Fundamentals of Acoustics
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Lecture – 29
Point Sources of Sound

Hello. Welcome to Fundamentals of Acoustics, today is the fifth day of this week. And in the last four days we had been discussing the concept of power and power flow per unit area; as it applies to the first one was power, as it applied to electrical networks and power flow per unit area as applicable to acoustical systems. What we plan to do going forward is we should gives and will go back to are one dimensional wave propagation equation, but now we will start discussing wave propagation in context of spherical waves.

So, in our earlier discussion of 1-D dimensional waves was specific to planer waves which you can analysis using the Cartesian frame of reference. But in general a lot of sound sources do not propagate wave just in one single direction rather the sound gets spread in all direction. For instance, you may have a bird tweeting on top of a tree and that is sound just does not go in one particular direction, but its spreads in all the directions. Now in this context I would like to say that as we start doing this we will initially assume that there is a point source of sound. So, there is one particular point which is emitting sound and it is directionally neutral. So, it emits sound in all the directions, its Omni directional. And the intensity of sound as observed by a loud microphone or a person it does not change with respect to direction.

So, those are type of point sources we are going to consider. So, we will be considering Omni directional point sources.

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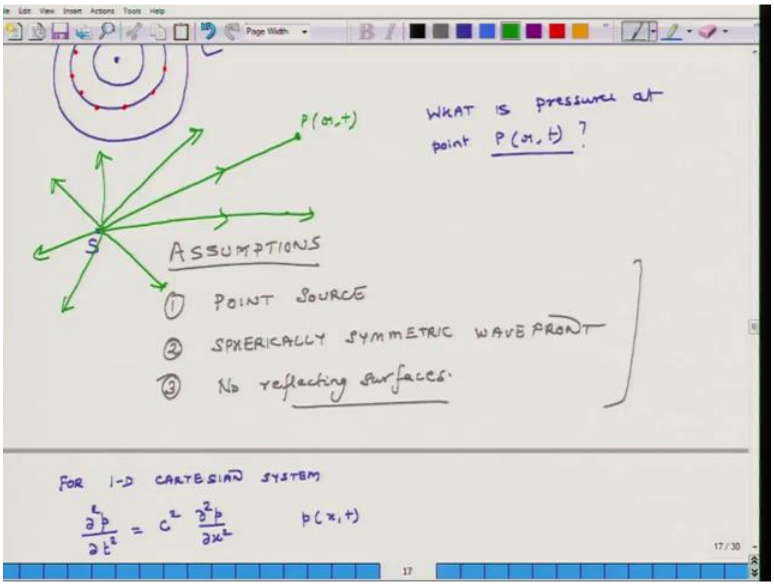

Point sources of sound; so you have point and let us say the sound source is located at the center of the sphere. If we measure sound on the surface of sphere then the intensity or the sound pressure level which you measure will be same. So, this is an Omni directional sound source. So it is Omni directional because it is spreading sound in all the directions. This particular sound source which is spherically symmetric that is if you have the source at the centre and if on a sphere the amount of sound which you perceive or which you measure is same then this type of sound source is called Spherically Symmetric Sound Source.

So, Omni directional sound sources may or may not be spherically symmetric, but spherical symmetric means that the sources is located in the centre and at all points on the sphere the sound which is been perceived is the same. Now that does not mean that if you make the sphere larger or smaller the sound level may be the same, but for a sphere if the source is located at centre for a particular sphere the intensity of the sound is same. You go to different sphere it is same on that sphere, but it may not be same as compared to sphere with a different radius.

So, this is what we are going to discuss. Now what is our aim? Our aim is that if there is a sound source let us say S and it is spherically symmetric then what we are interested in knowing is that suppose this person at point P and at a distance r away from the sound source; then what we are interested in trying to figure out is what is the pressure at point

P, P which is distance r away and at a given time. So, that is what our aim is; our aim is to be figure out how does pressure as I move away from a spherically symmetric sound source which is the point source how does pressure change.

So, in this context we will make some assumptions. So, what are the assumptions? The first assumption we are going to make is that it is a point source. Second is spherically symmetric; it emits spherically symmetric wave front. The third thing we know or assume is that whenever we are going to do these measurements there are no reflecting surfaces. So, what does mean no reflecting surfaces? I mean is that source may be located somewhere in the sky and there is no object in the vicinity that sound comes hits it and it get reflected.

So, in the case of Cartesian frame of reference we had also accounted for reflections. So, there was a forward travelling wave front and a reflected wave front, but here at least for starters we will not worry about reflections. And then later we will accommodate reflections, but for starting purposes we will not discuss reflections.

So, that is the background and these are the assumptions. With these assumptions we will work on the solution or of this problem that is how is pressure changing as I go away from the source.

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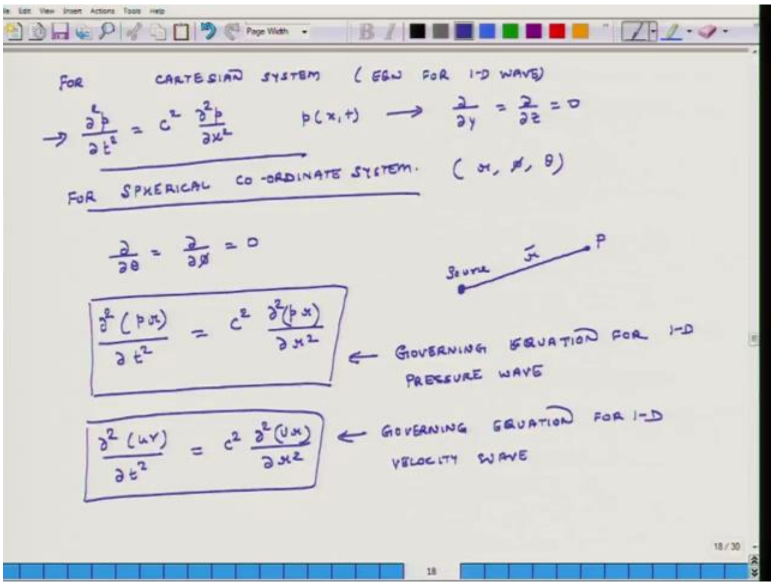

Now, for one dimensional Cartesian system we had developed the wave equation, and what did that wave equation say? It said that partial of pressure second derivative of pressure with respect to time is equal to c square, where c is the velocity of sound times second derivative of pressure with respect to x. So, here we had assumed that p is the function of x and t.

Now in our case; so here the problem is not Cartesian in nature so sound is travelling in all directions. So, here it is not a planer wave. In a Cartesian frame sound is travelling like this and there are planes, if at a given point of time if you take planer cross section the pressure across the plane are same, but in this case the sound is not travelling like this but the sound is travelling in all directions. So, here we will have instead of planer wave fronts here we will have a spherical wave fronts. So, these are the spherical wave fronts which have been drawn here. And on a spherical wave front at pressure is going to be the same.

So let us look at the spherical system; for spherical co-ordinate system. So, Cartesian system we have x y and z co-ordinates. And in this case for 1-D Cartesian system we have assumed that because the wave is propagating in only one direction, so any partial derivative with respect to y and with respect to z we had assumed that there were 0. Now if the wave front is spherically symmetric. Now in a spherical coordinate system we have three co-ordinates; one is r, other one is phi, and the third one is theta right. So, for a spherical co-ordinate system if the wave is spherically symmetric then we can say that partial derivative of any entity with respect to theta is equal to 0 and so is partial derivative of any entity with respect to phi. So, this is the assumption based on our understanding that the wave front is going to be spherically symmetric.

With that understanding I am just going into write the directly the equation which governs in a spherically symmetric system, I am not going to do the derivation. But if you are interested in derivation I encourage you to look at some of the texts which have been referred. So, in the 1-D wave equation was, so it should not be 1-D Cartesian system it is Cartesian system and this is the equation for 1-D wave. So, this equal to second derivative of pressure with respect to time equals c square times second derivative pressure with respect to x.

So, for spherical system a one dimensional wave can be represented by this equation looks similar, but it is not entirely similar. So, in Cartesian frame we had just partial derivative of with respect to partial derivative of pressure, but it here it is partial derivative of pressure times radius. So, this is the wave equation. So, second derivative of p times r in time equals c square equal c times second derivative t times r with respect to r. And what is r? R is the distance of point t from the source. So, this is source on this is r. This is the governing equation for pressure for 1-D pressure wave.

Similarly, the governing equation for one dimensional velocity wave is. So, in this particular course we are not going to develop the expression, but at this stage we will just write the equation and if you are interested you can go and look at the derivation from some of the standard texts. So, this is the governing equation for 1-D velocity wave and this is in spherical coordinate system.

Now let us look at the solutions, so we will just draw an analogy. So, for Cartesian system the solution for p was f any function f 1 which depended on t minus x over c and that was forward travelling wave.

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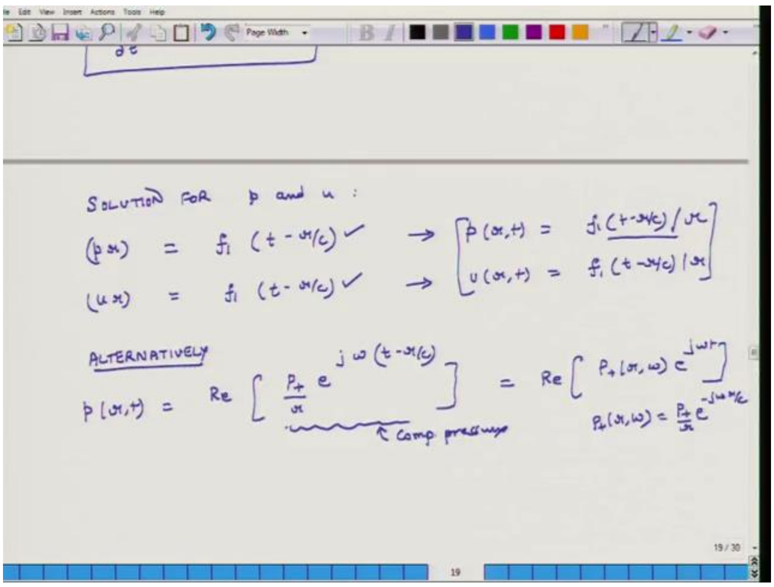

Similarly, the structure of the equation is very similar. So, a solution for p and u it can be written as; p times r, because here this is the entity which is being differentiated upon. So, this is p times r equals f 1 of t minus r over c. And similarly, for wave u times r is equal to f 1 t minus r over c. Or you can write it as p which depends on r and t is equal to f 1 t minus r over c divided by r. And from here you get u which is the function of r and t is equal to f 1 t minus r over c by r. So, these are this is the solution.

And you can prove it to yourself that if you plug in this relation in these differential equations you will see that the left hand side and the right hand side they satisfy, they become equal. So, such function generates functions they satisfy the differential equation. So, this function is our guess for this function is correct. Now, because this is t minus r over c it represents a wave which is travelling outwards. It represents a wave which travelling away from the source, because origin is located at the source. If there was a reflected wave I would have included n f 2 where it would have been t plus r over c. But in this course we are not going to discuss at least at this point of time reflected spherical waves.

So, this is the general solution, but if we have to express this solution in terms of you know in terms of complex variables then we can write it as; so alternatively we have to express them in exponential form as one we can write it has p of r t equals real of P plus e j omega t minus r by c divided by r. So, this is my complex pressure and if I take its real component I get the actual pressure. And I can also rearrange the term in the bracket as real of P plus r omega e minus e to the power of j omega t. Where, P plus is complex pressure amplitude that radius r and that equals P plus by r times e minus j omega r over c.

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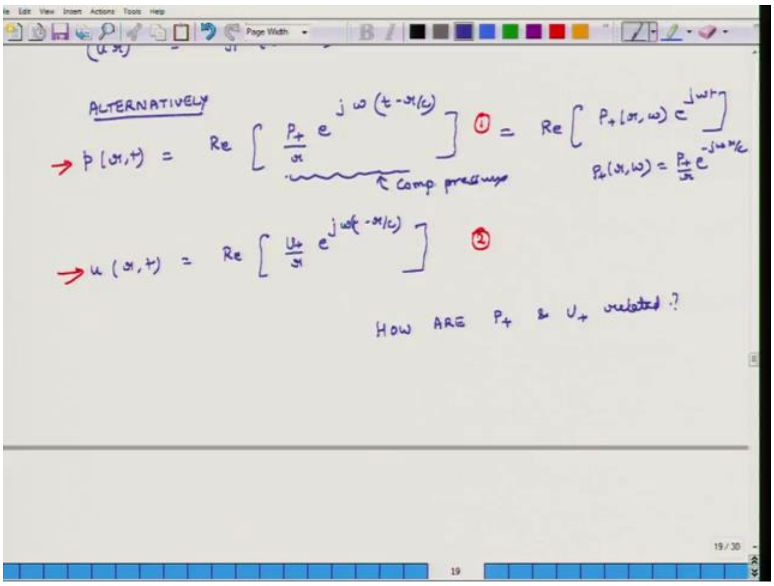

Similarly, velocity can be written as real of U plus over r e j omega t minus r over c. So, let us call this equation 1 and this as equation 2. Now, what we plan to do is; so this is the equation for pressure and this is the equation for velocity, and what we plan to do today is figure out how was U plus and P plus connected. So that is our question, how are U plus and P plus connected?

So, in 1-D Cartesian in system we had found the relationship between P plus and U plus by using the momentum equation which is essentially the statement of Newton's second law. So, will use the same momentum equation, but as expressed in the spherical system and using that equivalence we will connect the equations for pressure and equation for velocity.

So, that is what we planned to do in the next class. And that concludes our discussion for today on one dimensional spherical wave.

Thank you and have a great day. Bye.