Fundamentals of Acoustics
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Lecture - 32
Specific Acoustic Impedance for an Open Tube
& an Infinitely Long Tube

Hello, welcome to Fundamentals of Acoustics. Today is the second day of the 6th week of this course and we will continue the extension the discussion we were having yesterday. Yesterday we had developed an expression for impedance in a close tube, what we will do today is we will develop a similar expression for impedance as, but here geometry we will consider will be for an open tube.

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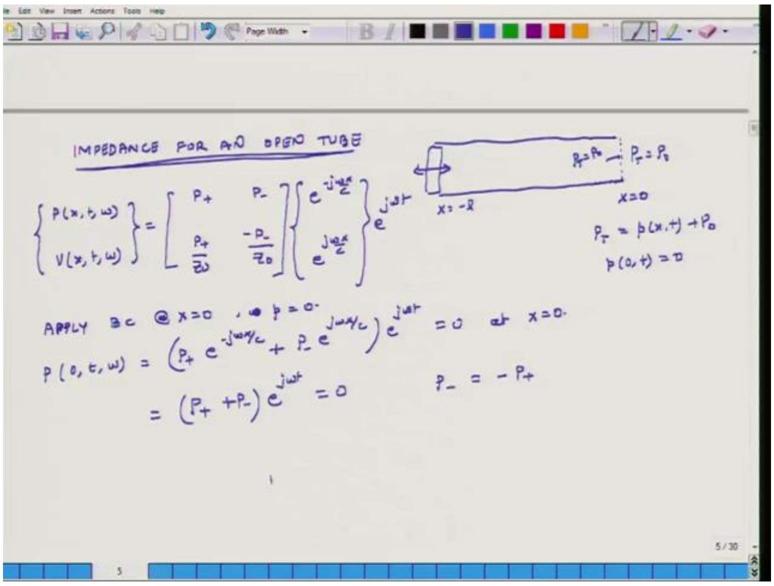

So impedance for an open tube; what is the condition in open tube? That is the tube this is the open end, this is my piston moving back and forth, x equals 0 at the open end, x equals minus l at the closed end; just outside the open tube we have pressure ambient pressure. So, P T is equal to P naught and because of continuity which means that just in a point inside also the pressure is going to be P T is equal to P naught. But P T is equal to p of x and t plus P naught and if P T is P naught which means that p at 0 and at all time is 0 pascals. So, that is the condition for an open tube.

So, with this understanding we will develop an expression for impedance for an open tube. So, how do we do that? We start with these transmission line equations, p x t omega x t omega. So, here capital EP and capital U are complex pressures and complex velocity; P plus P negative, P plus minus P negative over zee naught; e minus j omega x over c, e j omega x over c, e j omega t. Now we apply the boundary condition BC at x is equal to 0, which says that p is equal to 0. So, we take the second equation which is actually the first equation. So, complex pressure is equal to p plus e minus j omega x over c, plus e minus e j omega x over c, e j omega t is equal to 0, at x is equal to 0. So, why do we say that complex pressure is also 0 because the actual pressure is 0 and the real part of complex pressure is the actual pressure, so the complex pressure also has to be 0. So, we put in this relation x is equal to 0. So, we get P plus, plus P minus e j omega t equals 0, which means P negative equals minus of P positive.

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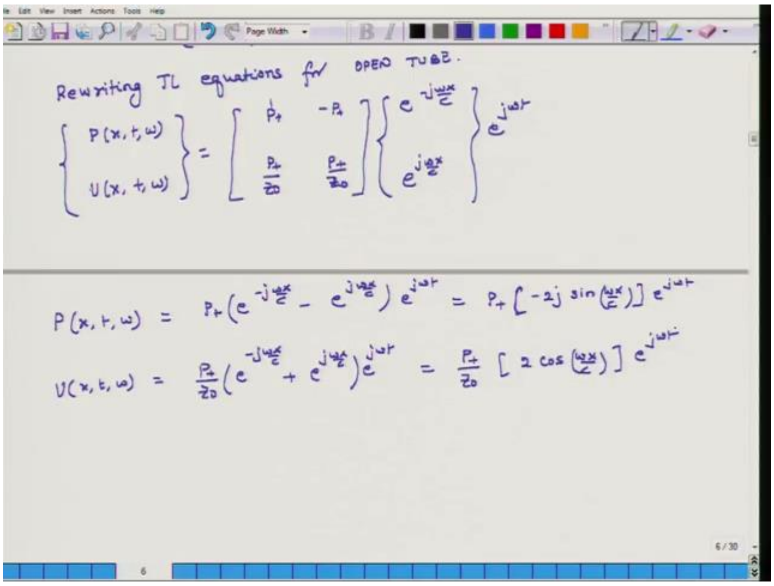

So, with that understanding we rewrite the transmission line equations, rewriting transmission equations for open tube. So, what do we get P x t omega, U x t omega equals P plus minus of P plus, P plus over Z naught and here also we get Z plus over Z naught, e minus j omega x over c, e j omega x over c, e j omega t. So, now, we expand these relations, pressure omega equals P plus e minus j omega x over c, plus e j omega x over c e j omega t and complex velocity is.

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So, I should have put a negative here. So, this is P plus over zee naught, e minus j omega x over c, plus e j omega x over c, e j omega t. So, if I simplify the terms in the bracket, what do I get? I get P plus times. So, e minus j omega x over c minus e j omega x over c is minus 2 j sin omega x over c. Then of course, exponent to the power of exponent j omega t and for the velocity relation I get P plus over Z naught and the term in the parenthesis 2 cosine omega x over c, e j omega t.

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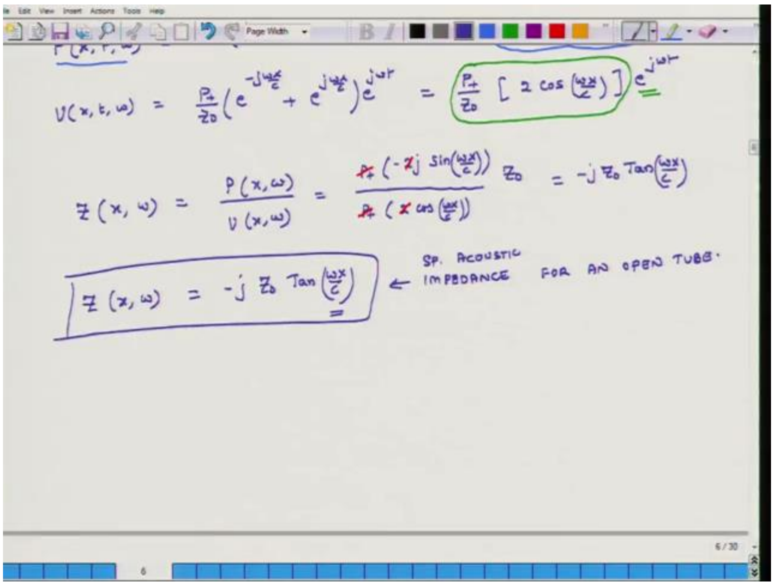

Now, impedance is specific acoustic impedance is what? It is defined as the amplitude of the complex pressure and we write it as P x omega and the amplitude of complex velocity. So, let us identify what is the complex pressure amplitude for this thing? So, if I look at this term in circle, this term in circle represents the amplitude of this complex pressures why is it? Because the amplitude of exponent j omega t is 1; so, the amplitude of complex pressure will be whatever is there is the blue circle. So, in the numerator for the expression for Z we will write P plus minus2 j sin omega x over c.

Similarly this term in green is the amplitude or the complex pressure amplitude of velocity because the amplitude of exponent of j omega t is 1. So, the complex pressure amplitude of U is P plus times 2 cosine omega x over c and there is a Z naught here which will come in the numerator. So, these 2 cancels out p plus also cancels out and what I am left with is minus j zee naught tan omega x over c. So, Z for an open tube is minus j zee naught tan omega x over c. So, that is the impedance for impedance and what type of impedance it is? It is specific acoustic impedance for an open tube.

So, again we can develop a plot for this also and the graph will be somewhat different, but the value of this acoustic impedance again repeats itself after lambda distance because each lambda distance corresponds to a phase difference of two pi. So, after every two pi value when omega x over c becomes two pi, the value repeats itself that is the second part.

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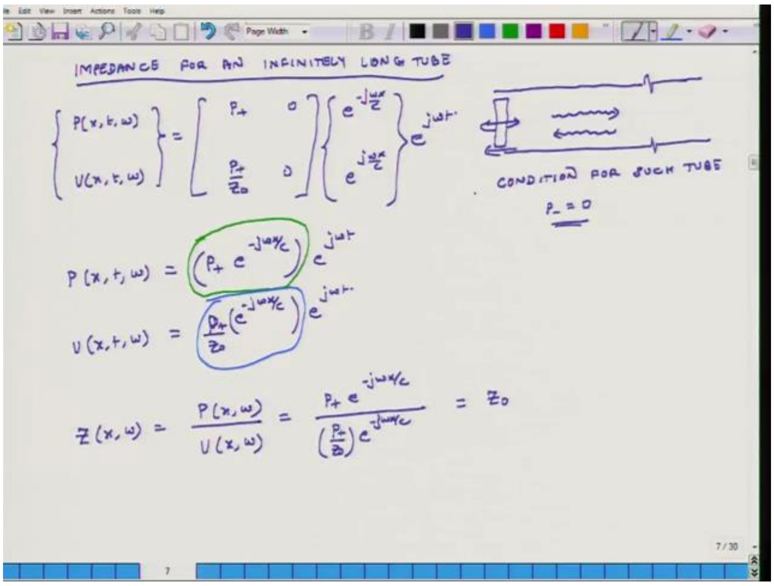

Third part we will discuss today is impedance for an infinitely long tube. So, if I have a infinitely long tube and at one end of this infinitely long tube there is a piston which is exciting some sound, then the condition for such tube, so P plus represents the amplitude of the wave travelling in the forward direction and P negative represents a negative a wave travelling after reflection and because this is a tube which is infinitely long P negative is 0. So, the transmission line equation for this one is plus P plus Z naught. So, if that is the case then P of x t omega equals P plus e minus j omega x over c times e j omega t and u x t omega is u plus e j omega x over c, oh excuse me P plus divided by Z naught, e j omega t. Now once again this represents the complex pressure amplitude. So, I will write the expression Z is the ratio of P complex pressure amplitude divided by complex velocity amplitude and that is equal to P plus e minus j omega x over c divided by, so the complex pressure velocity amplitude is this term in blue. So, that is P plus by zee naught, e minus j omega x over c and that becomes simply Z naught.

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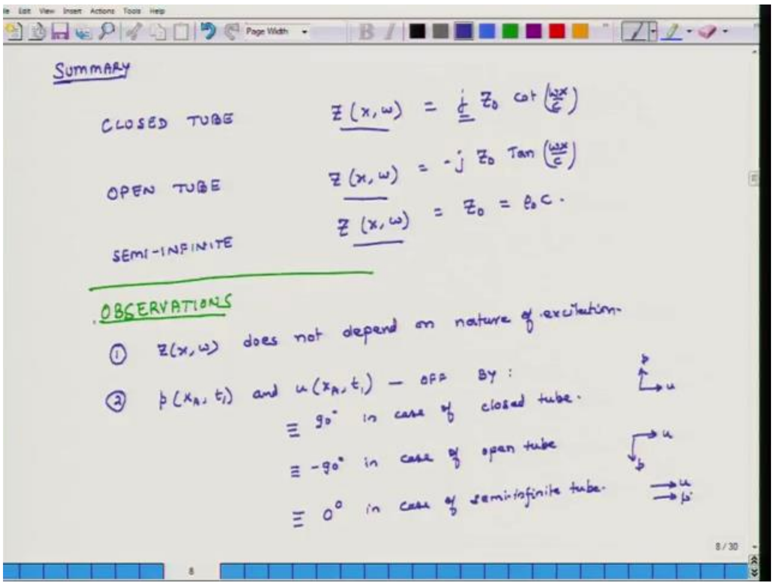

So, with this what we will do is summarize. So, you have a closed tube, then we have an open tube and then we have a tube which is semi infinite because we know its beginning where the source is pushing air in and out. So, for closed tube Z of Z which depends on x and omega is j zee naught co tangent of, omega x over c for open tube Z is equal to j, but it is negative zee naught tan omega x over c and for semi infinite tube it this is equal to Z naught and that is equal to rho naught c.

So, one important observation is observations. The first observation is that z x naught x comma omega, it does not depend on nature of excitation; what does that mean? What that means is when we were calculating this Z of x in all the cases, I did not even specify what conditions exist due to piston right this is one boundary condition which is there. So, I did not because the piston which is exciting the sound source, it generates some p plus and when we take when we calculate Z this P plus cancels out and this cancellation happens in case of infinitely long tube, it also happens in case of a tube which has a close a open end right P plus cancels out. So, the motion of the piston actually influences the nature of p plus, but p plus cancels out. So, whether this piston is moving at u it has a velocity u or it have a velocity something else or it is producing a known pressure it does not really matter, this p plus cancels out and we see the same thing in case of closed tube also. I never defined any conditions at x is equal to minus l because p plus cancels out. So, the Z naught not Z naught, Z it essentially depends whether the tube is closed or open or is it infinitely long this is one thing.

Second, pressure at a given point X A and at a given point and time, at a given point and at instant of time t 1 let us call it and velocity at the same point X A and at the same time they are off by how much? So, in case of thereof by let us write down these three things. So, the phase essentially they are off by 90 degrees, in case of what closed tube right because which means that u is at 0 degrees u then pressure will be at 90 degrees then only we will have this j component, then they are off by negative 90 degrees in case of open tubes. So, if u is at 0 degrees. So, these arrows they do not mistake depict the direction of u, they depict the phase. So, u is at 0 degrees then pressure will be at minus 90 degrees. So, if u is at 10 degrees then pressure will be at 100 degrees and so on and so forth. And in case of infinitely long tube, what is the phase difference? It is 0 degrees in case of semi infinite tube. So, pressure is like this and so is pressure as well as velocity having the same phase there is no phase difference. So, that is there.

So, with this we will now close the discussion for today, what we have done is we have captured how pressure and velocity they are related through this term called phase and we have explored this relationship in context of open tubes, closed tubes and semi infinite tubes.

So, tomorrow and day after tomorrow we will continue this discussion on Z, but in the next two days, at least in the next two days we will do this discussion in context of imperfect terminations, where its neither closed tube nor an open tube, but the situation somewhere in between. So, that is what we will discuss in next 2-3 sessions.

Thank you and have a great day, bye.