Fundamentals of Acoustics

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Lecture - 34
Kundt's Tube

Hello, welcome to Fundamentals of Acoustics. Today is the fourth day of this week and what we will do today is continue our discussion on tubes with imperfect terminations. Yesterday we had discussed the problem that if at the end of the tube the nature of termination is known, specifically if we know the value of impedance at the end of the tube then in terms of that impedance, we had developed expressions for complex pressure and complex velocity in the tube and using these expressions we can finally, calculate the actual pressure and actual velocity in the tube, if the boundary condition at the first point or the initial point in the tube, where it is getting excited it is known.

Today we will do the second problem and the second problem is that if there is a tube with an imperfect termination and if we do not know the nature of the termination, that is if we do not know the impedance of the material which is present at the end of the tube then our aim would be to find the nature of the terminations specifically the value of the impedance at the end of the tubes, in terms of known pressure values which are available throughout the length of the tube. So, here we are assuming that in the tube we know pressure by actual measurement through microphones.

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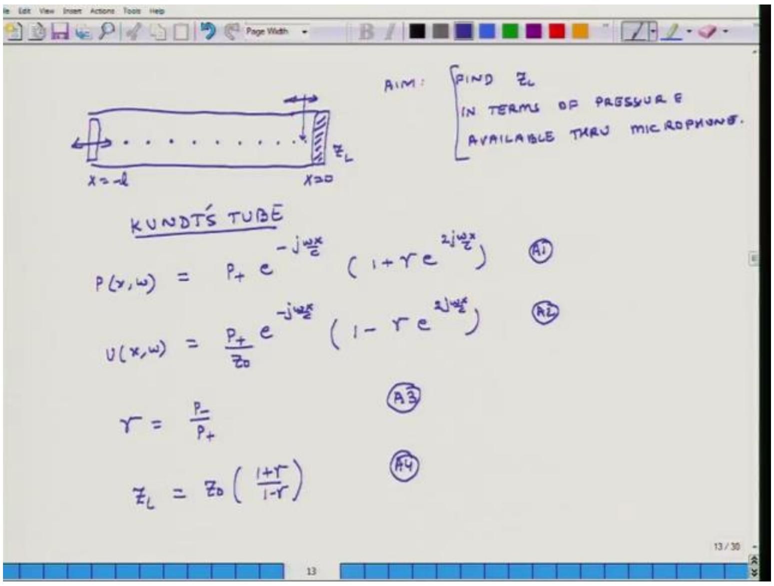

So, once again you have a tube x equals 0 at the end which is having imperfect termination, the tube is excited at this end by some pistonic source, x is equal to minus l here and our aim, let us say the impedance at x is equal to 0 is Z L, and this Z L is not known so aim is to find Z L in terms of pressure data available through microphone. So, what do I mean by pressure data available through microphone, I can take a microphone and I can measure pressure at all points inside the tube. So, I take a microphone and I can slide this microphone back and forth slowly and at all points in the tube I can measure pressure. So, I have a map how is pressure changing at every point over a period of time.

So, that is what I can do. So, this type of an apparatus is called a Kundt's tube. So, what is a Kundt's tube? Essentially it is a pipe and at one end I have a sound source. So, I excite sound in the pipe, at the other end I can block it with the material whose Z L we do not know and our aim is to find Z L and the way we find Z L is we take a microphone put it in the pipe and slide that microphone along the length of the pipe slowly, gather all the pressure data and from that pressure data we somehow calculate the value of Z L. So, Kundt's tube is used to find the impedance of the material which can be, which closes the tube. So, that is the procedure we will develop today.

Now we had developed these 2 relations earlier. So, the complex pressure amplitude was P plus e to the power of minus j omega x over c times one plus gamma e 2 j omega x over c. So, this is an expression which we had developed earlier that is yesterday and the amplitude of complex velocity we had figured out was P plus e minus j omega x over c and P plus is divided by zee naught times 1 minus gamma e 2 j omega x over c.

So, let us call these equations A1 and this is equation A2; the other thing was that we had defined gamma as the ratio of P minus over P plus and we had also shown that Z L can be written in terms of gamma using this relation. So, I call this equation A3 and I call this equation A4. So, these are mathematical relations and our aim is that Z L is unknown, if I know gamma I can calculate Z L. So, now, we have to figure out how to find gamma? Now we realize that gamma is a complex number.

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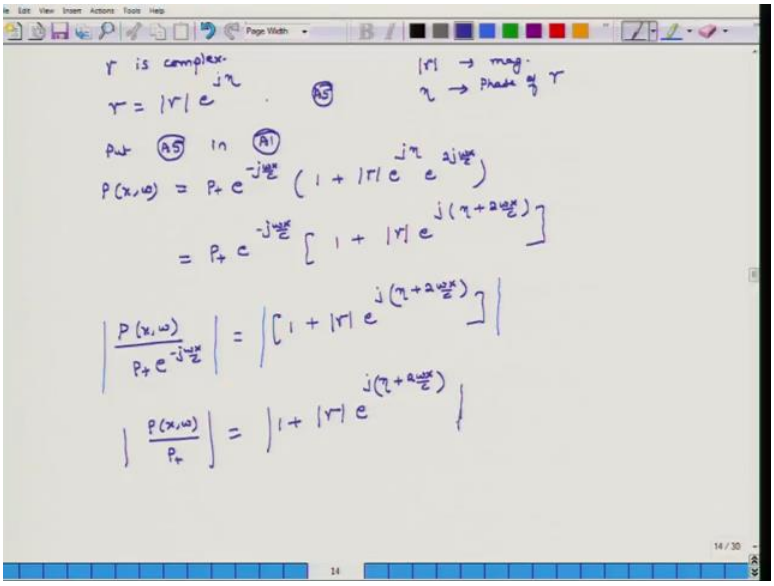

So, we know that gamma is complex. So, it will have a magnitude and it will have a phase. So, I can express gamma as its magnate portion modulus of gamma, times exponent j eta. So, eta tells us about the phase of gamma and modulus tells us its magnitude. So, if we put this relation. So, let us call this A5. So, we put A5 in A1. So, what is A1? It is the expression for complex pressure amplitude. So, what we get is P of P which is the function of x and omega equals e plus e minus j omega x over c times, 1 plus and gamma is its magnitude component e to the power of j eta and then of course, I have e 2 j omega x over c, e 2 j omega x over c, this can be expressed as P plus e minus j omega x over c. 1plus magnitude of gamma, exponent j eta plus 2 omega x over c.

So, I reorganize this last relation and write it as P x omega divided by P plus e minus j omega x over c, equals 1 plus gamma e j eta plus t omega x over c. Now I want to understand what is the magnitude of the left hand side of this equation? So, the magnitude of the left hand side is same as the magnitude of the right side; the other thing I would like to notice that the magnitude of the left side is magnitude of numerator divided by magnitude of denominator, and the magnitude of denominator is the magnitude of P plus times magnitude of e to the power of minus j omega x over c, but magnitude of e minus j omega x over c is 1. So, I can write it as this relation; so this is important relation and let us look at its physical interpretation. So what we will do is, this is our RHS and we will plot the RHS on a graph.

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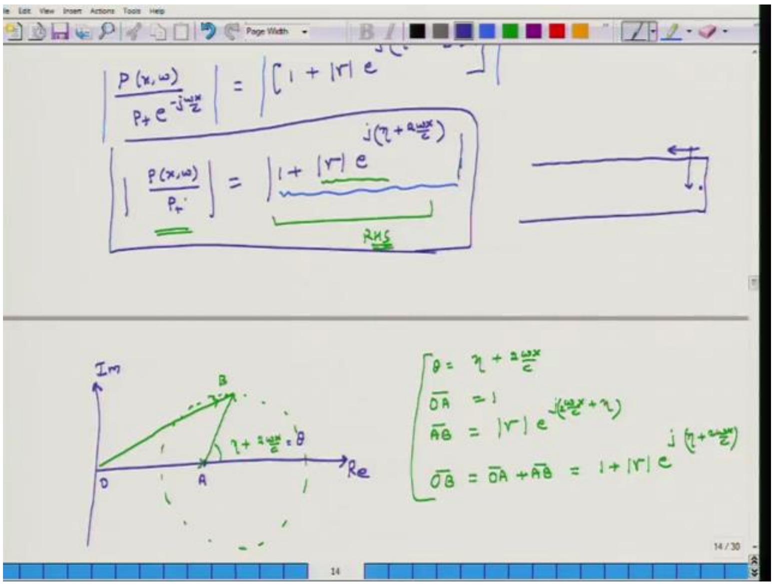

So, we will use polar coordinates. So, I have a real axis and I have an imaginary axis and I am going to plot the term which is in blue. So, the plot term which is in blue is having the first component is 1. So, let us say this is 1 this is 0. So, this is o and this is A; then the second term which is this thing has an amplitude of gamma and it is not it is at some angle and the angle is eta plus 2 omega x over c. So, the amplitude of the second component of the one thing RHS is gamma and this angle is theta and theta equals eta plus 2 omega x over c. So, this is point A this is point B and this is the overall term. So, we will make it explicit OA represents 1, OB represents not OB AB, AB represents gamma time’s exponents, j omega x over c plus eta there is a 2 here and the last thing is OA OB is equal to OA AB plus OA right. So, this is vector addition OA. So, this there is lot of scratch marks, OB equals OA plus AB and this equals 1 plus gamma exponent j eta plus 2 omega x over c.

So, this is there and what I will do is I will erase this graph again because I will make it more clear so this is OA, this is point B, this is the overall thing, this angle is eta plus 2 omega x over c, this is my real axis and that is my imaginary axis. Now eta is a constant, omega if I am exerting eta specific frequency that is not changing, but as I move in the tube x changes, So, as x changes this angle which is theta, theta changes and theta changes in such a way that it changes along the circle. So, now, it is not exactly a well drawn circle, but the centre of the circle is point A. So, there will be a minima associated with OB and there will be a maxima associated with OB. So, the maximum value of OB will be when this theta is equal to 0 degrees and when theta equals 0 degree then the maximum value will be 1 plus gamma, and minima value of OB will be 1 minus gamma. So, when theta equals pi radiance or minus pi radiance we will have minimum value of OB, when theta equals 0 degrees then we will have maximum values of OB.

So, now what we do is with this understanding, what we do is we measure the maximum and minimum values of pressure, in the tube if I have a microphone which is located at some point and as I move the tube back and forth I can identify the locations at which these minima and maximum values of pressures are happening that is one important thing. So, the next thing is at with this understanding our aim is what is our final aim that we have to compute Z L, now we said that Z L equals Z naught times 1 plus gamma divided by 1 minus gamma. So, if I know gamma then I can find Z L, but I do not know gamma and we have said that gamma has 2 components, its magnitude and its phase. So, if we want to know gamma we have to know its magnitude and we have to know its phase.

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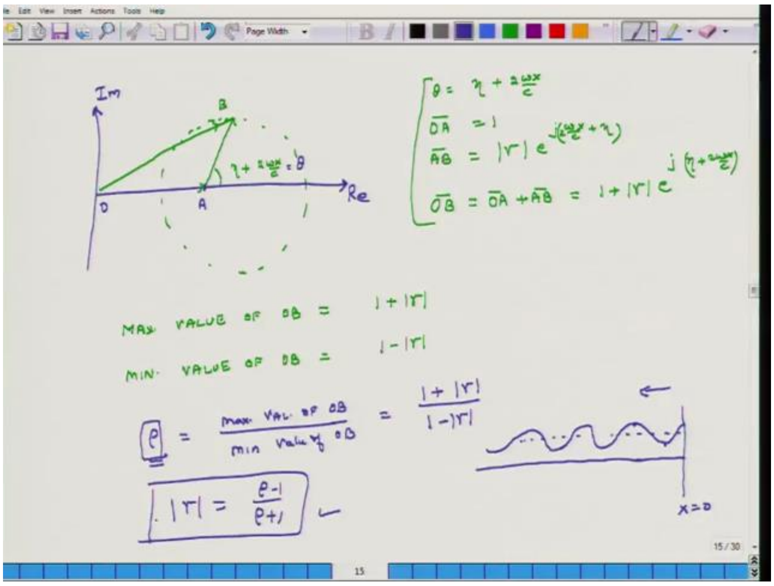

Now we define another parameter rho. So, this rho is not density it is something different, it is the maximum value of OB divided by minimum value OB, and this is equal to 1 plus gamma divided by 1 minus gamma; we can measure this rho, how do we measure this rho? So, in the tube suppose this is the length of the tube x is equal to 0 at this location, how will the pressure change? The pressure will change it will behave in some way like this, the pressure need not be maximum at x is equal to 0 it will have some value, but as I travel my microphone along the length, I can measure what is the minimum value of pressure and I can also measure the maximum value of pressure.

Once I know the minimum and maximum value of the pressure I can divide, then I can calculate rho and it is a constant real number. So, once I know rho, then I use this relationship rho equal’s 1 plus gamma modulus divided by 1 minus modulus of gamma and I can say that modulus of gamma equals rho minus 1 divided by rho plus 1. So, if rho is known I can calculate gamma. So, I can compute rho experimentally and once I have done that I have can calculate the modulus of gamma using this relationship.

So, if this is there done then 50 percent of my problem because if I have to know gamma, I have to know its modulus and eta. So, I have known modulus. So, the next thing is that now I have to find what is the value of eta?

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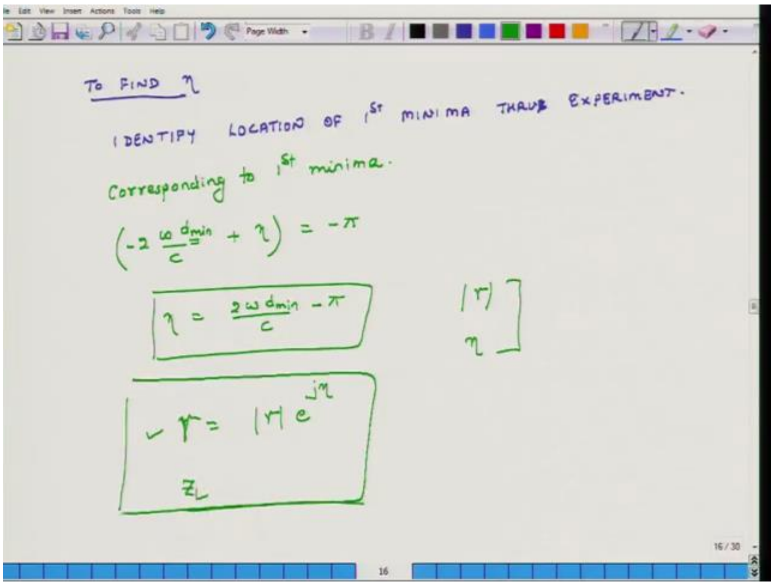

To find eta, we again do the experiment and what do we do? We identify the location of the first minima through experiment. So, we have our microphone and we move it in this direction and when we identify the location of first minima. So, let us say this is the first minima and let us say this distance is d min. So, what does this d min means? What this means is now let us look at this thing again, when at this d min the sign of this thing is negative one, the value of this thing is negative one because this corresponds to minima, and when this is minima OB will be at pi radiance AB will be at pi radiance and when it is at pi radiance, e to the power of j pi will be minus 1.

So, corresponding to first minima omega x over c and x I call it as d min which is the location of first minima. So, I will replace x by d min and there is a 2 here plus eta, this equals minus pi and why am I using minus pi because I am going in the negative direction. The other thing is that this d min our positive axis is rightwards. So, the coordinate associated with d min is negative, I have to put a negative sign here. So, experimentally I can find out the location d min, omega is already known, c is known. So, eta equals 2 omega d min divided by c minus pi.

So, with this understanding I have computed or measured gamma, I have also experimentally determined eta. So, with this information I know that gamma equals its modulus time's exponent j eta. So, I can calculate gamma, through gamma I can calculate Z L. So, this is the overall approach and through this approach using a Kundt's apparatus or a Kundt's tube, we can compute Z L if we are able to measure pressures in the tube at all the locations.

So, that is the conclusion for today's lecture. Tomorrow we will go back to spherically propagating waves and we will again revisit one dimensional wave in a spherical frame of reference.

Thanks and have a great day, bye.