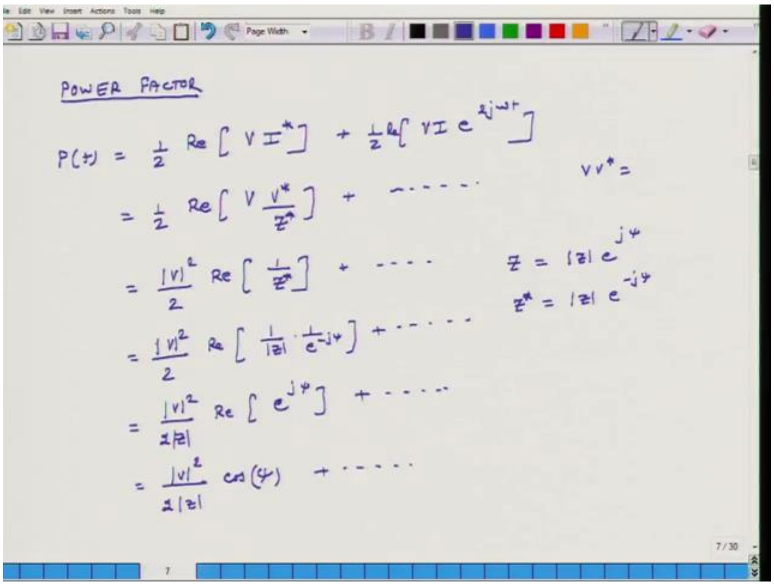
Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 27
Power factor, and Acoustics Power

Hello. Welcome to Fundamentals of Acoustics, today is third day of the 5th week of this course. In the last two lectures we have been discussing about instantaneous power; and then we have just started discussing the concept of power factor. And what we had mentioned in the last class was that if compared to a purely register circuits if there is a electrical system were we have an inductor parallel to the register then it draws in more current, and in such cases you tend to have higher power transmission losses. And the amount of power which is consumed because of these additional effects could be related to a factor known as power factor and that is what we are going to discuss today.

(Refer Slide Time: 01:16)



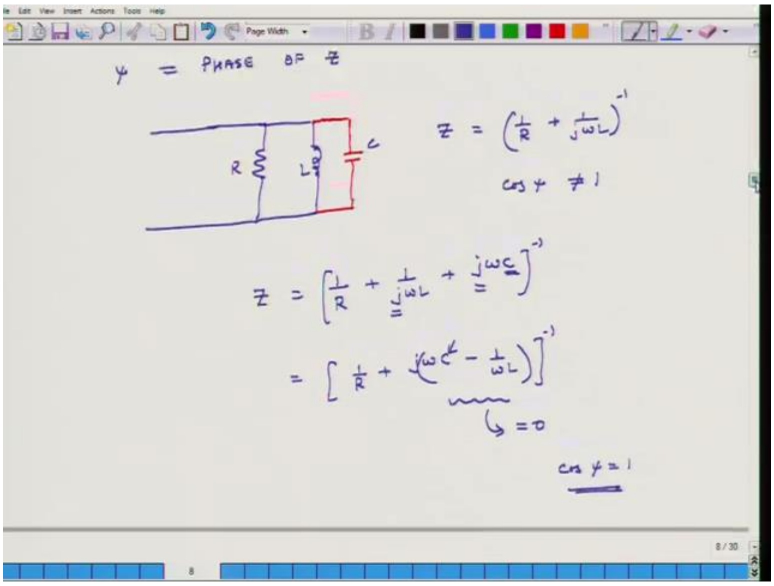
So, our theme is Power Factor. We know that power is equal to half of real of V I star plus half of V I e 2 j omega t. So this is equal to half of real of V, and I star is the ratio of complex voltage and complex impedance and their conjugates so it is going to be V star divided by Z star and then of course I have the additional term. Now V, V star is what? It is the magnitude square of magnitude a complex number multiplied by its complex conjugate that gives us the complex magnitude square. So, this is equal to modulus of V whole square divided by 2 times real of 1 over Z star.

Now, Z the overall impedance of the system would be having some magnitude let us call it Z modulus, and it will have some phase to e to the power j psi. So, Z star would be modulus of Z e to power of minus j psi. So, I can rewrite expression for power as modulus of V whole square divided by 2 times real of 1 over modulus of Z times 1 over e to the power of minus j psi plus the remaining parts. So, this is equal to V square by 2 Z times real of e j to the power of psi plus all other parts.

Students: (Refer Time: 04:13).

So, I have e to the power of minus j psi on the denominator. So, when I take it up it becomes e to the power of positive j psi. And the real portion of this is equal to V square over 2 mode of Z cosine of psi plus the cyclic terms.

(Refer Slide Time: 04:46)



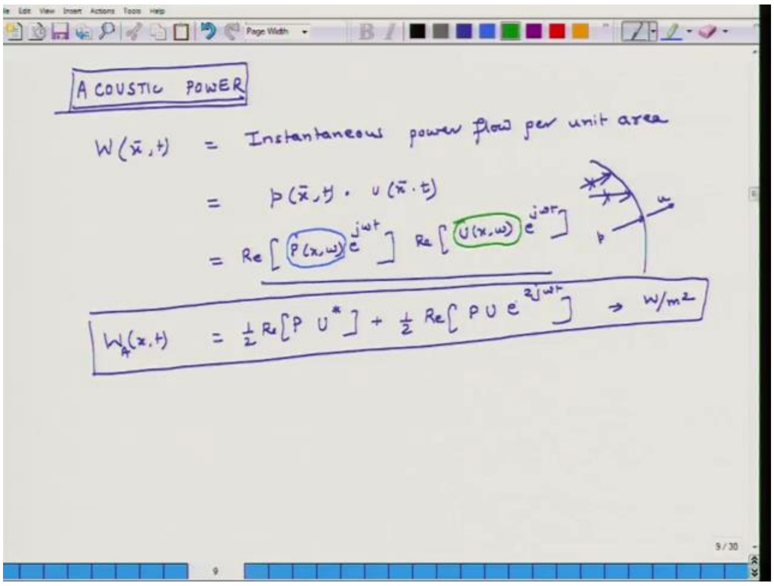
So power factor, it is defined as the cosine of psi. If this cosine term is 1, essentially what it means is that the impedance is purely resistive. If it is not exactly 1, then there is some reactive component in the system. So, if I want to minimize my transmission losses I have to make sure that cosine of psi should be 1. And psi is what? It is the phase of Z phase of complex impedance.

Now, how do I we make cosine of psi 1. Suppose I have a system which is something like this, and so in this case the overall impedance will be 1 over R plus 1 over j omega L and I have to take the inverse of this. And in this case there will be cosine of psi term and it will not be equal to 1, because of the presence of L. Now if I have to make this system such that it is purely resistive in nature, we know that the capacitors behave in a way which is somewhat opposite of an inductor. So, I can put a capacitor in parallel to this.

So, in that case my Z will be 1 over R plus 1 over j omega L plus the impedance offered by a capacitor is 1 over j omega C; so its inverse is j omega C and I take the inverse of it. And I choose this C in such a way, so here I have j in the denominator and here I have j in the numerator. So, this is equal to 1 over R plus j omega C minus 1 over omega L minus 1, because 1 over j is equal to minus of j. So, I can choose C in such a way that this term in the system it becomes 0; and when this term becomes 0 there Z becomes once again purely resistive and cosine psi it becomes 1; so by doing this I can set the power factor of my system to close to 1 and that helps in reduction of transmission losses which helps the economy of the country.

So, with that we conclude the discussion on power factor and electrical power, and what we will do now is we will now move to Acoustical Power.

(Refer Slide Time: 08:28)



Acoustic power; now in context of electrical engineering we had voltage times current and when you multiple these two entities you get watts. The fundamental entities in acoustical world are pressure and velocity, and when you multiply them dimensionally you do not get watts but rather what you get it is watts per square meter. So, even though we use this term acoustical power essentially what we are playing with is instantaneous power per unit area or alternatively you can call it intensity.

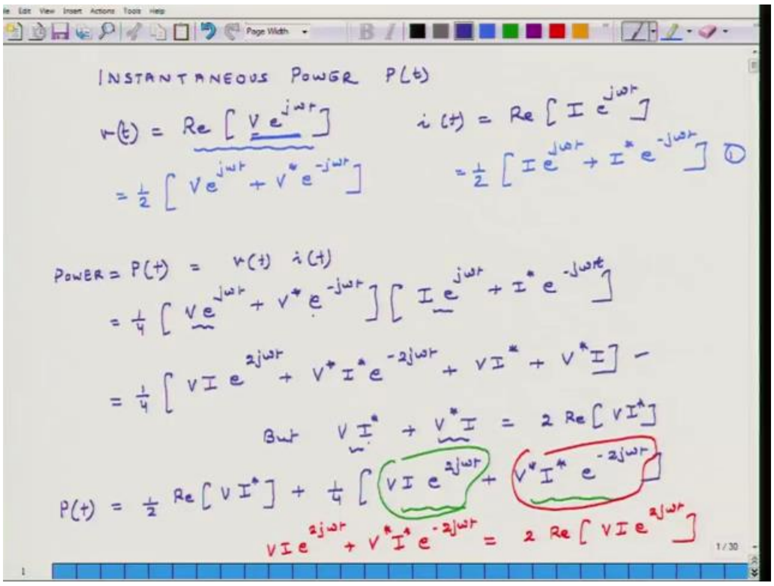
So intensity, I will call it W it depends on what it can depend on the position how far you are from the source which is x bar right; so x bar is a vector it can have x y and Z components and time. So, this is equal to instantaneous power flow per unit area. And that so in electrical world it was voltage times current here it is pressure, and pressure can change with respect to position, and then I multiple it by the normal velocity so that is why I have a dot product U x t. So, what does this mean? What does this dot product mean?

So, suppose I have a imaginary surface and the pressure is like this right. So, then I have to multiply it, so this is my surface, so whatever velocity. So, pressure is always going to at normal to the surface, we will draw better picture. So, pressure is not going to act like this or it will not act like this it will always act like; so it will not at in these sorts of ways pressure always at normal to a surface. So, pressure will always act normal to this.

And what I am interested in what is the velocity of the fluid particle at this in this direction. So this is my direction of pressure and this is my u. So, pressure corresponds to force per unit area and U is the velocity, so when I multiply force times velocity I get power when I multiple pressure which is force per unit area times velocity I get power per unit area which is getting transmitted.

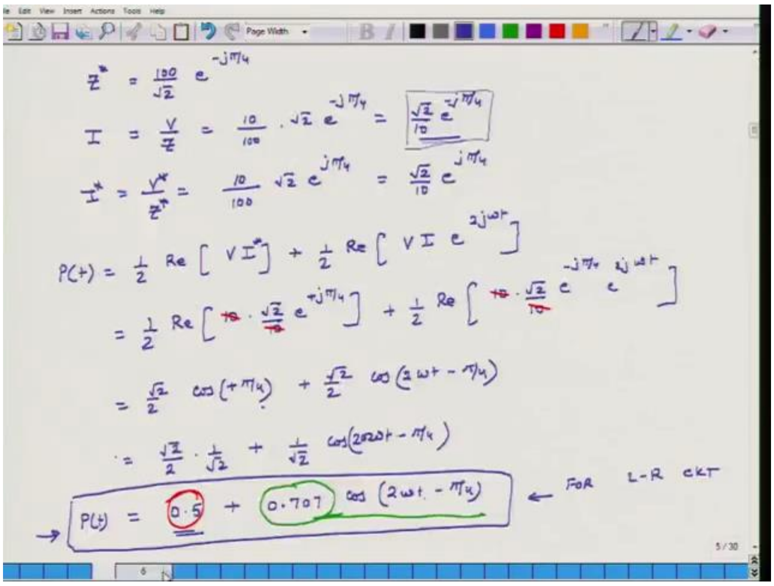
So, now for a one dimensional system we had defined after lot of analysis, we had expressed this pressure as real portion of complex pressure P x omega e j omega t, and velocity was real of its complex amplitude e j omega t.

(Refer Slide Time: 12:20)



So, this is identical or very similar to the relation which we have developed here; V here this is real V real of complex voltage and I is real of complex current.

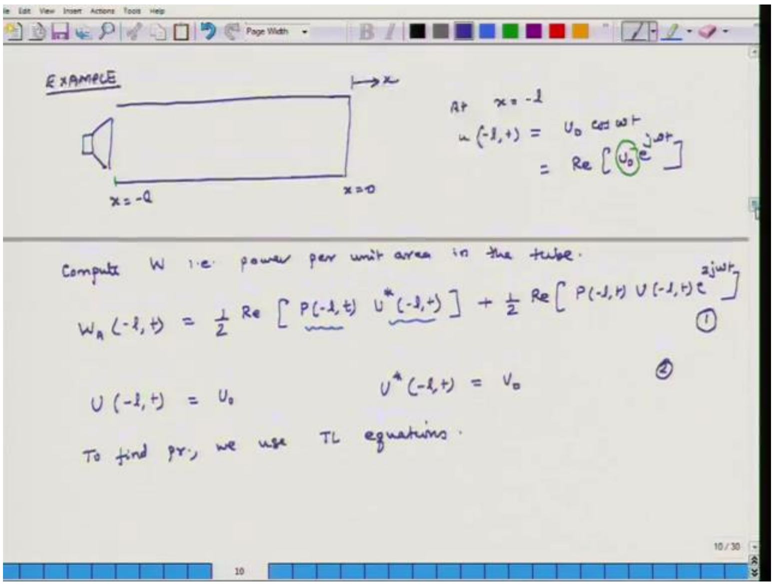
(Refer Slide Time: 12:37)



So I use the similar thinking here, and using similar principles of mathematics I can write it as half of P. So, P is the complex amplitude at position x times U star, where U is the complex conjugate of the velocity amplitude plus half of real- oh I missed a real here no I should not have anything real. And here I have half of; so sorry for the confusion. So, I have to have the real of P U star times half of real of P times U times e 2 j omega t.

So, here the units are watts per square meter. So that is acoustical power W A, actually its acoustical power per unit area. And what is P? P is the entire term. And what is U? U is this entire term and its conjugate is the conjugate of this function. This P and U can change from location to location and they can also vary with omega. So, we will do two examples.

(Refer Slide Time: 14:46)



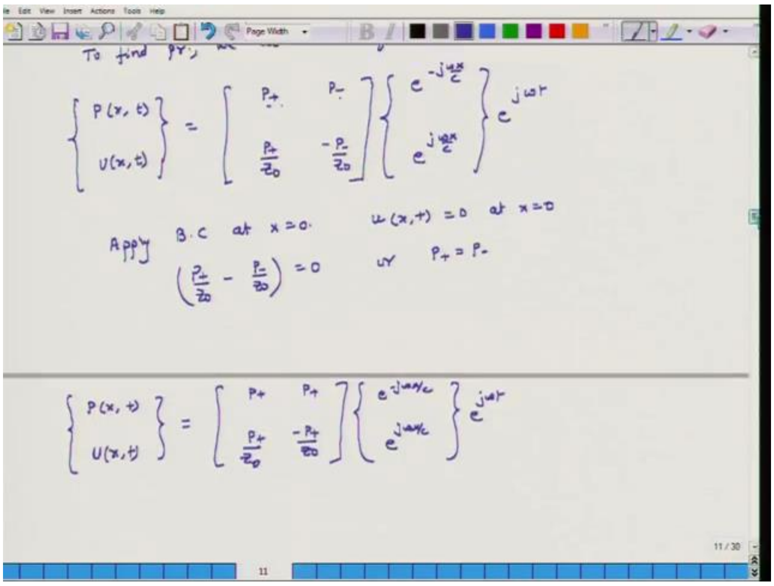
The first example is that of a closed tube. So, here I have a speaker and in a closed tube which is our length l it is closed at x is equal to 0; this is my direction of x positive direction. At x is equal to minus l, I know the velocity of the diaphragm. So, whatever is the velocity of deform will be the velocity of air at that location. So, I know that U at minus l t is equal to some constant U 0 which is a real number cosine omega t. Or I can also express it as real of U naught e j omega t. So, our aim is to compute pressure W. So, compute watts or power flowing per unit area and the tube; that is the power per unit area in the tube.

So, let us first write down the expression. So at x is equal to minus l, what am I interested in? I am interested in finding out this expression for power per unit area which is equal to half of real of P minus l t U star minus l t plus half of real of P minus l t U minus l t e 2 j omega t. So, let us call this equation 1. And if we are able to calculate all these individual terms P at minus l t complex amplitude of velocity conjugate at minus l t and so on and so forth then we can calculate the expression for W.

Now, we know it is given that; so just to remove confusion this is the actual velocity. So, it is lower case and this is complex amplitude, no excuse me sorry. So, from the boundary condition we known that U at minus l t is what? It is equal to U naught; this is U naught complex amplitude of velocity at minus l t which is at this location is equal to U naught. Similarly its conjugate at minus l t is conjugate of U naught so it is still U naught. So, this much we know.

So, the next thing is we have to find pressure; pressure at minus l t. Now, how do we find pressure? To find pressure we have to use the transmission line equation. So this is equation 2, and for finding pressure we use transmission line equations which we had developed earlier. So, let us look the transmission line equations.

(Refer Slide Time: 19:35)



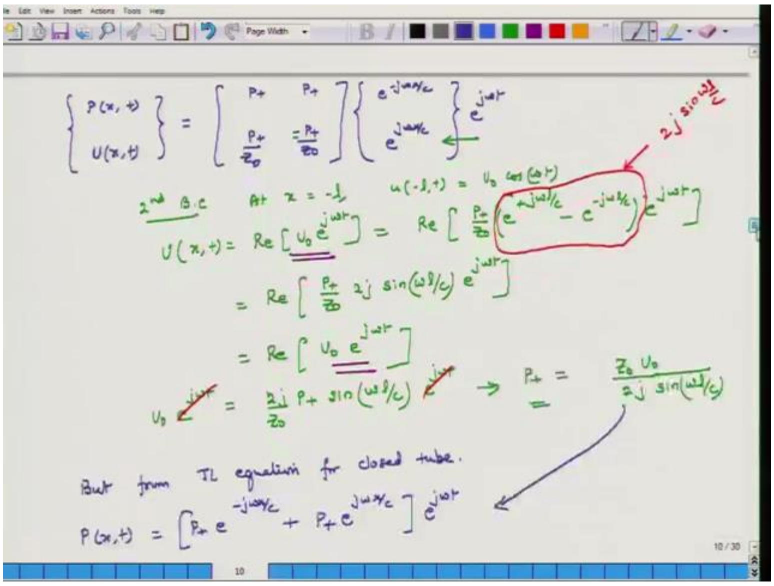
So, our transmission line equations say that complex pressure and complex velocity they can be expressed as P plus P minus P plus by Z naught and minus P minus by Z naught divided by e minus j omega x over c e j omega x over c times e to the power of j omega t. So, we have to find P plus and P minus and this case. If we can find P plus and P minus then we can find the pressure.

So, first is we apply B.C at x is equal to 0. So, what is a boundary condition at x is equal to 0? It is a close boundary, so no air can go out or no air can come out. If no air is moving at the boundary it means that U x t is equal to 0 at x is equal to 0. So, which means that P plus over Z naught minus P minus over Z naught equal 0 or P plus is equal to P minus. So, we put this boundary condition back into our system of equations. So, P x t and this is complex velocity is equal to P plus P plus P plus P plus and this divided by Z naught e minus j omega x over c e j omega x over c times e j omega t.

Student: (Refer Time: 22:11).

Z 0.

(Refer Slide Time: 22:17)



So, we have applied one boundary condition, and then the second boundary condition we have to apply is the condition for x is equal to l; x is equal to minus l. So, at x is equal to minus l, so now we apply second boundary condition, second B.C. At x is equal to minus l U minus l t is equal to U naught cosine omega t; so complex velocity is equal to what real of U naught e j omega t. So, this is from the boundary condition and this I equate from the second equation, so I get real of P plus over Z naught e minus j omega L over c, and in the first equation x is minus l so this becomes plus, and then I have a minus because of this minus sign and then P plus Z naught is already out and here I put x as minus l again. So, I get e minus j omega L over c e j omega t.

So, our aim is to find P plus in this case. So, this is equal to P plus over Z naught and e to the power of j omega L over c plus e to the power of minus j omega L over c, what does this mean? When I add them up it becomes; so this thing, so this becomes 2 j sin omega

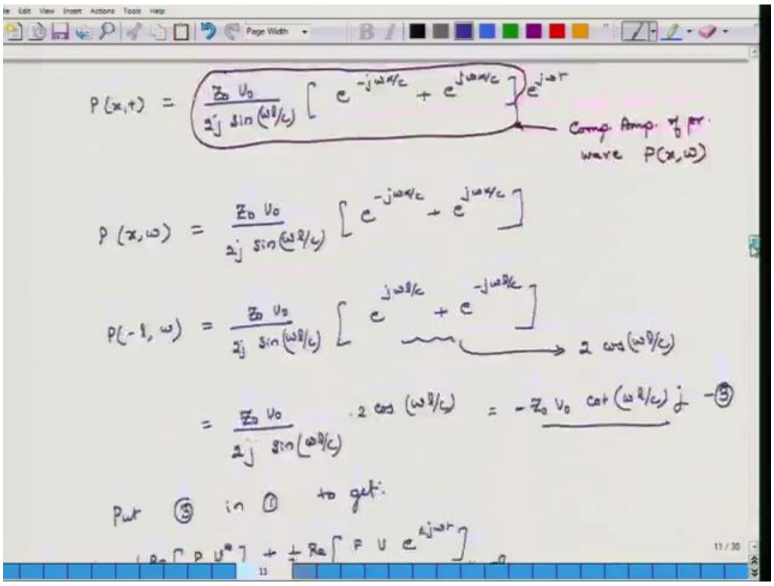
L over c. So, I write 2 j sin omega L over c e to the power of j omega t. So, that is equal to real of U naught e j omega t, from here.

So, if the real parts of both sides are equal then the term inside the bracket also has to be same, because this equation is valid for all periods of time for all values of omega and for all values of c. So what that means is? U naught, and U naught is a real positive number e j omega t is equal to 2 j over Z naught P plus sin omega L over c which give; excuse me times e j omega t. So, e j omega t goes away from both sides and I am left with P plus is equal to Z naught U naught by 2 j sin omega L over c.

So, that is my P plus. So if I put this P plus; where, so our aim is to calculate what complex pressure at minus l. This complex pressure is not same as P plus, because this complex pressure depends on P plus as well as P minus. So, I have to ultimately compute P of minus l t. U I already know but I have to compute P of minus l t.

So, we know but from TL equation for closed tube P of x t is equal to P plus e minus j omega x over c plus P minus e j omega x over c e j omega t, but we know for a closed tube P plus equals P minus so I will change this to P plus. So, now what I do is I substitute this in this relation.

(Refer Slide Time: 28:08)



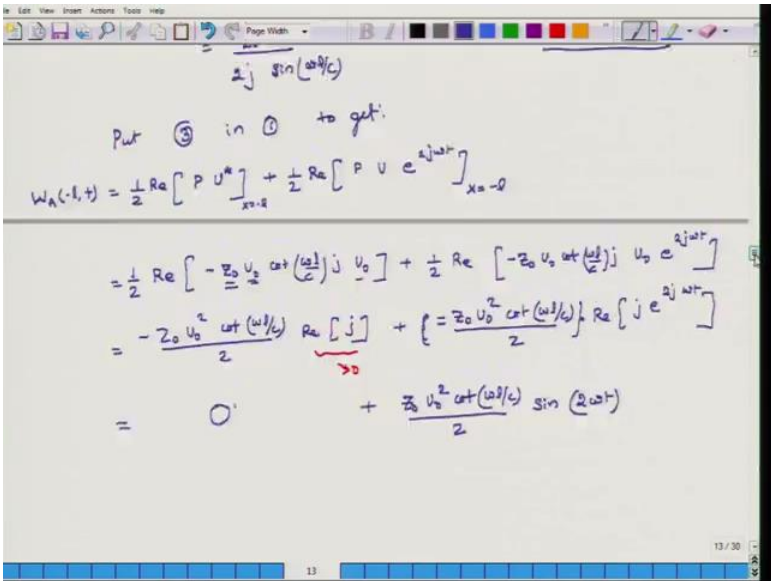
So, I get complex pressure is equal to, and P plus is Z naught U naught divided by 2 j sin omega L over c e minus j omega x over c plus e j omega x over c e j omega t. This term is the complex amplitude of pressure wave and it can be return as P x omega; so P x omega equals Z naught U naught by 2 j sin omega L over c into e minus j omega x over c plus e j omega x over c.

So, let us see our goal is; so we have developed an expression for P of x and omega and our aim is to find the value of this P of x and omega at x is equal to minus l. Once I find that then in this equation 1 I plug in this value I already know U naught then I will be able to calculate acoustical power. So, let us look at this.

So, P at minus l omega is equal to Z naught U naught divided by 2 j sin omega L over c, and here I put x is equal to minus l. So, I get e j omega L over c plus e minus j omega L over c and this thing equals two cosine omega L over c. So, I get Z naught, U naught over 2 j sin omega L over c cosine omega L over c times 2. So, this is equal to Z naught U naught cotangent omega L over c. And there is a j there, so I bring this j up and because I brings this j up there is a negative sign. So, this is equation 3. So, now we put 3 in 1 to get. So, what is the equation? This is what we are trying to calculate. Power flow per unit area that is equal to half of real of; half of real of?

Student: (Refer Time: 32:05).

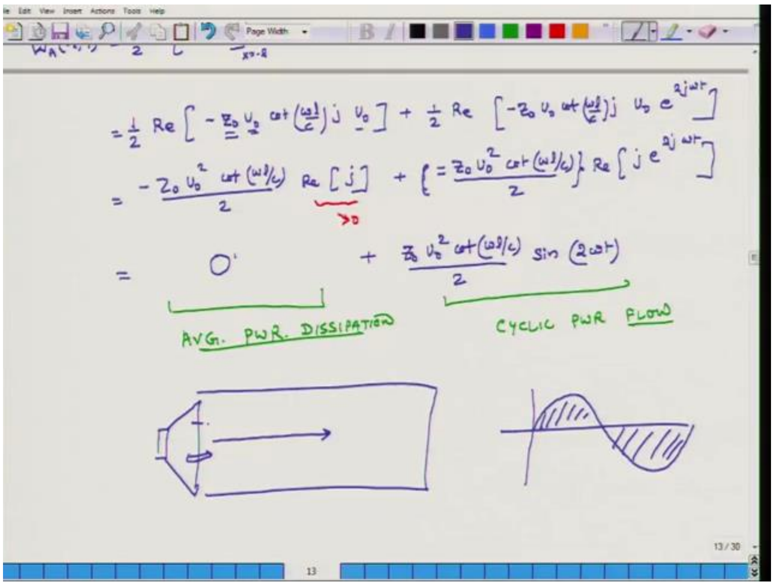
(Refer Slide Time: 32:14)



P U star plus half of real of; so this is evaluated x is equal to minus l. And P U, I am sorry e 2 j omega t. And this is also evaluated at x is equal to minus l. So, now we just put in the values. So this is equal to half of real of, the value of P at x is equal to minus l is this thing; so minus Z naught U naught cotangent omega L over c j this is important times U start, and what is the value of U star? U star we had calculated or realize that at the same as U naught; so U naught, plus half of real of P evaluated x is equal to minus l so it is minus Z naught U naught cotangent omega L over c times j times U and value of U is also U naught e to the power of 2 j omega t.

So, this is equal to minus Z naught U naught square cotangent omega L over c by 2 real of j, because Z naught is positive Z naught equals rho naught c, U naught is positive we know it. So, everything comes out real of j plus half of; so excuse me I will just erase this. So, minus Z naught U naught square cotangent omega L over c entire thing divided by 2 times real of j e to the power of 2 j omega t.

(Refer Slide Time: 36:00)



So, this value real of j is 0; so I get 0 plus. And what is the real of j times e to the power of 2 j omega t e to the power of 2 j omega is cosine twice of omega t plus j sin of omega t. So, I multiply that by j I get minus sin of 2 j omega t, that minus and this minus they interact and then they get eliminated. So, I get Z naught U naught cotangent omega L over c divided by 2 sin 2 omega t.

So what does this term represent? Average Power, so this is the power dissipation; and this is cyclic power flow. In half of the cycle you have positive power flowing and another half you have power flowing back to the system. So, what this means is that if you have a closed tube and I have a loudspeaker which is moving back and forth the amount of power which is getting dissipated in this is total amount is 0; total amount of power which is getting dissipated, which is getting absorbed by this tube is 0. That is what it means. What is happening is that your power in one-half the cycle it is positive. So, in half of the cycle the speaker dumps in power in to the system, and in other half the cycle the system dumps in that power back into the loudspeaker. That is what this analysis means.

So, that I think concludes the discussion for today. Tomorrow we will do one more example for an open tube, and we will see that the contrast between open tube and a closed tube. So, with that I conclude the discussion and we will meet once again tomorrow.

Thank you.