Fundamentals of Acoustics
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Lecture – 26
Instantaneous Power in a L-R Circuit

Welcome to Fundamentals of Acoustics, this is the 5th week of this course and today is the second day of this particular week, yesterday we had started our discussion on power and we had developed an expression for power in terms of its voltage and current amplitudes and also we had dissolved this expression for power into a steady state comp1nt and a cyclic component. So, what we are going to do today is continue that discussion.

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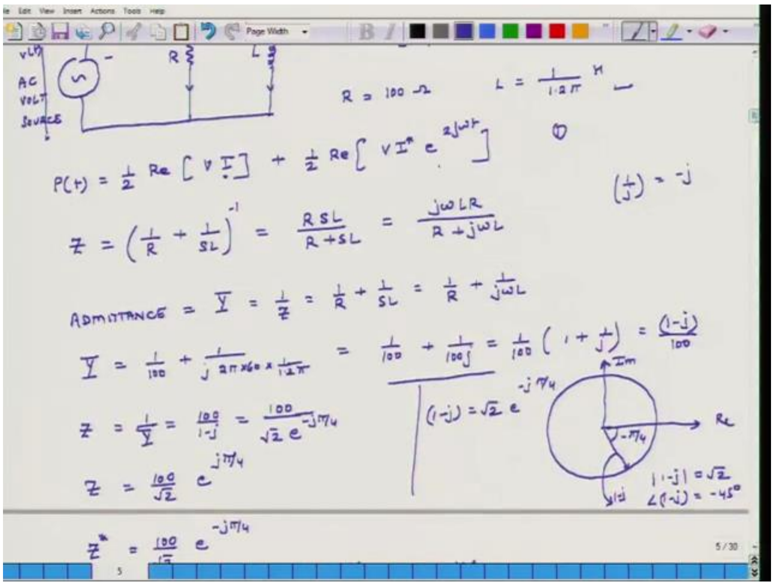

And essentially we are going to find out how much power is dissipated by a system which has a resistive as well as an inductive element. So, let us consider an electrical system. So, the system as an AC voltage source, it has a resistor which has a value of R. So, the inductance of this element is L. So, the voltage which is supplied by this AC source is equal to 10 cosine 2 pi times 60 t or this is equal to 10 cosine 120 pi t.

The current is something we do not know, but what we do know is, what is the value of R? And what is the value of L? So, R is equal to 100 ohms and L is defined as 1 over 1.2 pi henrys. So, our aim is to find an expression for power and power we will again write is equal to half of real of V times I, plus half of real of V times I square e 2 j omega t. So, at this point it is important to understand what is this capital I? So, here if the voltage is V t and the total current flowing in the circuit is small I as a function of time, then capital I is the complex amplitude of the corresponding complex current which corresponds (Refer Time: 03:28) total actual current flowing in the system. So, it is not the current in R or in L, but it is it corresponds to the complex amplitude of the total current and V is the external voltage on the overall system.

So, we know that expression for V. So, I can also express it as real of 10 e h omega t where omega is 2 pi times 60 hertz. So, now, what we have to find out is, what is the value of I and what is the value of I star? The value of capital V is 10, but we do not know the value of I and I star, our aim is to find out I and I star. So, this is one way we can do it. So, overall impedance of the circuit across the voltage source is, across the voltage source there are 2 impedance elements they are in parallel. So, I can say that the overall impedance is 1 over R plus 1 over SL where S is the complex frequency. So, S times L is it is overall impedance and then I take the inverse. So, that equals R SL over R plus SL and if S equals j omega, then I can write it as j omega L R divided by R plus j omega L. So, that is my first expression this is expression 2.

So, actually what we will do is we will actually can compute this. Now the inverse of impedance is known as admittance. So, this comes from electrical engineering. So, it is admittance and this I designate as dis scripted form of this symbol y. So, this is 1 over Z. So, Z is 1 over R plus 1 over SL. So, 1 over Z is equal to 1 over R, plus 1 over SL and S is equal to j omega. So, it is equal to 1 over R plus 1 over j omega L and now we put the values. So, admittance is equal to R is 100 ohms. So, it is 1 over 100 and L is 1 over 1.2 pi henrys, and omega is 2 pi times 60. So, it is j times 2 pi times 60 into 1 over 1.2 pi. So, I add these 2 up I get 1 over 100 plus. So, what do I get from here pi cancels out, 2 times 60 is 120, 120 divided by 1.2 is 100. So, it is 1 over 100 j. So, this admittance I can write it as y 1 over 100 times 1 plus 1 over j, and we know that 1 over j is equal to minus of j (Refer Time: 07:47) is equal to minus of j. So, it is equal to 1 minus j divided by 100, that is my admittance, then impedance is inverse of admittance and that equals 100 divided by 1 minus j.

Let us look at this 1 minus j term in a complex plain, in a complex plain this is my real axis, this is my imaginary axis and the amplitude or the magnitude of this 1 minus j is the square of real part plus square of imaginary part and it is square root. So, magnitude of 1 minus j is root 2 and what is the phase of 1 minus j? Phase of 1 minus j is minus 45 degrees, how do I get that? It is the inverse of tangent inverse, tan inverse of the imaginary part which is minus 1, divided by the real part which is plus 1. So, minus 1 over 1 minus 1 it is tan inverse is minus 45 degrees. So, on this complex plain if I have a vector, which is root 2 long and which is at minus pi over 2 orientations then this corresponds to 1 minus j ha pi by 4; sorry.

So, excuse me. So, that is my real axis and this angle if I t is minus pi over 4, then that corresponds to this thing. So, I can also write it as 1 minus j is having a magnitude of root 2, times exponent of minus j time's pi over 4. So, Z is nothing but 100 divided by root 2, e to the power of minus j time's pi over 4 or it is 100 divided by square root of 2, e to the power of j pi over 4.

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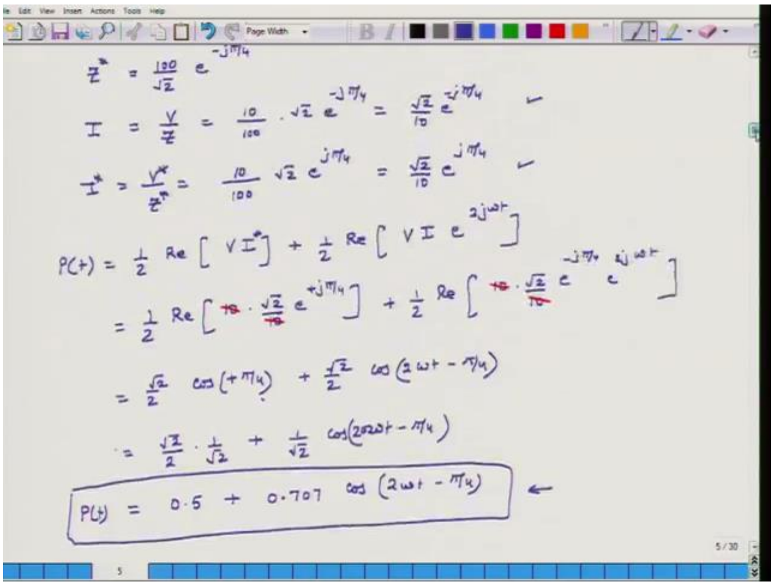

So, Z is 100 divided by root 2 e to the power of j time's pi over 4 and it is complex conjugate Z star is going to be 100 divided by root 2, e to the power of minus j pi over 4.

So, why are we doing all this? We are doing this because we have to calculate I and I star in this relation 1. So, now, we do the calculation for I, I equal V over Z and V is 10 volts, V is the complex amplitude of the voltage signal. So, that is 10 divided by Z 100 times root 2 e to the power of j pi over 4. So, this is equal to root 2 divided by 10 e to the power of j pi over 4 and I star is V star divided by Z star. So, I missed a negative sign here. So, I is root 2 divided by 10 into e to the power of minus j pi over 4 and I star is going to be. So, V star V is 10 it is a real number. So, it is complex conjugate is also 10 and Z star 100 divided by root 2. So, it is 100 root 2 e to the power of j pi over 4 and that gives me root 2 divided by 10 e to the power of j pi over 4.

So, now I know what is I and what is I star, I put these back into my relation for power and I will be able to calculate the relation for power. So, P t equals half of real of V I plus half of real of V I e to the power of 2 j omega t. So, this is equal to half of real of V was 10 volts I is times root 2 over 10 e to the power of minus j pi over 4, plus half of real of and I think here this was I star. So, this should be I star. So, V is 10 volts and I star is times root 2 over 10, e to the power of j pi over 4 and then e to the power of j omega t, 2 j omega t. So, this 10 and this 10 cancels out these 2 raise also cancels out. So, what I am left with is half and then root 2 comes out and then. So, the real of exp1nt of minus j pi over 4 is cosine of minus pi over 4 and then here I get half. So, our equation is for power is not half of real of V I, but it is actually half of real of V I star.

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And in this part it should be just V times I. So, there is a star here and there is no star here. So, because of there is negative sign it becomes a positive sign and this becomes a negative sign. So, it is cosine of plus pi over 4 plus. So, we will look at second part it is root 2 over 2 time's cosine of 2 omega t minus pi over 4. So, what is cosine pi over 4? It is 1 over a square root of 2. So, it is root 2 divided by 2 times 1 over root 2, plus root 2 by 2. So, I can simply, it is 1 over a square root of 2 times cosine omega t minus pi over 4. So, that is equal to 0.5 plus and 1 over a square root of 2 is 0.707 cosine 2 omega t minus pi over 4. So, that is my expression for power.

So, this is the expression for power, the expression for power for the earlier circuit which had only resistive elements here, it was similar, but where was a significant difference. So, it was half plus 0.5 cosine 240 pi t. So for the purely resistive circuit and where the resistance was same as what we had in this case, this expression is for L C circuit no excuse me L R circuit and here L and R are in parallel.

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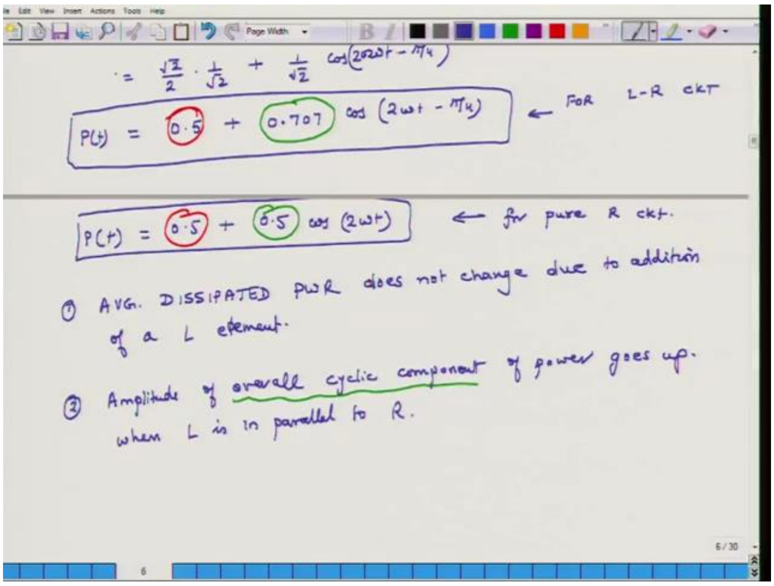

And the expression for power was 0.5 plus 0.5 cosine 2 omega t and this one was for pure R circuit, when we look at these 2 expression we can make some important conclusions.

So, the first conclusions is that average dissipated power which is this thing, it does not change due to addition of a L element, does not change due to addition of an inductive element. The second thing is that the amplitude of overall cyclic component of power. So, what is that amplitude? It is this thing which is underlined. So, this amplitude is goes up when L is in parallel to C. So, when L is in parallel to C it now it is even it is in parallel to the resistor; third thing is let us look at current, let us look at the amplitude of current, in an L c circuit the amplitude of current was root 2 over 10 times e to the power of minus j pi over 4 and so this is there, but the current also changes, the amplitude also changes, but what does that imply that is what we would like to see. So, what we will do is we will plot this equation on x y plane.

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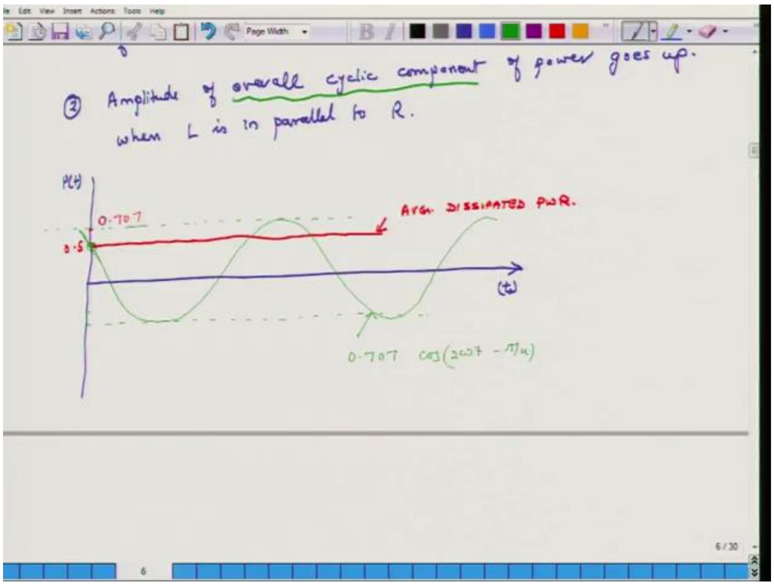

So, this is my time axis and this is my power and the first thing I am going to plot is the average dissipated power. So, average dissipated power let say this is a half so this does not change. So, this is average dissipated power; the second thing is that you have this cyclic portion, but it is limit is are 0.707 on the positive side and 0.707 on the negative side and at time is equal to t it is value is 0.707 cosine of minus pi or 4. So, cosine of pi or 4 is 1 over square root of 2. So, it will be half, but it will have a higher peak limit.

So, this I am going to plot it in green this entire thing. So, this is 0.5 and this is 0.707 that excuse me. So, this is. So, that is 0.707 and at time t is equal to 0 it is value will be here, but at an earlier time it will the negative time it will go up right and it will fluctuate between positive 0.707 and negative 0.707. So, it will fluctuate like this. So, this is 0.707 cosine of omega t minus pi over 4 to omega t times minus pi over 4. So, in the purely resistance circuit, the green dotted line and the sharp red line they where same at the same level and as the consequence when you add up these 2 curves, the curve would shift only on the positive side, but in this case that is not the case. The third important thing is that you have this R and L they are in parallel.

So, when there are 2 elements in parallel, the overall impedance of the system it goes down, when you have 2 elements in series overall impedance goes up because you have more resistance to the system, but here when you have 2 things in parallel then it is easier for current because now it has current has two more than 1 waves. So, the overall impedance goes down and when overall impedance goes down the amplitude of the current which flows through the system it also goes up and that is reflected in the fat that are value of I which is this is more. In the earlier case in a purely resistance system, the value of current was 0.1 here it has gone up. So, what; that means is when current is flowing through a system which has R as well as L this current is going to be having larger amplitude.

Now, in this example we assumed that these transmission wires had no resistance. So, when these transmission wires have no resistance the heat loss for transmitting a higher amount of current will be 0, but in reality we get electricity from a power station to our home and sometimes that power station can be sometimes kilo meters and in another case 100 or even more than thousand kilo meters away. So, if I have in my home just a resistive circuit, then the amount of current which will come through the transmission wires will be red less, if I have a circuit which is having R and L in parallel then more current is going to come in I will consume the same amount of power, which is half which I have calculated, but the current the heat loss due to transmission in the second case, where I have R and L in parallel it will be higher.

Now, it turns out that most of the equipment which we use in our homes, what are they? They are fans, air conditioners, refrigerators, coolers and all of these equipment they have some resistance and then there is a winding and that winding is an inductor. So, most of these products which we use in our homes, they are something like R and L in parallel. So, because of the very nature of these products they tend to draw more current and that causes a loss of heat or energy due to transmission. So, in that context there is a term defined as power correction factor. So, we will discuss what is power correction factor in the next class and then once we have finished that discussion then we will go to acoustical power.

So, with that I would like to close the discussion for today and we will meet once again tomorrow.

Thank you, bye.