Fundamentals of Acoustics
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Lecture – 30
Relations for Outward Travelling Spherical Acoustic Wave

Hello, welcome to Fundamentals of Acoustics. Today is the last day of this particular week which is the fifth week of this course and what we plan to do today is conclude the discussion which we initiated yesterday and what we will specifically start today lecture is by developing are relationship between P plus and U plus and once we are able to develop that relationship then we will compute the impedance of spherically symmetric outward travelling wave.

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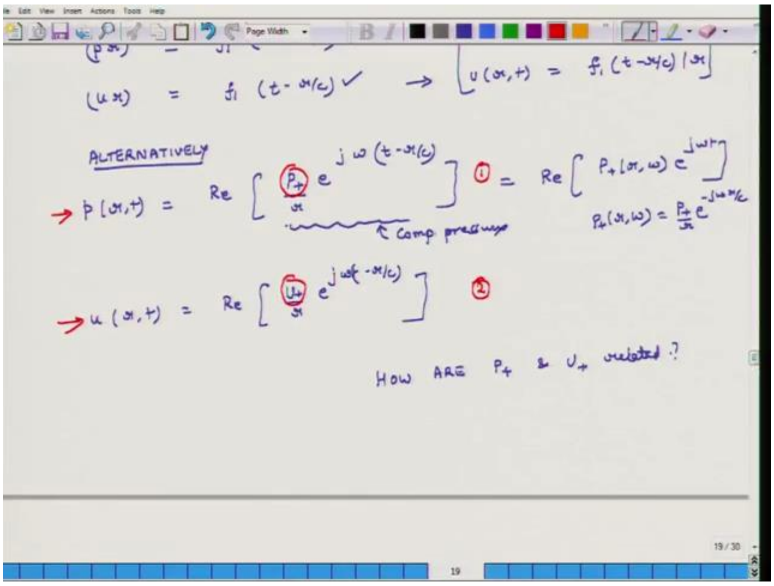

When we are doing one dimensional waves moving in Cartesian frame of reference that is planer waves, we had used Newton's second laws that is momentum equation to connect pressure and velocity. So, we will you use similar relation, but which is applicable for spherical symmetric systems to develop the pressure velocity relationship.

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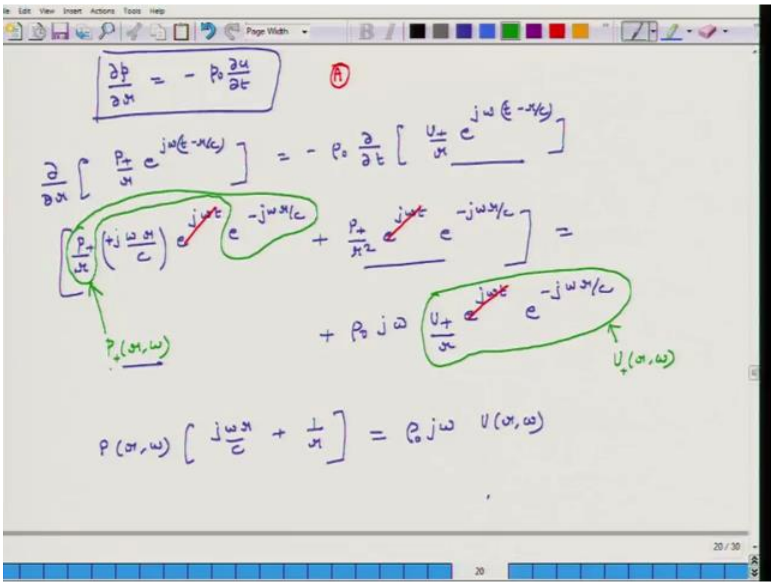

The momentum equation for 1-D is spherical. So, momentum equation for this spherical system or spherical reference frame and of 1 D verity is what is it? It says that partial derivative pressure with respect to radius is equal to minus rho naught del u over del t. So, we look at equation 1, this is the equation for pressure equation 2 is the equation for velocity we plug it equation 1 and 2 in I call this equation A and we equate them. So, what does it mean? It says 1 side do that my expression is partial derivative with respect to radius of the pressure. So, pressure is P plus over r e to the power of j omega t minus r c is equal to minus rho times partial of u with respect to t and u is U plus over r e to the power of j omega t minus r over c. So, this is what the terms are on both sides. So, we do the differentiation on both sides. So, on the LHS I get people as constant, it may be complex, but it is constant. So, I get P plus over r and first I differentiate the exponential ha P plus and if I differentiate the exponential I get j omega r over c and in the negative sign, e j omega t e minus j omega r over c. So, I will just differentiate the exponential part.

Now, I will differentiate the part related to 1 over r. So, I get another term minus P plus over r square, e j omega t e minus j omega r over c this is equal to negative of. So, I am going to write this in the next line, negative of rho naught and here I am going to differentiated this with respect to time. So, I get j omega U plus e j omega t e minus j omega r over c and there is a r in the denominator. So, this negative sign from the both sides it gets canceled out. So, this becomes plus this becomes plus and. So, does this the other thing we note is e j omega t it gets canceled out I can also cancel out e minus j omega r c, but I will not do that because it is all some useful purpose. So, the next thing is that this term is defined as U of r omega complex amplitude of r omega.

It is U plus divided by r time's e minus j omega r c similarly this multiple is p of r omega and I can rewrite this entire equation as P of r omega and then here I get j omega r over c plus 1 over r because here also I have P of r omega. So, this is equal to rho naught j omega U of r omega and remember that this waves are travelling in the outward direction and there are no reflected components. So, I will also add up positive sign here. So, this is the complex amplitude which varies with r and it just travelling the outward directions I have put up positive sign, if they also reflected component then I will not be doing this.

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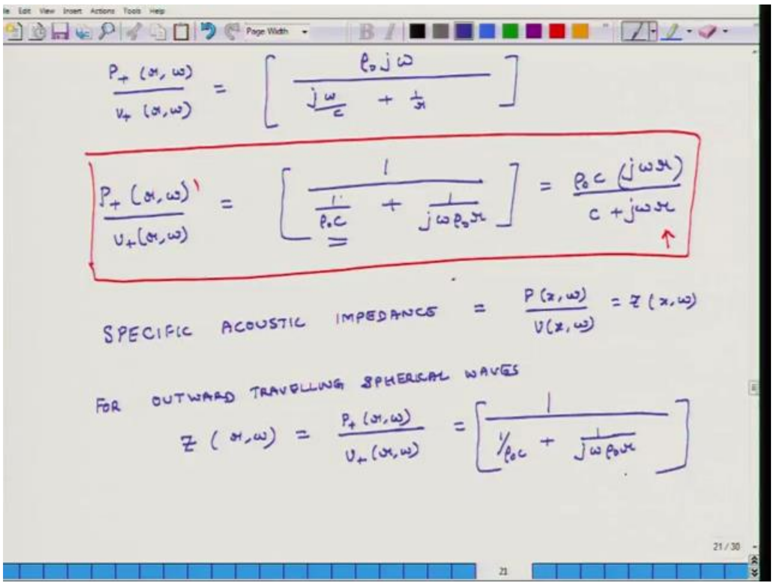

That is there. So, I put the positive sign here also. So, P plus r omega over U plus r omega equals 1 j omega c no; j omega r divided by c and then there is the rho here and also 1 omega. So, P plus. So, I get rho naught j omega and if I rearrange this terms I can write this entire thing as 1 over. So, if I divide the numerator and denominator by rho j omega I get numerator just 1 and here I get 1 over rho naught c plus 1 over j omega rho naught r or another way to express the same thing is rho naught c j omega r divided by c plus j omega r.

This is an important relation. So, very important relation and what this means is that the complex amplitude of the pressure wave and the complex amplitude of velocity wave they proportion is this complicated function and this function changes with r it changes with r, it is important to understand, I missing some r somewhere in.

Student: (Refer Time: 10:32) which is equal to j omega r by c.

You are saying here r should be there.

Student: yes

No, it should be there, I am missing otherwise there will be problem way did I do a mistake.

Student: and we have already (Refer Time: 10:52) differentiating with respect to r. So, that r will be there in when differentiate the omega r divided by c j omega by c will be there.

Student: (Refer Time: 11:00)

This r should be not there ha. So, this r is not there j omega by c then this c r will be not there (Refer Time: 11:25) we start again. So, while doing this differentiation there was a small error which I committed, this r should not be there.

It should be just j omega over c and if this r is not there then it also goes away from here and it also goes away from here opus and it is then that I get this expression r over rho naught c so.

Now earlier in one of our earlier classes we had defined is specific acoustic impedance and what this was complex pressure amplitude divided by complex velocity amplitude and this were definition was for the planer wave travelling in one direction in just x direction and this complex pressure amplitude as both reflected as well as forward travelling component; now in the clerical wave there is no reflected component at list in this case which we have developed; so for outward travelling is spherical waves Z or his specific acoustic impedance. So, this is Z and this is x and omega Z which is the function of x on omega. So, not x spherical wave it is r and omega this function can be expressed as P plus of r and omega divided by U plus of r and omega complex pressure amplitude and complex velocity amplitude this ratio and this ratio is equal to 1 over 1 by rho naught c plus 1 over j omega rho naught r.

This is the very important relation.

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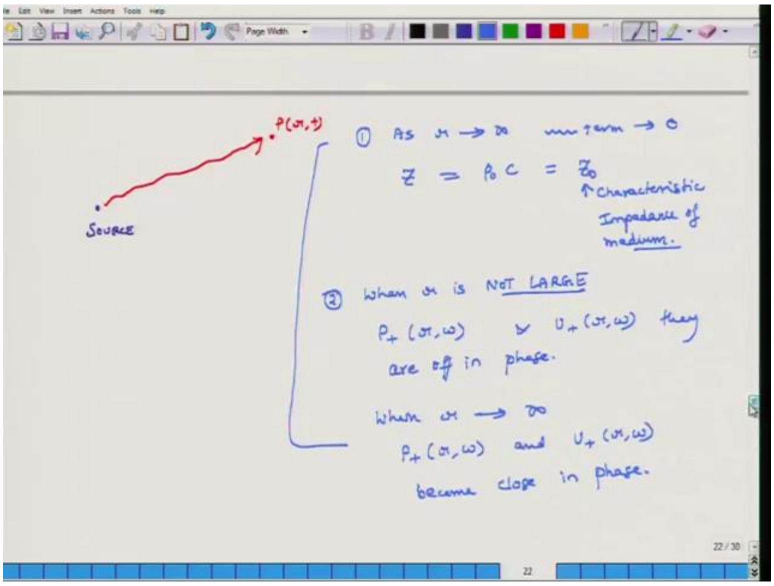

Now, just look at considered spherical wave. So, this is my source and I am at distance here, some point P which is r then t away. So, as wave travels from here to here the ratio complex amplitudes of pressure and u is going to be like this and when we make some important observations first observation as r goes to infinity this term 1 over j omega rho naught r inverse of this term, this thing it goes to 0, which means that Z approaches rho naught C and what is rho naught C? It is Z naught which is characteristic impedance of medium.

It becomes Z which is the specific acoustic impedance which varies with r it approaches rho naught c which is the characteristic impedance. So, this is 1 important observation, second thing, when r is not large what happens P plus r omega and U plus r omega they are off in phase because they ratio is a complex number, the ratio of these 2 thing is the complex function because there is j term, but as r becomes large are r approaches infinity then P plus r omega and U plus r omega become close in phase it become close in phase. So, this is important, what this means is 1 and 2 collectively they say that as I go away from spherically symmetric source in the wave when it is for away from this spherical source, it behaves as the planer wave because in planer wave where there is no reflections, see here there is no reflection when there is no reflection in planer waves if you look at the complex pressure amplitude and complex velocity amplitude there is no phase difference that is one thing and the characteristic impedance and Z is same.

As r becomes very large is spherical waves start becoming planer waves and physically also it makes since because the radius of cover of this waves so large that they are almost just travelling in a Cartesian kind of a fashion in just one specific direction. So, this is important to understand.

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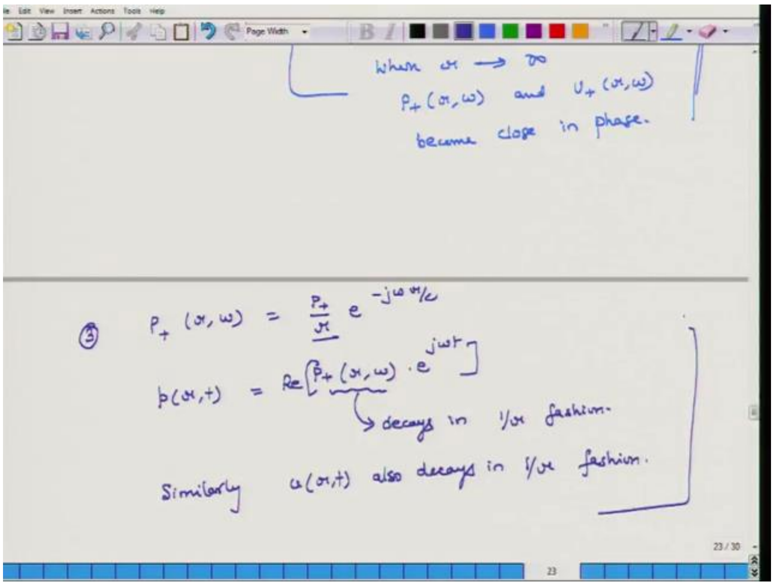

The third thing, the third observation is what is P of x P plus of r omega? What is this? we have defined it as P plus over r e minus j omega r over c and the actual pressure is equal to P plus r omega times e j omega t, if I take its real thing then I get the actual pressure now P plus decays in 1 over r fashion why because of this term 1 over r which means that as I move away from the source the pressure which I micro phone will sense it will fall in 1 over r fashion. So, if I got 10 times away from the source, the pressure is going down by factor of 10. So, pressure decays in 10. So, pressure decays in 1 over r fashion similarly u of r t also decays in 1 over r fashion. So, both pressure velocity and pressure the fall following this 1 over r rule. So, this are the 3 important observations which we have to understand as we as dealing with spherical wave which are travelling just outwards, first is that there phase difference between the pressure and velocity is not 0 in 1 dimension forward traveling wave if it is exactly 0. So, this is 1 important difference, second thing is as you move away from the source, the pressure with some going down is the velocity. So, this is the second difference and the third difference is that when become r become is very large is spherical wave start behaving as a planer waves.

This concludes our discussion for today and for the week; we will continue this discussion in the next week also till then have a great weekend and bye.