Fundamentals of Acoustics
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Lecture – 24
Transmission Line Equations - Part V

Hello, welcome to fundamentals of acoustics, today is the last day of the 4th week of this course and we will conclude this week is discussion with consideration of open tubes and we will develop relationships for pressure as well as velocity in these tubes.

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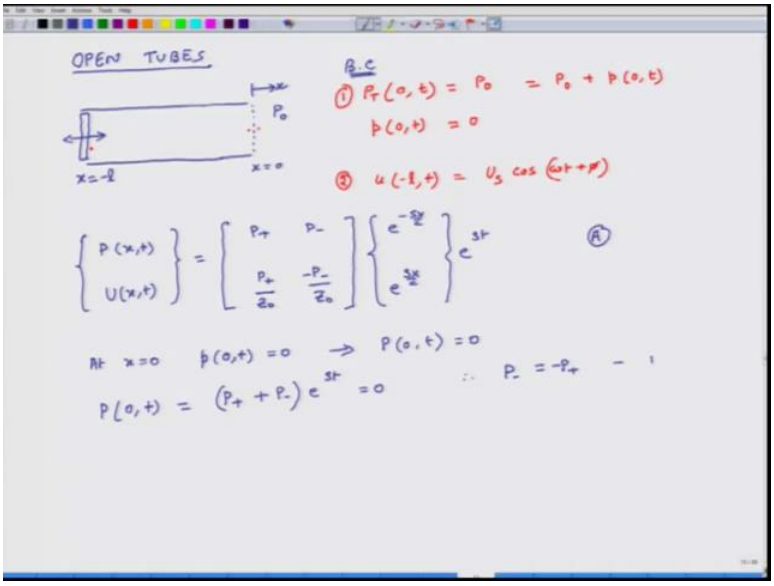

So, our theme is going to be open tubes. So, before we start discussing in detail let us look at the problem statement. So, I have a tube, which is open at one end to the atmosphere. So, the pressure here is P naught, outside the tube and the other end of the tube is having a piston, which is moving back and forth the coordinate system is such that this is my positive X direction and the open at the open end of the tube, the value of X is 0, the tube is l meters long.

So, at the other end of the tube, where I have a piston X is equal to minus l. So, the first thing is that we have to look at the boundary conditions. So, the first boundary condition is that at X is equal to 0 pressure, just outside the tube is P naught which is atmospheric pressure. So, what; that means, is that if there is a point just outside here the pressure is P naught, the same pressure is going to be at a point just inside the tube as well. So, pressure just outside the tube is going to be P naught, which means pressure just inside my tube is going to be P naught, as well which means that P T at X is equal to 0, at all times is equal to P naught, because P T is the total pressure, is equal to P naught.

But P T we had defined in such a way that it was equal to P to P naught, plus P of 0 and T, where 0 is the coordinate of the location. So, which means that the value of incremental pressure at X is equal to 0, is equal to 0. So, pressure at the open end of the tube is 0, the incremental pressure or the disturbencing pressure, this is my first boundary condition. The second boundary condition is that, in this case I define that there is a piston moving with a certain velocity and I know the velocity of the piston and whatever is the velocity of the piston is the velocity of the air particle adjacent to it. So, in this case velocity at minus l and at time t is known, because I know the velocity of the piston and that equals U S which is velocity of the source, times cosine omega t plus phi, where U S is the amplitude of the motion of piston and it is a real positive number.

So, these are the 2 boundary conditions and we will use these 2 boundary conditions to develop expressions for pressure and velocity in this open tube. So, how do we go around it, doing is we start with transmission line equations and for complex pressure and for complex velocity, these equations are P plus, P minus, P plus, P minus minus over Z naught, E minus s x over C, e s x over c, e s t. Now, we apply the first boundary condition. So, at x is equal to 0, P of 0 t is equal to 0, this means, so this also implies that complex pressure, when x is equal to 0, is equal to 0. So, that means, P 0 t is equal to P plus, plus P minus e to the power of s t is equal to 0. Therefore, P negative is equal to minus of P positive. So, this is relation. So, I call this relation A, I call this relation B.

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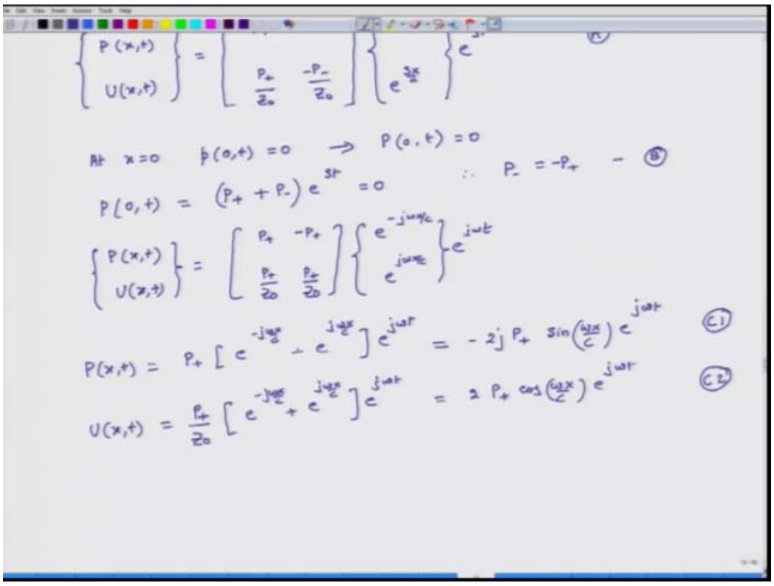

So, I know now P minus. So, I develop, I put this back in my transmission line equations. So, my transmission line equations look like P plus minus P plus, P plus over Z naught and P plus over Z naught, e minus s x over c, e s x over c, e s t. Now based on the fact that we have a linear system and this linear system is being excited, by angular frequency omega it implies that s is same as omega. So, what I will do is, I will erase this and I will re-level this as, j omega x over c j omega x over c, j omega t.

So, from these relations, we develop these relations further. So, complex pressure equals P plus, e minus j omega x over c, minus e j omega x over c, e j omega t. And this if I resolve it, expand e j omega x over c into x sin and cosine, what I get is minus 2 j P plus sin omega x over c, e j omega t so that is my complex pressure; and complex velocity is equal to P plus over Z naught, e minus j omega x over c, plus e j omega x over c, e j omega t and this equals 2 P plus cosine omega x over c, e j omega t. So, I call this relation C 1. And I call this C 2. And then at this stage, now that we have expressions for pressure complex pressure and complex velocity, we are ready to impose the second boundary condition.

So, our second boundary condition is that velocity at x is equal to minus l is equal to U S cosine omega t, plus phi what does that give us?

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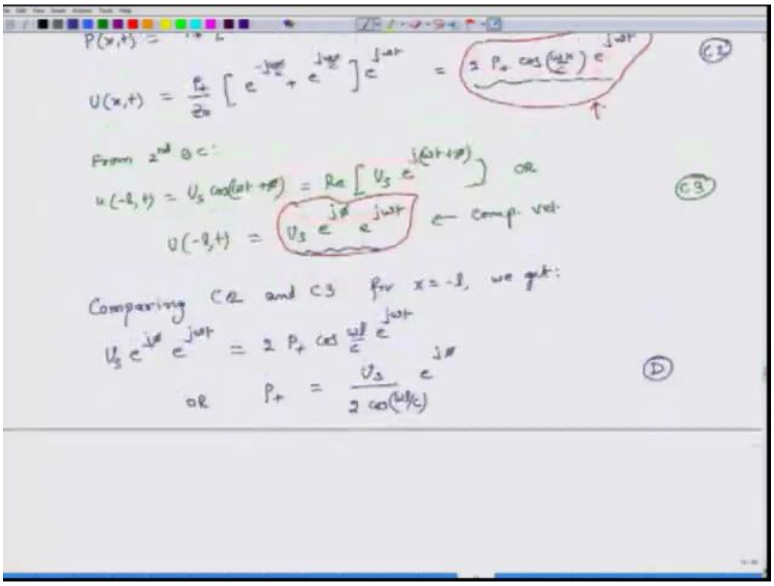

So from second boundary condition; second boundary condition what do we get? So, u at minus l, t equals U S cosine omega T plus phi, this I can rewrite it as, real of U S e j omega t plus phi or I can write it as or U at minus l, t is equal to U S e j phi, e j omega t, this is the relation for complex velocity. So, this is C 3.

So, now we compare C 2 and C3, we get. So, when we compare C 2 and C3, this term and they should be the same, this term and this term they should be the same, because this is when there should be same when x is equal to minus l.

So, I will make this a little more accurate comparing C 2 and C 3, for x is equal to minus l ,we get. So, what we get? U S e j phi e j omega t equals 2 P plus cosine omega l over c e j omega t or P plus equals U S over 2 cosine omega l over C, e j phi. So, this is equation D. So, now, I know P plus. So, I know P plus I know expression C 1 and C 2 which are relations for complex pressure and velocity. So, I can plug them back end to C 1 and C 2 equation D.

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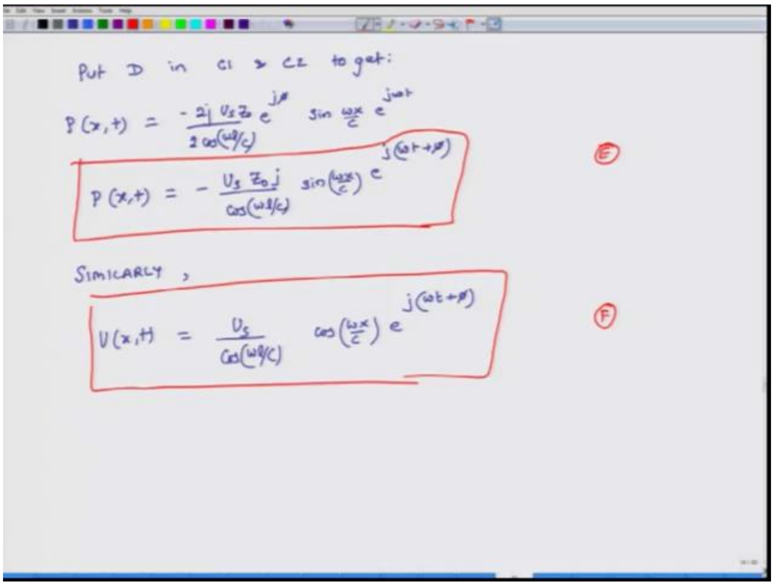

So, finally, what I get is, so put D in C 1 and C 2 to get complex pressure. So, complex pressure is minus 2 j P plus and instead of P plus I will write this expression hope. So, I think I missed parameter here and the parameter was that they should be a Z naught here.

So, if there is a Z naught here. So, there is a Z naught in the numerator and I miss the Z naught in the term. So, I am just accounting for that. So, this is U S Z naught e j phi divided by 2 cosine omega l plus c, l over c times sin omega x over c, which is. So, our complex pressure is equal to. So, 2 cancels out minus U S Z naught divided by cosine omega l over c sin omega x over c, e j omega t plus phi.

So, this is my expression for complex pressure and there is a j there, which is j there sorry and similarly. So, we can develop the relation for velocity similarly, U complex pressure, complex velocity U equals U S divided by cosine omega l over C cosine omega x over c, e j omega t plus phi.

So, you have not done the derivation for U complex velocity that by now we should be familiar with how we will get this. So, these are the expressions for complex pressure and complex velocity and the actual velocity real pressure.

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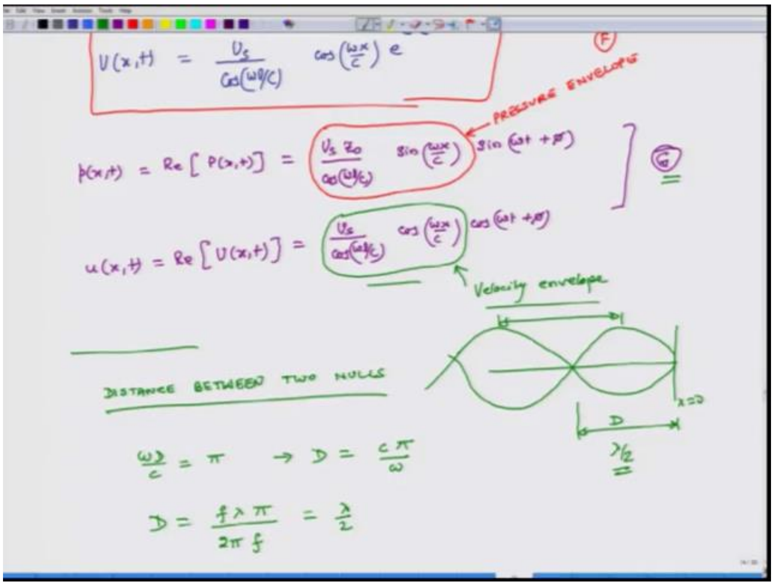

P at of x and t, is the real of complex pressure and if we use this equation E, then what we get is U S Z naught over cosine omega l over C times sin omega x by C times sin omega t plus phi and similarly, velocity actual velocity is real of complex velocity and this is equal to US divided by cosine omega l over C cosine omega c over C cosine omega t plus phi. So, these are the expressions. So, these are the expressions for pressure in a tube, which is open at one end and the consequence of that situation is that pressure incremental pressure small case P this is 0 and at the other end it is excited by a piston whose velocity is US cosine omega t plus phi.

So, once again, we see that the pressure relationship as a complex amplitude and I am in circling this not complex it has a pressure envelop, which is this thing and so is the case with velocity, velocity also as a velocity envelop and so this pressure envelop and a velocity envelop and we again see, that when the pressure envelops value is 0 at that location, the envelops value of the velocity envelop is maximum, because the pressure envelop as a sin term and the velocity envelop as a cosine term, the other important difference is that if we compare equation G with the results for our closed tube.

In the closed tube at x is equal to 0, pressure was maximum and velocity was minimum in this case pressure is minimum that is 0, at the open end of the tube and velocity is maximum at open end of the tube. So, this is the other important difference, but in besides that again these are standing waves we have already discussed standing waves earlier in context of close tubes. So, the same observations hold it just that wherever pressure is maximum in close tube at (Refer Time: 20:17) locations velocity is maximum in open tubes and vice versa.

The last thing I wanted to talk about is the distance between two nulls. So, what does two nulls mean that you have submerse, you have let say the pressure envelop. So, pressure is 0 at x is equal to 0 and it behaves like this.

So, what we are interested in finding out is what is this distance and let us call this distance as D. So, this distance will correspond to situation, where omega x over C is equal to pi radiance or so this value of x is D. So, I can write it as omega D over C is pi radiance or D equal's C time's pi, this distance divided by omega or D equals now C is frequency time's wavelength of the sound. So, C is f time's lambda time's pi and omega is two pi times f. So, this is equal to so f cancels out, pi cancels out lambda over 2. So, the distance between two nulls is lambda over 2, where lambda is the wavelength of the sound which is getting transmitted.

Similarly, the distance between two peaks is also the same, which is lambda over 2. So, at every half wavelength, you get either a null or maxima, the same thing is true for standing waves and close tubes, there also the value of D which is the distance between two nulls is half of lambda. So, with this discussion, I think we are done with discussion on transmission line equations, at least in context of open tubes and close tubes and infinitely long tubes and we will do one more case in the next week and till then have a great week end and we will meet each other on Monday.

Thank you bye.