Fundamentals of Acoustics
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Lecture - 33
Specific Acoustic Impedance for a Tube with Imperfect Termination

Hello, welcome to Fundamentals of Acoustics. Today is the third day of this week and in next two days that is today and tomorrow and may be even day after tomorrow, we will have a discussion on the impedance for tube which has an imperfect termination.

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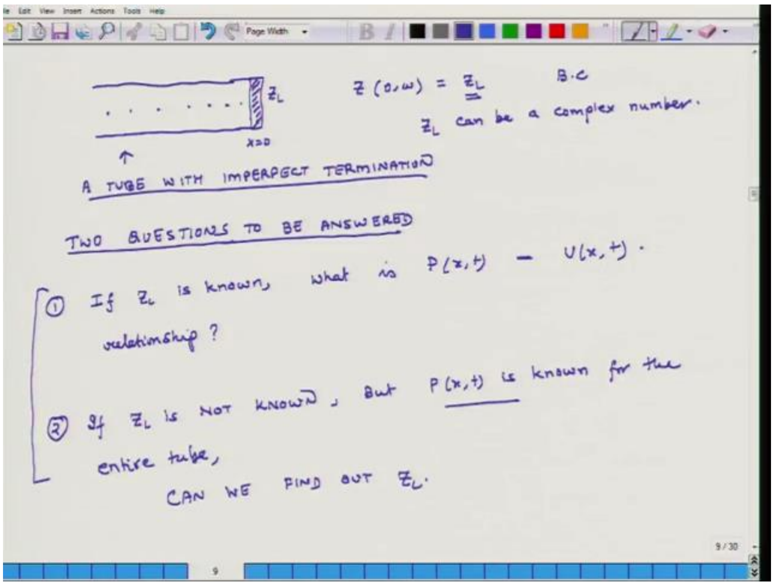

So, let us first explain what imperfect termination means, suppose I have a tube, till so far we have discussed some ideal discussions one is that we have a rigid termination and what that means is that the end of the tube is very rigid such that the wall does not move at all. So, the velocity of the air particles which are next to the wall is 0, so that is one termination. The other one is that the tube is perfectly open so whatever pressure fluctuations are happening they get dissipated because there is atmospheric pressure outside the tube. So, those are the situations where we have perfect terminations, but here we have an imperfect termination, so the tube is not rigid, but it is not fully open also. So, we can put foam at the end of the tube or some flexible membrane and the nature of this membrane is such that the overall impedance at the end of the tube is Z L. So the boundary conditions, this is x is equal to 0 if the boundary condition is such that Z at 0 omega equals Z L.

So this is the boundary condition and please notes that Z L can be a complex number. So, this is a tube with imperfect termination, it is a tube with imperfect termination, it is imperfect because it is neither a rigid structure at the end nor it is a perfectly open thing it is somewhere in between. So, there is some stiffness some damping happening in the end and the overall impedance is Z L and that is it can be a complex number. So, with this background, we will like to do two things in the discussion the first thing is we will address two questions. So, the first question we will answer is, if Z L is known what is p? So, how is complex pressure varying because if we know the pressure, I take it is real part and I get the actual pressure. So, how is complex pressure? What is complex velocity in the tube? So, if Z L is known, then at all these points what is the relationship between what is the. So, what is the P x t, U x t relationship, this is one thing we want to know.

The second question is that there could be a situation that the tube is ending with some termination and I do not know it is Z L. So, if Z L is not known, but P x t is known for the entire tube then can we find out Z L. So, if I know the pressure microphone and measuring pressure at all points in the thing. So, I know the real portion of the complex pressure that is from that I can figure out what is the imaginary portion. So, if P x t is known and we know P we can know P x t by using microphone, then from that data can we figure out what is Z L this is the second thing. So, these are the two questions and these are the two questions which we will address. So, in the first question we know Z L and then using that information we want to develop a relationship between pressure and velocity. In the second case we do not know Z L, but we know pressure and from that understanding we want to figure out Z L. So, these are the two situations.

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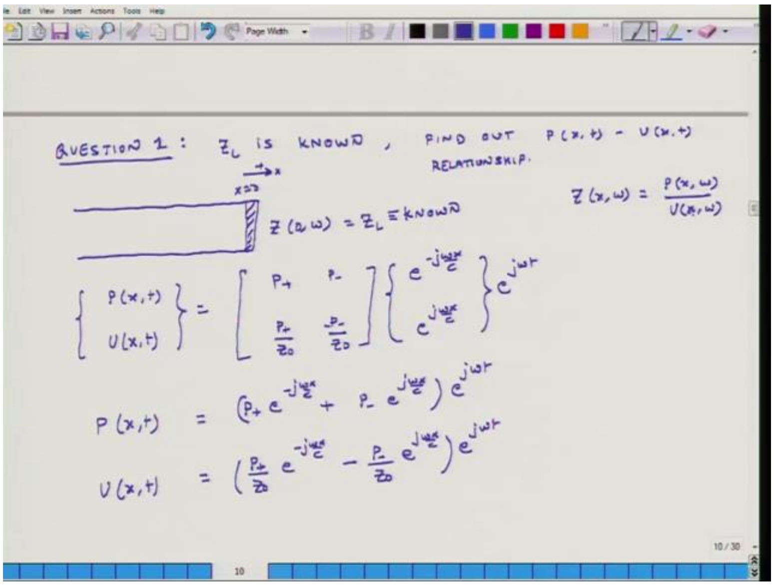

So, question 1; here we are trying to answer the first question, Z L is known find out P x t, U x t relationship. So, once again we will draw the tube and at x is equal to 0, Z 0 omega is equal to Z L is known this is known. So, that is why x is equal to 0 and this means positive x. So, how do we start, we go back to our transmission line equations and we start from the transmission line equations. So, earlier in last two days I have just written P x t omega and complex amplitude, but for purposes of gravity not that this is mathematically inaccurate actually this is mathematically the most accurate expression, but for purposes of gravity, I will just write P x t and U x t is equal to P plus P minus, P plus P minus zee naught divided by zee naught, e minus j omega x over c, e j omega x over c, e j omega t. So, I can also write it as P, complex pressure is equal to P plus e minus j omega x over c. So, this is P negative exponent of j omega x over c, e j omega t and complex velocity is P plus by z naught, e minus j omega x over c, minus P negative divided by z naught, e j omega x over c, e j omega t.

Now, we look at these relations and we know that z which is the impedance at x is equal to 0 is known, at x is equal to 0 we know the impedance. So, we know that Z x omega is equal to complex pressure amplitude divided by complex velocity amplitude, this is the definition. So, in these expressions we have to identify what is the amplitude of pressure and complex amplitude of pressure and complex amplitude of velocity.

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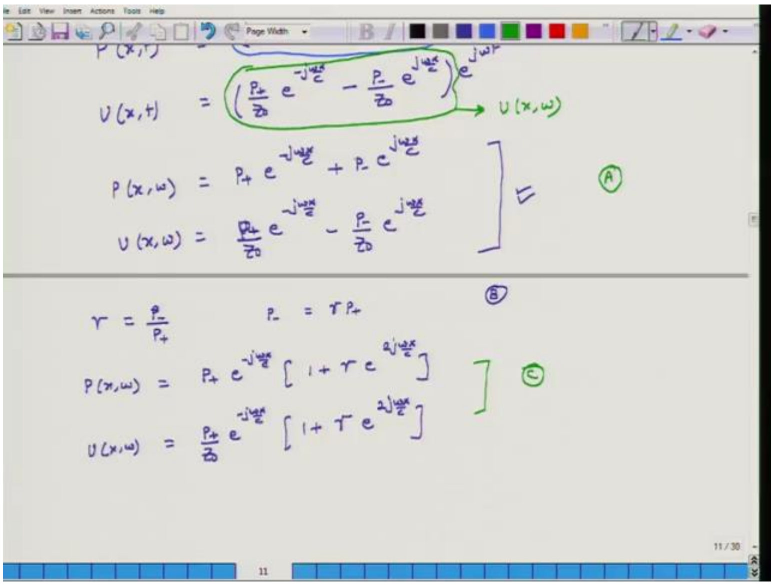

So, we look at these expressions and this is the complex pressure amplitude because the complex amplitude of pressure is equal to amplitude of this thing in blue time’s exponent j omega t amplitude and exponent of j omega t is 1. So, this is P of x omega.

Similarly, this is the complex pressure amplitude of velocity. So, I can write these expressions as P of x omega equals P plus e minus j omega x over c plus, P negative e j omega x over c and the complex velocity amplitude minus P minus divided by z naught e j omega x over c. So, these are the two relations for complex pressure amplitude and complex velocity amplitude.

Now, these expressions are very general, we have not applied any boundary condition till so far. So, these expressions are very general and they are applied to any one dimensional sound propagation situation as long as it is happening in a partition frame. So, these are good for open tubes, closed tubes infinitely long tubes and also for the case we are considering here. So, let us call this equation, now we define a parameter gamma and we say that gamma is the ratio of P negative over P positive. So, P negative is a complex number P positive is a complex number so gamma is also a complex number. So, if that is the case then P negative is equal to gamma times P positive. So, this is equation B, if I put B in A, I get P x omega equals P plus and what I am going to do is I am going to take e minus j omega x outside the bracket. So, 1 plus gamma e 2 j omega x over c and the amplitude of complex velocity is P plus over zee naught e minus j omega x over c, 1 plus gamma e 2 j omega x over c. So, these are equations c.

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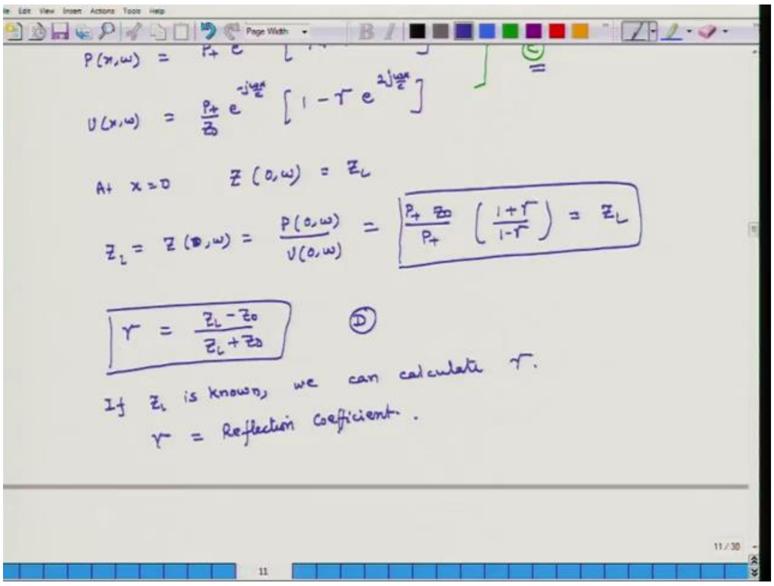

Now, at x is equal to 0 the impedance is Z L. So, Z 0 and this is known because that is our problem definition that if Z L is known we have to develop the relation between pressure and velocity. So, Z L is this and there should be a negative sign here. So, at x is equal to 0 the impedance is Z L. So, we can write Z L equals Z at x equals 0 omega and this is equal to P at 0 omega divided by U 0 omega and what is the value of P and u at 0 can compute by equation c. So, this is P plus and I am putting x is equal to 0 in this case. So, e to the power of minus j omega x over c becomes 1. So, I am left with this zee naught divided by P plus times one plus gamma divided by one minus gamma.

So, from this, so this is equal to Z L. So, if I redo this maths from here I can write gamma as Z L minus Z naught, divided by Z L plus z naught I am just rearranging the terms. So, this is equation D. So, what does that mean? So, what; that means is that if Z L is known we can calculate gamma. So, gamma is the reflection coefficient and this is also a complex number because Z L is a complex number. So, gamma is reflection coefficient and it is the ratio if P negative and P positive. So, what this says is that if Z L is known then we can calculate gamma and in our case Z L is known. So, gamma is known once we know gamma, then we can use equation C to compute complex pressure amplitudes for P and U.

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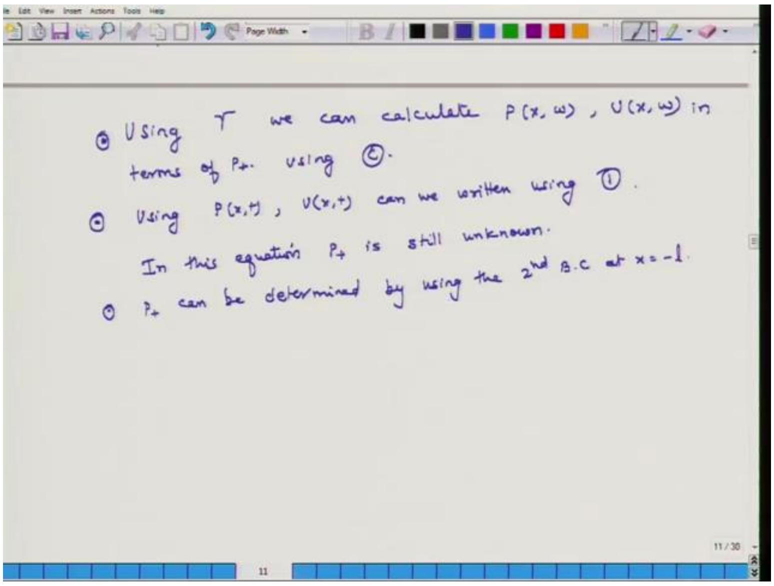

So, using gamma we can calculate complex pressure amplitude P x omega and U x omega in terms of P plus. So, this was the first step this is the second step in terms of using which equation was that C and then once we know C then we can go back to. So, using C complex pressure as a function of x and t and complex velocity as a function of t can be written using equation one, in this equation once we do that in this equation there will be still something unknown. So, look at C, in equation c gamma is known because we know Z L from Z L we can calculate gamma. So, gamma is known, but we do not know P plus. So, when we write an expression for P and x u we still have to calculate P plus we can calculate P minus in terms of P plus, but we still need P plus. So, in this equation P plus is still unknown, how will we get that value of P plus? We will get that value of P plus I mean if there is some sound producing system here, then at this point we have to know what is the pressure or velocity or something, in all the situations we need to have two conditions, till so far we have applied only one condition, which is that x is equal to 0, we also have to know the condition at x is equal to minus l.

So, the value of P plus. So, P plus can be determined. So, P plus can be determined by using the second boundary condition at x is equal to minus l. So, in this way what was our original question? Our original question was if Z L is known, can we develop pressure velocity relationship which we have done; which we have done and the way to do it is first you find out gamma in terms of Z L, use that gamma to compute complex pressure amplitudes using equation A then go back and use equation 1 to develop expressions for P, complex pressure complex velocity and then find out P plus using the second boundary condition at x is equal to minus l.

So, that terminates our discussion for today, tomorrow we will address the second question and the second question is that if Z L is not known, but we know pressure at different places in the tube, then can we calculate the value of Z L which is the impedance at the end of the tube and it is the material property of the termination point. So, that is the second question and that is what we will discuss tomorrow.

Thank you and have a great day, bye.