

Fundamentals of Acoustics
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Lecture – 28
Power flow into an Infinitely Long Tube

Hello. Welcome to Fundamentals of Acoustics, today is the fourth lecture of this week. Yesterday we were discussing acoustical power flow per unit area and we have developed the concept and we had applied that concept to calculate power which is flowing into a closed tube. And what we were finding through our analysis was that the total amount of power which actually flows into the tube and gets dissipated is 0 watts per unit area.

So, we will now contrast this particular relation and results with the same the same parameter as it that is power flow per unit area for a tube which is infinitely long.

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POWER FLOW INTO AN INFINITELY LONG TUBE:

$$u(0, t) = U_s \cos \omega t = \operatorname{Re} \left[U_s e^{j\omega t} \right]$$

$$U(0, \omega) = U_s \quad U^*(0, \omega) = U_s$$

$$\begin{Bmatrix} P(x, t) \\ U(x, t) \end{Bmatrix} = \begin{bmatrix} P_+ & 0 \\ \frac{P_+}{Z_0} & 0 \end{bmatrix} \begin{Bmatrix} e^{-j\omega x/c} \\ e^{j\omega x/c} \end{Bmatrix} \begin{Bmatrix} e^{j\omega t} \\ e^{j\omega t} \end{Bmatrix}$$

Find P_+ by applying B.C. at $x=0$.

$$U(0, t) = U_s e^{j\omega t} = \frac{P_+}{Z_0} e^{j\omega t} \Rightarrow \boxed{P_+ = Z_0 U_s}$$

$$P(x, t) = P_+ e^{-j\omega x/c} e^{j\omega t} - P_- e^{j\omega x/c} e^{j\omega t}$$

$$= Z_0 U_s e^{-j\omega x/c} e^{j\omega t}$$

So, here power flow into an infinite long tube. So, this is my tube which is infinitely long and I am interested in finding out how much power flow is happening in this tube. So, here this is my x positive x direction and x is equal to 0 at this location. And the boundary condition at x is equal to 0 is that we know that u at 0 t is equal to U s which is a positive number we are given it is given cosine of omega t. So, I can also express it as real of U s e j omega t. So, this is the boundary condition which we know.

So, once again we have to calculate the power flow per unit area, and to do that what do we have to do? We have to use this relation and in this relation we have to plug in the values of u at x is equal to 0 at t and u at x is equal to 0 at t , and if we are able to do that we will be able to calculate power flow for this tube. U at 0 is U_s and u at 0 is again U_s , that does not change.

Now what we have to find out is the pressure at x is equal to 0 . So, for that we go back to our transmission line equations we say that complex pressure and complex velocity can be expressed as $P^+ + P^-$ over $Z_0 e^{-j\omega x/c}$ and $P^+ - P^-$ over $Z_0 e^{j\omega x/c}$. Now this is infinitely long tube, so once sound goes in this direction it has no chance to get reflected and come back which means that P^- is 0 ; so this is 0 . So what I can do is, I can modify this equation by putting that value P^- to be 0 . So, this is my equation for an infinitely long tube.

The second thing, so now we have to find the value of P^+ , we do not know P^+ . So, we eliminate P^- with the understanding that it is an infinitely long tube, so no reflections. Now we calculate the value of P^+ by applying this boundary condition. We find P^+ by applying B.C at x is equal to 0 . So, what is boundary condition at x is equal to 0 , what do we say complex velocity 0 at t . So, this is a complex velocity at x is equal to 0 . So, this is equal to $U_s e^{j\omega t}$ and that equals, we find it from this relation. So, this is equal to P^+ by $Z_0 e^{-j\omega x/c}$. So, I put x is equal to 0 . So, I just get 1 times $e^{j\omega t}$. So, from here I get P^+ equals $Z_0 U_s$.

So, I get P^+ is equal to $Z_0 U_s$. So, P at x ; excuse me so our aim is to find P at x is equal to $P^+ e^{-j\omega x/c} e^{j\omega t}$ from this equation. And P^+ is $Z_0 U_s$, so this is equal to $Z_0 U_s e^{-j\omega x/c} e^{j\omega t}$.

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$$\begin{aligned}
 p(x,t) &= p_+ e^{-j\omega t/c} e^{j\omega t} \\
 p(x,t) &= Z_0 U_s e^{-j\omega t/c} e^{j\omega t} \rightarrow P(x,\omega) \\
 &P(0,\omega) = Z_0 U_s \\
 \\
 w_A(0,t) &= \frac{1}{2} \text{Re} [P(0,\omega) U^*(0,\omega)] + \frac{1}{2} \text{Re} [P(0,\omega) U(0,\omega) e^{2j\omega t}] \\
 &= \frac{1}{2} \text{Re} [(Z_0 U_s) U_s] + \frac{1}{2} \text{Re} [(Z_0 U_s) (U_s) e^{2j\omega t}] \\
 &= \frac{Z_0 U_s^2}{2} + \frac{Z_0 U_s^2}{2} \cos(2\omega t)
 \end{aligned}$$

So now, let us look at the relation. For power we had defined power per unit area is equal to what. So, we are going to evaluate at x is equal to 0, so I will put that thing here. So, that is equal to half of real of P complex pressure amplitude times velocity its conjugate 0ω plus half of real of $P 0 \omega U 0 \omega e^{2j \omega t}$.

So in this relation now this thing; so this is equal to p of x and ω it is the complex pressure amplitude. So, p of 0ω I put x is equal to 0 is Z naught U_s . So, this is equal to half of real of Z naught U_s and U star is again U_s plus half of real of $p 0 \omega$ is Z naught U_s and this is again $U_s e^{2j \omega t}$; so this is entirely real, this is also real so I get Z naught U_s square over 2 plus Z naught U_s square over 2 cosine $2 \omega t$. So, in an open tube this is the power per unit area which gets dissipated, so many joules of energy get dissipated per unit area every second. And the reason it is happening is that once the speaker has excited air the wave travels and it never gets chance to reflect and give it back to the loudspeaker.

So, energy just keeps on getting dissipated. So, here you have a resistance system, and because of this you have a constant energy which is getting dissipated per unit time per unit area and its value is this, and this is the cyclic portion. So, at some point air gets compressed in the next cycle it gets it expands. So because of that this portion of energy it just recycles, but this is the energy which gets dissipated per unit second.

So, this completes our discussion on power and these are important concepts so please read these. And we will be using these concepts very exhaustively in maybe a couple of weeks later, but I wanted to cover this. And this concludes our discussion for today. Tomorrow onwards we will start discussing point sources. And with that I would like to conclude the discussion for today, and I look forward to seeing you tomorrow.

Bye, have a good night.