r naught. The other thing I do is I divide the numerator and denominator by c, so what I end up with this c goes away and this c goes away and it gets replace by 1 and here I have in the denominator c. Now we defined volume velocity of the source. So, the point is let us suppose this is the source of the sound, this pulse rating is sphere is the source of the sound then volume velocity of the source what is it? It is the dot product of velocity times the surface area.

What does this mean the dot product? It means that suppose this is the surface area, then the surface area at each point has a normal right. So, that is the normal and you may be in this direction, but what we are really interested in computing is U which is alloying to the normal which is x component. So, that is what it means. So, this is velocity this is complex velocity time’s normal area. So, if the pulse rating is sphere is just moving in an out uniformly, if it is then the U and it surface area are mutually normal anyway. So, this dot product will be. So, mathematically this is U modulus surface time mod of A; modulus of U times mod of A times cosine of theta and cosine of theta if it is uniformly expanding on trending that is will be 1.

So, in that case it will be U times 4 pi r naught square and this is the complex velocity. So, what does this show?

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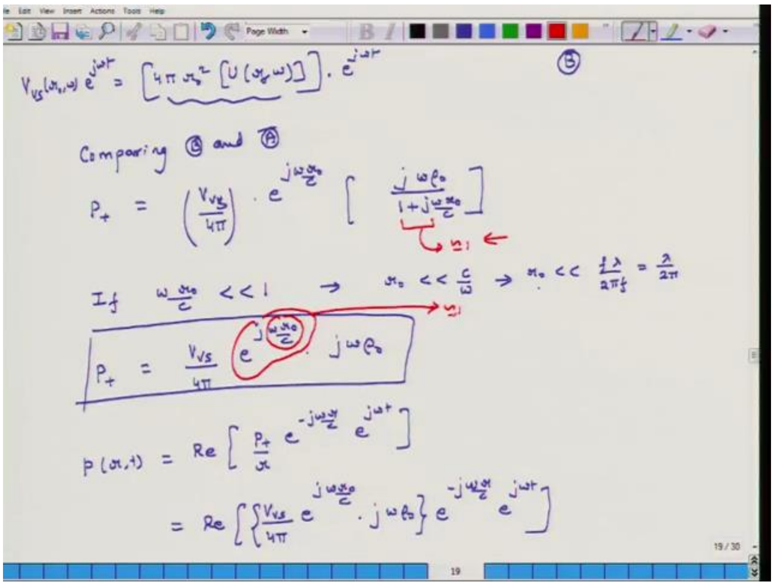

V v s, so V v s will have 4 pi r naught square, times complex velocity amplitude, times e j omega t right that is what how it is going to behave. Now this term looks very close to this term except that in this term in the red term there is no 4 pi. So, what we can write it as. So, let us call this equation A and lets call this equation B and I say that comparing and this V v s I am sorry, I should express it is that. So, V v s if I compare B and A, we can write that P plus equals complex velocity amplitude divided by 4 pi, because this is 4 pi r naught square times u r naught omega. So, this is complex velocity amplitude, times e j omega r naught over c, into j omega rho naught divided by 1 plus j omega r naught divided by c.

Now, we make a simplification; if omega r naught over c is very small compare to 1; what are we doing? We are looking at these two terms and we are saying that omega r naught c is very small compare to 1 that is r naught is extremely small compare to c over omega, which means r naught is very small compare to c is frequency times lambda and omega is 2 pi frequency. So, r naught is very small compare to lambda over 2 pi. So, if r naught is very small compare to lambda over 2 pi, then this term it approximates to 1. So, in such a case P plus equals complex amplitude a volume velocity, divided by 4 pi, e j omega r naught over c times j omega rho naught.

So, if I know the velocity of a pulse rating sphere and I know its radius, I can compute its volume velocity and from that I can calculate the value of P plus; once I know the value of P plus I can say that actual pressure is what real of P plus e minus j omega r over c, e j omega t. So, this we go back to the original relation we going to back to this relation. So, whatever we have done, we have done we have calculated P plus in terms of volume velocity and we are plugging that relation back into the expression for pressure. So, with that goal I can write the pressure is real of volume velocity complex amplitude, divided by 4 pi e j omega r naught over c times j omega rho naught.

So, that is my expression for P plus. So, this is V v s over 4 pi e to the power of j omega r naught over c times j omega rho naught times. So, that is P plus over r, times e minus j omega r over c e j omega t, but we had said that omega r naught over c is very small compare to 1 and that will happen when r naught is extremely small compare to lambda over 2 pi.

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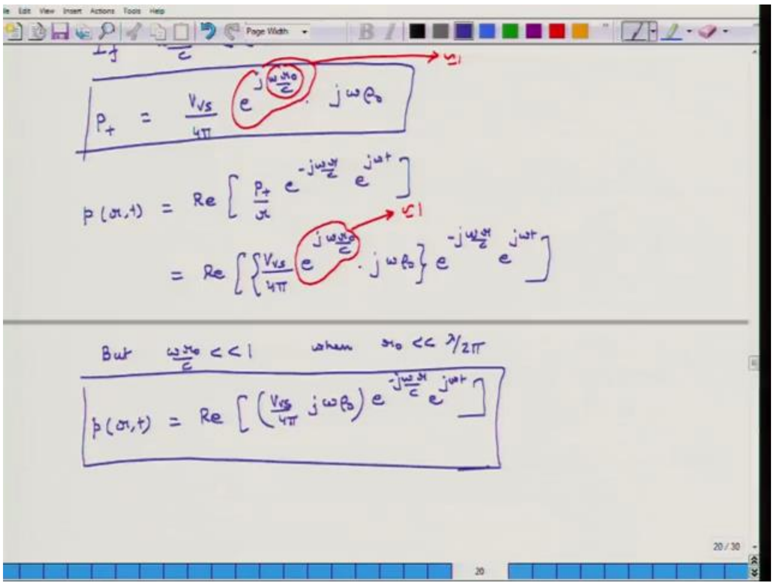

Now because of this assumption we had made one simplification which was here, but we could have made one more simplification that when omega r naught over c is very small, then this entire term this also approximates to 1. So, in that case this term this approximates to 1. So, for the condition when omega r naught over c is extremely small, we can write that pressure equals real of V v s over 4 pi j omega rho naught, e minus j omega r over c e j omega t.

So, this is our expression for pressure in terms of volume velocity. So, that concludes the discussion for today on volume velocity and tomorrow we will extend this discussion and then we will actually apply some of these concepts to find out what happens if there are two point sources close to each other and they are emitting sound, then how does the overall sound pattern for away from those two per sound sources looks like. So, with that I close for today and we will meet once again tomorrow.

Thank you, bye.