**Fundamentals of Acoustics**
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**Lecture – 31**
**Specific Acoustic Impedance for a Closed Tube**

Hello. Welcome to Fundamentals of Acoustics. This is the sixth week of this course, and in this week we will extend the discussion which we having in the last week. So, last week we had concluded by introducing the concept of spherical waves for related to monopoles. And in that context we had also developed an expression for the impedance of wave which is travelling is spherically in outward direction.

So, will continue this discussion on spherical waves, but before we continue that discussion we will take a small detail because we had in introduce this motion of impedance for these spherical waves. So, we will have a little bit more discussion on impedance for a spherical as well as for waves travelling in one direction, Cartesian frame that is for planer waves and also how we can actually measure this impedance. So, that is how going to start, and then once discussion over then will go back to this spherical propagation of one dimensional wave in spherical co-ordinate system.

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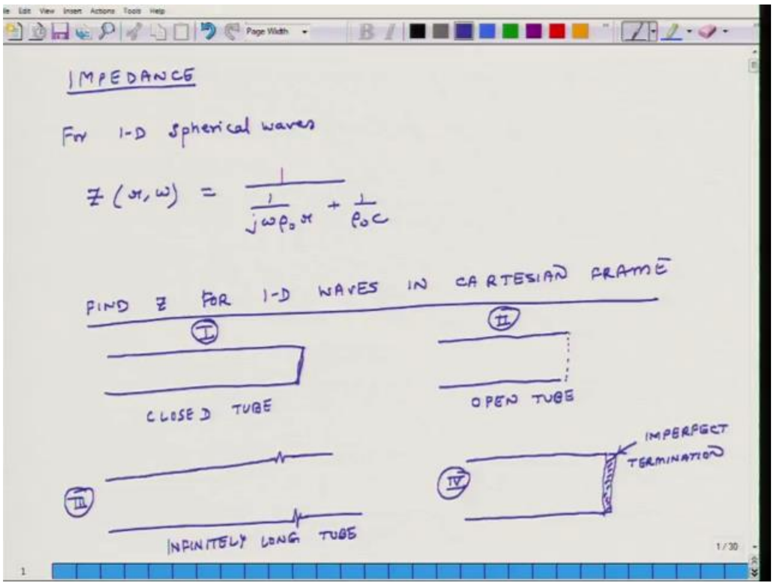


Our theme will be impedance; now for one dimensional spherical waves we had defined or we had shown that the impedance of a wave which is travelling outwards is Z and it depends on r and omega and that can be written as 1 over 1 over j omega rho naught r plus 1 over rho naught c.

Now, let us look at the impedance for one dimensional waves travelling in Cartesian frame that is one dimensional planer waves. So, our aim is to find Z for 1-D waves in Cartesian frame. So, specifically we considered for four scenarios. First is, we will have closed tube this is scenario I. Then we will also explore how Z behaves in open tube, so that is the scenario II. Then the third scenario will be; how what is the nature of Z in a tube which is infinitely long. So, infinitely infinite long tube so that is scenario III. And the fourth scenario will be a tube, and here in scenario I the tube is closed with rigid end. So, here the tube is closed, but it is not perfectly rigid. So, this is imperfect termination, so this is four. These are the force scenarios were we will see how Z behaves.

So, then we will overall picture how Z behaves in 1-D Cartesian frame and also how it been use for spherical waves which are in one dimensional spherical waves. So, that is there.

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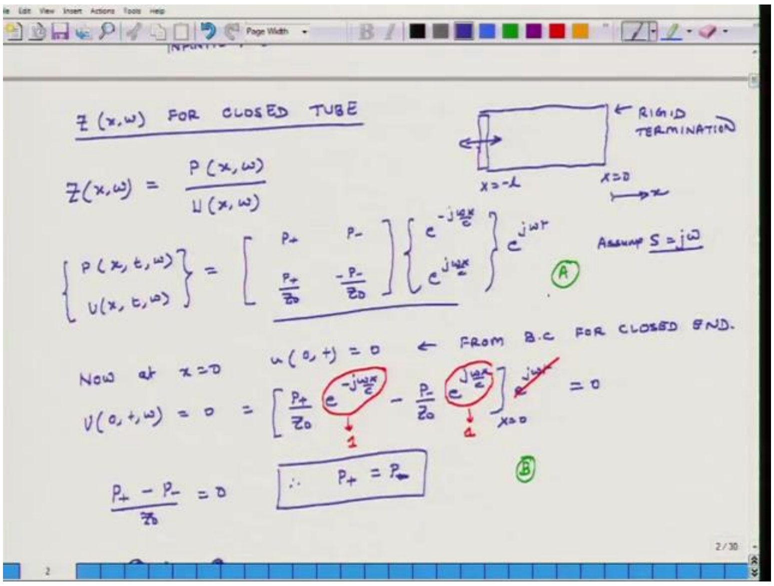


So, close tube look something like this at x is equal to 0 this is the rigid termination, this is x is equal to minus l and our positive x direction is going towards the write starting from rigid termination end we should know what is if we trying achieve. So, are aim is calculate Z which is the function of x and w and earlier we had defined Z as, so Z is a function of complex pressure amplitude and complex velocity amplitude. So, it is a function of P x omega, U x omega it is not ratio of d because this P and U are function, so the change with frequency and change with x. So, it is not a number or rather Z is the function and it depends on P and x.

So, our aim is figure out P x of omega P of which depends on x omega and u which depends on x and omega, and if then we can calculate Z. So, now we go to transmission line equation. So P; I am being strict mathematically then P essentially depends on x time and omega x time and omega. In earlier lectures I had not written like this x time and omega I have just written x and t, but if you are mathematically more precise then this is the way it should be written. So, this equals this are the transmission line equations P plus P minus P plus P minus. So, P plus and negative of P minus divided by Z naught, e minus j omega x over c e j omega x over c. So, here what I am doing is I am right away assuming that s is equal to j omega, and then e to the power of j omega t. So, here assuming that s is equal to j omega.

Now, at x is equal to 0 velocity is 0, this is from boundary condition for closed end. So, velocity 0 then u which is the complex and velocity 0 t omega is also 0 which means; so P plus by Z naught e minus j omega x over c minus P minus over Z naught e j omega x over c e j omega t and this thing in the brackets if it is evaluated x is equal to 0 it is 0 this is 0. So, one thing is that e j omega t cancels out and when x is equal to 0 this term become 1. So what I get is, P plus minus P minus divided by Z naught is equal to 0; therefore P plus equals P minus. So, let us call this equation A and this equation B.

Now what we do is we put B in A, so are modified transmission line equation. So, equation A is the general transmission line equation and it is valid for all sorts of boundary conditions, but now the momentum we put B into A the modified for framer transmission line equation will be specific to the close tube.

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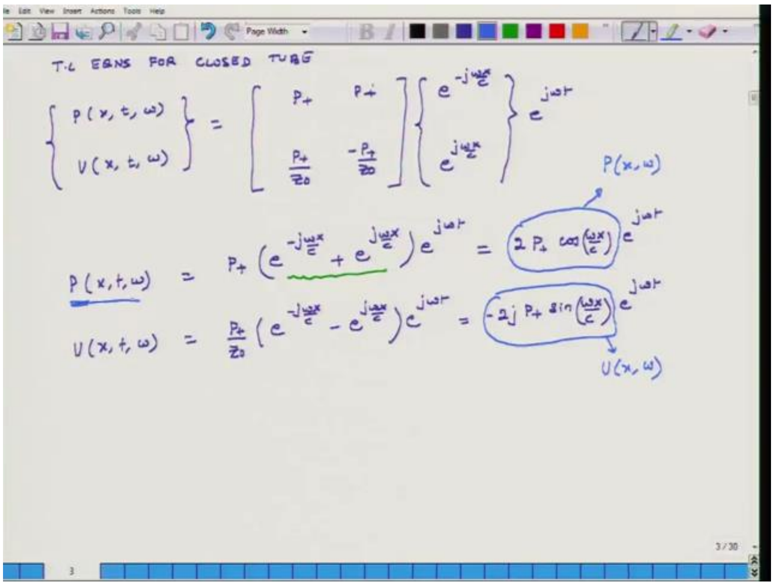


So T L equations for close tube; so this is complex pressure x t omega complex velocity x t omega equals P plus P minus, so I am just P minus so I am putting that equality and this is negative P plus over Z naught this is also divided by Z naught e minus j omega x over c e to the power j omega x over c e j omega t. So what? I can rewrite this as P x t omega equals P plus e minus j omega x over c plus e j omega x over c e j omega t. And complex velocity is P plus divided by Z naught e minus j omega x over c minus e j omega x over c exponent j omega t.

So, the relation for pressure I can further. So, this term in parenthesis is exponent minus j omega x over c and exponent j omega x over c. So, when I resolve them to real and imaginary parts the imaginary parts canceled out so left with this 2 P plus co sin omega x over c e j omega t. And likewise for velocity I get 2 j, and this negative here P plus sin omega x over c exponent j omega t. Now, this part is the complex pressure amplitude P of x and omega and this part is complex velocity amplitude of which depends on x and omega only. Why do I say it is complex pressure amplitude? So, the amplitude of this pressure is amplitude of this term in the circle times amplitude of exponent j omega t write. And what is x bar amplitude of e to the power of j omega t it is 1, because what is e j omega t? Sin of omega t plus t is co sin omega t plus j times sin of omega t and its amplitude is 1.

So, whatever is the amplitude of this P of x t omega will be actually same as the amplitude of P of x omega, because the amplitude of this component is unity. Likewise the complex amplitude of velocity is this term in the circle; it is this term in the circle.

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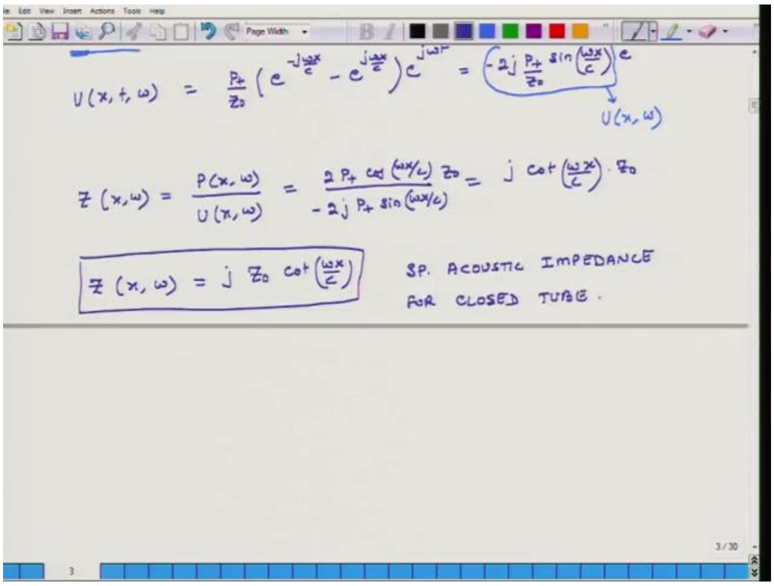


So, with that understanding we say and then we have defined Z which is the impedance is specific acoustic impedance and that is defined as complex amplitude of pressure divided by complex amplitude of velocity.

So, this equals 2 P plus cosine omega x over c divided by 2 j P plus sin omega x over c and this negative here. So this becomes, so P plus cancels out two cancels out and if I take j and the numerator if I multiply and den divide this numerator in denominator by j I am left this negative sin also goes away and I am left with j this there should be a Z naught here. So, they is Z naught in the numerator. So, Z naught co tangent of omega x over c times Z naught. So, Z of x and omega equals j Z naught co tangent omega x over c. So, this is the relation for specific acoustic impedance for closed tube.

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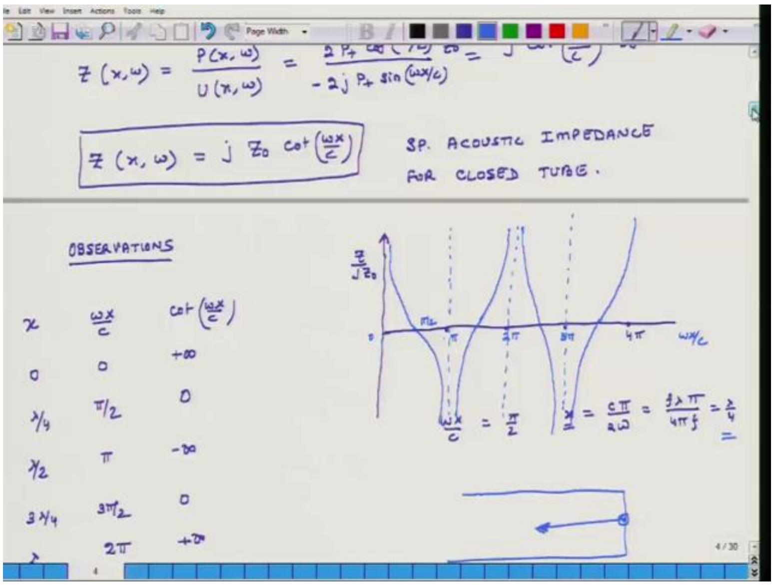


So, what does mean? So some observations; so first what we will do is, we will plot the magnitude of Z for using this relation. So, what is the value of? So, magnitude will be Z naught, actually I am not going to plot the magnitude of z, but I will the other plot Z over j. So, that I just vary about the real component. So, at x is equal to and I can normalize it by Z naught. So, at x is equal to 0 the value of this thing is going to be infinite, right at x is equal to 0 the value of this is going to be infinite. And so let us calculated when omega x over c equals pi over 2 then x is equal to c pi over 2 omega is equal to f lambda pi over 4 pi f, so this is equal to pi cancels out lambda over 4.

So, when x is equal to lambda over 4 omega x over c is going to be pi over 2, and co tangent of pi over 2 is going to be 0. So, x omega x over c and co tangent of omega x over c; when x is 0 omega x over c is 0, co tangent omega x over c is positive infinite. Lambda over 4 omega x over c is pi over 2, this is 0. When x is lambda divided by 2 half of the wave length then this is pi and co tangent of omega x over c is minus infinite. Then we have three lambda over 4 this is 3 pi over 2 it is 0 and then lambda; this is 2 pi and this is positive infinite.

So, the figure is going to repeat itself after lambda; and how is going to look like? So, at 0 is going to be very large and then at pi over 2 it becomes 0. And then at, so let us draw some; so here I am going to the x axis I am going to plot omega x over c. Let us say this is 0 pi over 2 pi 2 pi 4 pi and so on and so forth; x will may I will not call this. So, this is

3 pi and the next one is 4 pi. So, my curve is going to look like; so its start from positive infinite at pi over 2 it becomes 0 at pi it becomes negative infinities. And then it comes back and this is the asymptote, it is come back like this right then from 2 Pi so it goes like this.

So, this is how the impedance looks like they are the impedance. It repeats after every wave length - impedance. This is the closed end as I am travelling in this direction in closed tube at this point the impedances positive infinity then its goes down to 0 then it becomes negative infinity and so on and so forth. And then again so that is how it behaves. So, this is how impedance behaves for a closed tube. And what we will do in our next class is we will develop a similar expression for an open tube.

So, with that we close the discussion for today and we will meet tomorrow again.

Thank you. Bye.