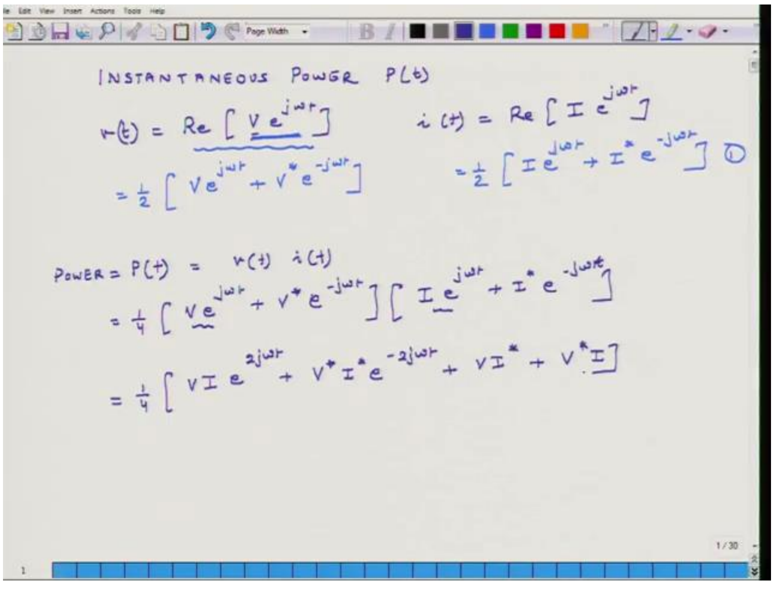
Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 25
Instantaneous Power

Hello, welcome to fundamentals of acoustics, this is the 5 th week of this course and you know this week, we will learnt essentially about two important concepts. The first concept will be related to acoustical power and to understand this concept we will first revisit the course of the concept of power in context of electrical engineering. So, we will understand how is electrical power measured and defined and then we will map that understanding to the acoustical world. And the second concept, we will learn about will be related to one dimensional waves but not in Cartesian frame of reference, but essentially in spherical coordinate system. So, that is what we plan to cover this week and let us start.

(Refer Slide Time: 01:18)



We will start by discussing the concept of power and instantaneous, so let us call this P as a function of t. So, in if we have to consider the beam of electricity, then there is voltage and this is our real signal for which we can measure to understood what is voltage. And this we can explicit as real of complex voltage signal, real of V e j omega t. So, please remember that, here V the capital V is a complex number, it is a complex amplitude and e j omega t omega is the angular frequency and t is time, similarly current which can vary with time, could be written as real of complex amplitude I e j omega t, that is our definition for current,

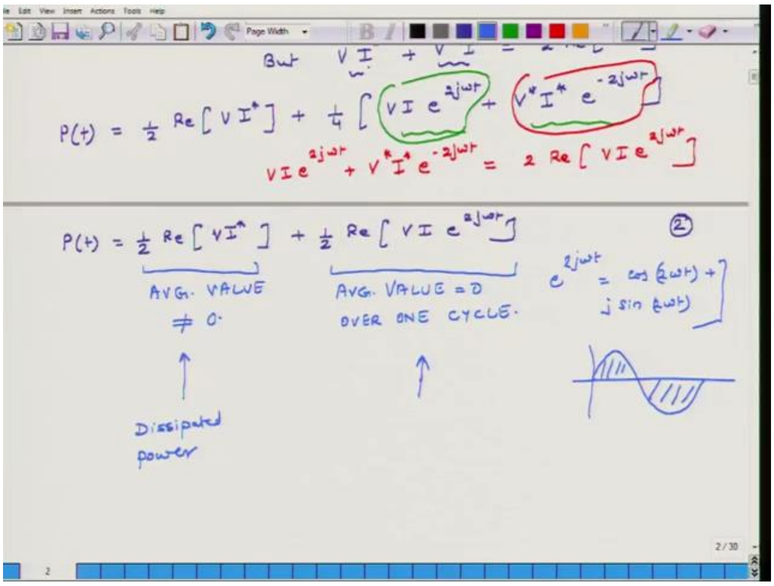
Now we know that this term can also be written as half of V e omega t and it is complex conjugate, which is V star and the complex conjugate of e j omega t is minus e j omega t, but with the negative sin. So, this term V e j omega t it is real component is nothing, but V e j omega t plus, it is complex conjugate which is V star times e minus j omega t. And similarly, I t which is current, I can also explicit as half of I, e j omega t plus it is complex conjugate. So, I star e minus j omega t.

So, let us call this equation 1 and then we define power. So, I define it as capital P and it can change with time. So, mathematically, it is equal to the product of voltage and current. So, what a current is flowing and as a function of time, if I multiply it by voltage of function of time, then it for each instance of time I get power. And if I use equation one in this expression, I get 1 4th of V exponent j omega t plus V star e minus j omega T, times I, e j omega t plus I minus j omega t.

So, I now expand it. So, I get 1 4th of. So, I multiply with this. So, I get V I e to the power of 2 j omega t and when I multiply the two complex conjugate entities I get V star I star e minus 2 j omega t and then, when I multiply V e j omega t with I star e minus j omega t, I get V I star and then I get a fourth term V star I.

(Refer Slide Time: 06:17)

.



So, now, we realize so, we can say that is my relation for complex power, but then we realize that V times I star plus V star times I, so is equal to so what is V times I star is the complex conjugate of V star times I? So, both these terms are complex conjugates of each other. So, when I add up 2 entities which are complex conjugates of each other essentially, what I will get is twice of real of V I star.

So, we plug this back into our relation for power. So, my relation for power becomes P t equals half of real of V I star plus 1 4th of V e 2 j omega t plus V star I star V minus 2 j omega t. Now we again compare this term and this term in the parenthesis and we note that this term is complex conjugate is the term and circled in red the complex number and it is complex conjugate I end up with the real portion of it. So, we can say that V I e to the power of 2 j omega t plus V star I star e to the power of minus 2 j omega t, equals twice of real of V I e 2J omega t. So, once again we plug this relation back into our relationship for power.

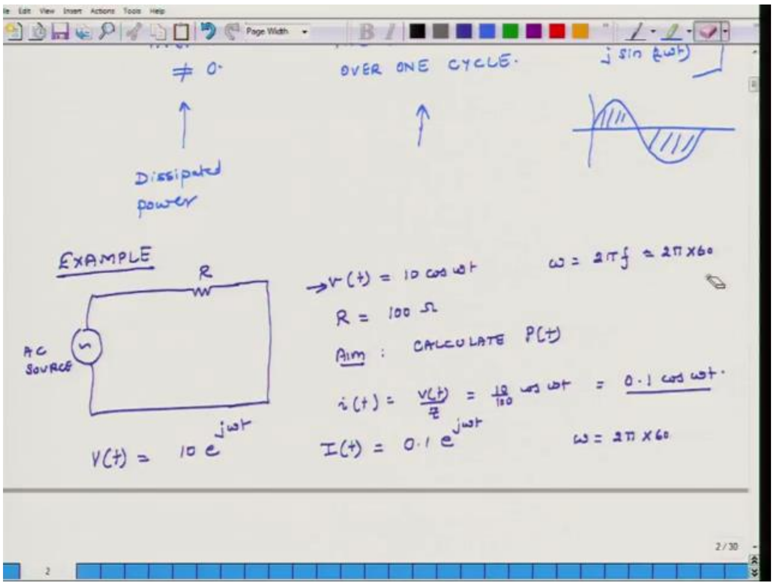
So, power equals half of real of V I star plus half of real of V I e to the power of 2 j omega t. So, we will call this equation number 2. So, that is equation 2, now we make some observations. So, you note that the average. So, here V is the constant it is the complex number, but it is a constant and I star is also a constant and if both of these are non zero then the first component this will not be equal to 0 right, if v is non zero if the magnitude of v is non zero and I star is non zero, then the first component which is half of real of v i star will not be equal to zero.

Now, let us look at the second component here, V is the constant number it may be complex i is the constant number. So, e to the power of 2 j omega t is equal to cosine of twice of omega t plus j sin of 2 omega t. So, if I plot the cosine term it is a cyclic function and same is the sin term and over one entire cycle, which means when omega t 2 twice of omega t is between 0 from 0 to 2 pi radiance over 1 entire cycle, the average of this e to the power of 2 j omega t will be 0.

So, for instance over 1 cycle cosine omega t is cosine omega t is something like this and if I take it is average this area will cancel out this area. So, average of this from the sin term is 0 and average of this cosine term is also 0 and V and I are constants. So, the overall average of this term over one cycle is 0. So, we can write it average value equal zero over one cycle and; however, in the first case average value is not equal to 0 over 1 cycle. So, this is our expression for power.

So, what this relation tells us is that the first term is not equal to 0 and it does not change with time. So, this is basically it is dissipated power and if the amplitude of voltage and amplitude of current does not change with time, when this dissipated power remains constant with respect to time, and the second component, the average value of this second component over as cycle is 0.

(Refer Slide Time: 13:03)

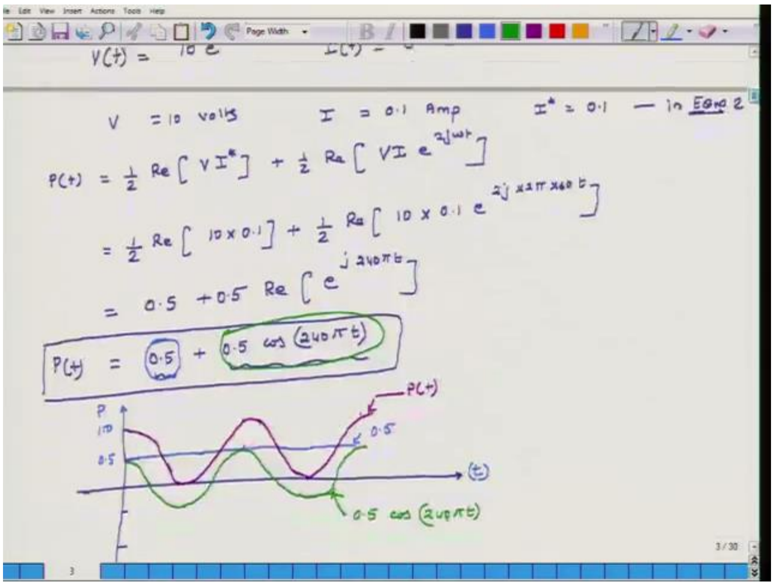


So, now let us do 1 example. So, we have an AC source and of the overall circuit of the system are very simple. So, it is AC source is connected to an external resister and we say that this AC source. So, this is an AC source of voltage is such that the value of voltage is equal to 10 cosine omega t, when omega equals 2 pi f and that equals 2 pi times 60 hertz. So, frequency is 60 hertz also R equals 100 ohms. So, our aim is calculate power as a function of time.

So, in the relation for power, we have to know V which is the amplitude of complex voltage, we have to know I star and we have to know omega. So, if I know V star and omega N I then I can calculate the relation for power. So, i t is equal to v t overall impedance Z and the overall impedance of the Z circuit is 100 ohms. So, this is 10 over 100 cosine omega t and that is equal to 0.1 cosine omega t.

So, complex voltage V t is equal to 10 e to the power of j omega t, with and if I take the real of this capital V t I will get this expression the real portion of; similarly complex current is equal to so the real current is 0.1 cosine omega t it is complex in the complex domain it will be 0.1 e to the power of j omega t, where omega is equal to 2 pi into 60. So, 60 is the frequency and the unit of omega will be radiance per second. So, 2 pi into 60.

(Refer Slide Time: 16:22)



So, this means that V the complex amplitude is equal to 10 volts and I complex is 0.1 amperes right and what else we need you know I star is again 0.1. So, we put all this in equation 1 or equation 2.

So, what is equation 2? P t equals half of real of V I star plus half of real of V I e 2 j omega t. So, this equals half of real of 10 into 0.1, plus half of real of 10 I is 0.1 exponents 2 j into 2 pi into 60 t. So, this equals 0.5 plus 0.5 real of exponent j times 240 pi t and that equals 0.5 plus 0.5 cosine 240 pi t.

So, what does that mean? That is my power and this is the dissipated portion and this is the cyclic portion of the power. So, what we are going to do is, we are going to plot it. So, the first thing I want to plot is the time in blue. So, my X axis is time and on the Y axis I have P which in watts. So, my average dissipated power ok the dissipated power is 0.5. So, this is 0.5, this is 1.0 and the cyclic portion is this thing and it is amplitude is half and it will be varying from plus half to minus half and at time t is equal to 0, it is value is half. So, it will vary like this, it is a cosine wave and the sum of both of these terms is going to look like this. So, that is my expression for p t and the green line this line is 0.5 cosine 240 pi t.

So, what we will do is, we will do one more example and then move on to the concept of acoustical power. So, the next example we are going to do is, this thing this circuit was purely resistive in nature this purely resistive in nature.

In the next example, we will develop an expression for power which has an inductive component and also resistive component and then we will compare that is else for this particular system which is purely resistive in nature and the system which as and inductive component also so but that particular example, we are going to do in the next class and that pretty much completes are discussion for today. So, I look forward to seeing you tomorrow.

Thank you and have a great day bye.