Fundamentals of Acoustics
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Lecture – 23
Transmission Line Equations – Part IV

Hello. Welcome to Fundamentals of Acoustics. Today is the fifth day of the current week, and what we will discuss today is continuation of what we are discussing in the last class which is wave propagation in closed tubes.

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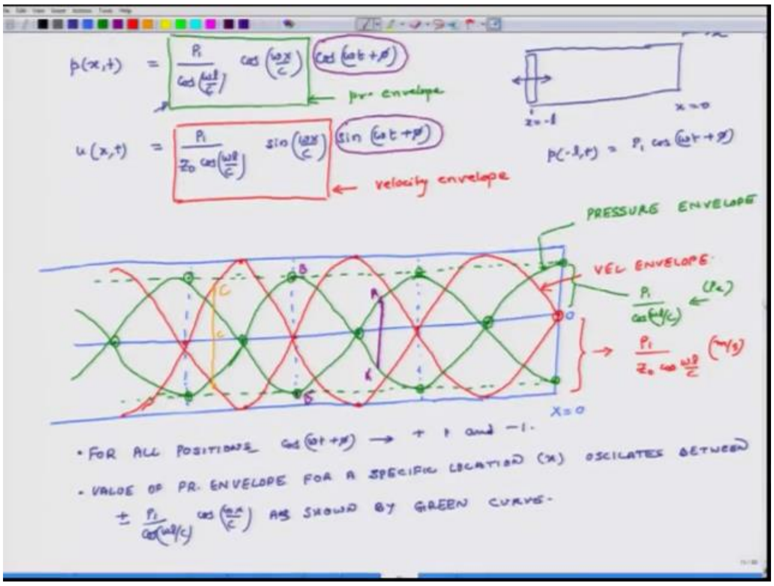

So, what we had shown through mathematics is that in a close tube the expression for pressure, so before we write the expression for pressure it is important to understand the topology of the tubes. So, x equals 0 at the closed end my positive x axis is in this direction the length of the tube is l. So, here at the other end x is equal to minus l and there is a piston here at x is equal to minus l and because of this pistonic motion P at minus l t equals P 1 cosine omega t plus some phase phi.

So, with this understanding we had developed expressions for pressure and velocity, and the expression for pressure was P 1 divided by cosine omega l over c cosine of omega x over c cosine of omega t plus phi. And velocity if function is defined as P 1 divided by Z naught cosine omega l over c sin of omega x over c sin of omega t plus phi. So, these are two equations which we have developed. Now what we plan to discuss today is the physical meaning of these two expressions.

So, the first thing I will like to say is that this term in green is called envelop for the pressure, so this is envelop pressure envelop. And why is it we will talk about it. Similarly, this term in red is velocity envelops; so we have a pressure envelop and we have a velocity envelop. And what we will do today is we are going to right now plot it. So, this is the location x is equal to 0. On the vertical axis this is 0 and I will make some graduation points with the hope that using these I will be able to draw into figure. So, first we are going to draw the pressure envelop, and what we see is that the value of this pressure the term in the green box is P 1 over cosine omega l over c at x is equal to 0; at x is equal to 0 its cosine omega l P 1 divided by cosine omega l over c. And the value of the pressure envelop for velocity is 0 at x is equal to 0.

So, first we will draw the velocity envelop. So, velocity envelop is 0 at this location and when omega x over c becomes 90 degrees or minus 90 degrees it becomes maximum, so it maximizes these locations. So, my curve is going to look like this. This is my velocity envelop, and this value is P 1 divided by Z naught cosine omega l over c. Now on the same graph I am also going to draw the pressure envelop. So, pressure is going to be maximum where velocity is going to be maximum, because pressure envelop has a cosine omega x over c term, and the velocity envelop has a sin omega x over c term.

So, the pressure envelop is going to be minimum here where velocity envelop maximizes and it is going to be maximum here. And let us say the maxima of this is; so the green dotted line is the downed within which is pressure envelop is going to fluctuate, so these are the points of its extreme. So, we will draw this. So, this may not be the greatest graph, but it is important that you get the idea. So, this green curve is the pressure envelop pressure envelop.

Now let us try to understand what these glass mean. So, let us just look at the green curve only for starters. So this term cosine omega t plus phi, now time is changing and time goes from 0 to whatever number it can go up to. So, the value of this cosine term will fluctuate anywhere between plus 1 and minus 1. So, for all positions for all positions cosine of omega t plus phi moves between plus 1 and minus 1, regardless of the position because of time is changing, so it can move between plus 1 and minus 1. But, the value of pressure envelop at a specific location x it oscillates between plus and minus of P 1 divided by cosine omega l over c times cosine omega x over c, and as shown by green curve.

So, this term green curve, this term in the box at a given location. So, for considering this location A at this location the pressure envelop fluctuates between these two position, these two values in this range; and cosine omega t at this location at point A its fluctuates between plus 1 and minus 1. And the overall pressure is the multiple of the term in the green box and the term in the purple circle. So, the overall pressure fluctuates between A and A prime corresponding to location A. Now look at another to curve point B, let us look at point B. Here the value in green box it fluctuates between its maximum n minimum possible value and cosine omega t fluctuates from plus 1 to minus 1, because time is just so it fluctuates. So, the overall pressure fluctuation and B is larger than the overall pressure fluctuation between A and A prime.

So, overall fluctuation of pressure at location B is larger than the overall fluctuation at location A. And at some locations this pressure fluctuation is as low as 0 at these nodes. And at other locations it is maximum. So, this is what we see for pressure. The same things we see for velocity envelop also. It just that, once again in this case the velocity is multiple of the velocity envelop times sin omega t plus phi, sin omega t plus phi oscillates between plus 1 and minus 1 regardless of the position right it depends only and time. So, as time changes it just fluctuate between plus 1 and minus 1,

But the velocity envelop, let us look at another some point; let us look at point C. So, corresponding two point c it fluctuates between these two limits this is point c. So, velocity envelop also fluctuates and the amount of fluctuation at these location is changing from one place to other place. So, this is very important to understand. So, the other thing is that this amount is P 1 divided by cosine omega l over c, this is the amplitude.

Now, I have drawn this graph in such a way that the vertical axis is having two different units, there is one system of units for pressure which is in pascals and another system units for velocity which is in meters per second. So, just the fact that visually it looks that the amplitude of pressure is more compared to that of velocity does not mean it is in reality more, it is that their just having different scales. So, that is important to understand. But the point is that the fluctuation of pressure is not the range of fluctuation of pressure is not same at all locations it is minima at some location it maximizes some location and intermediate at other locations. And the same is to for velocity also.

So, these types of velocity and pressure patterns are known as standing waves.

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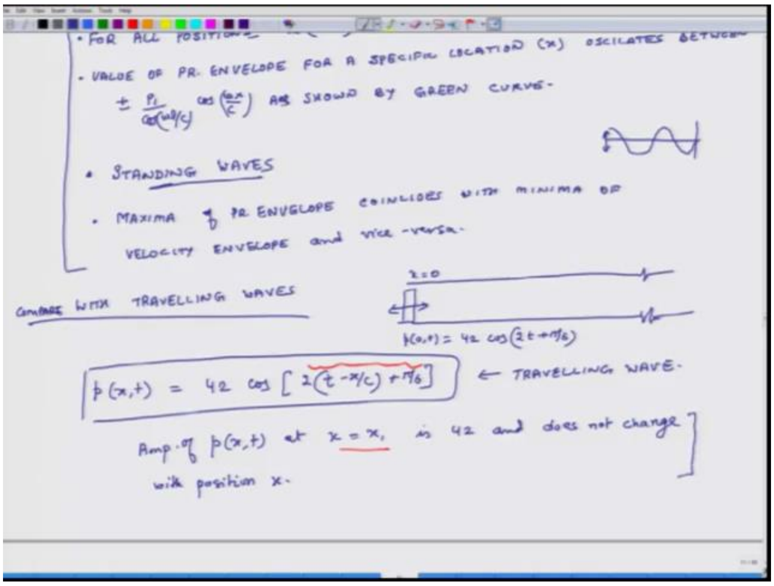

And the reason they are known as a standing waves is that when you look at these waves or if you take a somehow a snapshot, if suppose you have a rope for instance and your vibrating this rope up and down then the rope appears like this. So, the wave does not appear to travel in this case rather it appears that kind of a wave is standing and that perception is there, because there is interference of the forward travelling wave which we had seen and there is a reflected wave; the forward travelling wave and a reflected wave and both of these waves interfere in such a way that we get this type of a standing pattern. So, that is why these waves are known as Standing Waves.

The other things to notice that maxima of pressure envelop co insides with minima of velocity envelop and vice versa. So this is important from this, pressure of velocity envelop maxima of pressure envelop co insides with minima of velocity envelops wherever you have the maxima of pressure you will have minima of velocity. And the physical reason for this kind of phenomena is that at these locations where pressure is maximum velocity is minimum because the total amount of energy in the system is same.

So, wherever you have maximum of potential energy kinetic energy is going to be minimum and so on and so forth.

So, this is another important aspect to understand. Now the last thing I will like to do is contrast these types of standing waves with travelling waves. So, we will compare with travelling waves. So, in the last of the class before that we had, so what we are going to do here is we are going to relook at this tube which was infinitely long and it was exited at x is equal to 0 through a piston and such that the pressure here P had 0 t we had defined was 42 cosine 2 t plus pi over 6.

Now the solution for this was P of x t, we had calculated the solution we had develop the solution is equal to 42 cosine 2 t minus x by c plus pi over 6. Now this was a travelling wave. So, when we look at this the amplitude of P of x t at some location x is equal to x 1, what is the amplitude. So, it will be what it will be for the amplitude of 42 cosine 2 t minus x 1 over c plus pi over 6 right, whatever is the amplitude. And what is the amplitude of that? Now t is changing from 0 to infinity and for some t this entire term in the bracket will either be 0 or pi 2 pi or something.

So, regardless of the position of x the amplitude of the wave is same and it remains same which will be 42. So, amplitude of P x t at x is equal to x 1 which is any location in the tube is 42 and it does not change with position x. So, this is the very fundamental difference. So, in travelling ways the amplitude is constant, as exchanges in standing waves the amplitude of the vibration or of the oscillation it changes from point to point. So, this is the other thing I wanted to cover.

I will now talk about couple of more things. So now that we have discussed this I will introduce 2-3 new terms and close the discussion for today. So, what we have seen is that we have developed expressions for pressure and we have developed expressions for velocity.

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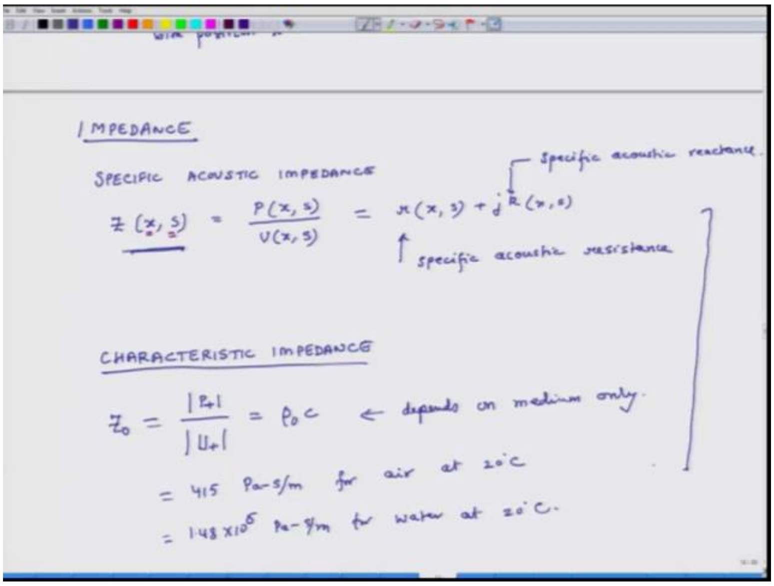

So, now I define a new term its impedance, and their several types of impedance. The first one is, specific acoustic impedance and I defined it as Z and please note it changes with position and it changes with frequency. And how do I define it? I call it as the ratio of complex amplitude of pressure wave and complex amplitude of velocity wave. So, please note this is the ratio of complex amplitudes it is not the real part. So, what is the complex amplitude of pressure wave for the standing wave which we had discussed?

So, let us look at these. So, this is the relation for complex pressure. So, to find this ratio we have to look at complex pressure relation and complex velocity relation. This term is the complex amplitude for the pressure wave time is out. And actually in this phi should also be in it. And this term is the complex amplitude for velocity. So, when e j c or should be inside this thing, but when I take the ratios e j phi goes away I get ultimately is specific acoustic impedance. So, this specific acoustic impedance is a function of x which is position. So, it changes from one point to other and it is also function of frequency or complex frequency.

So, this will be a complex function and I can also express this complex function as a function with real part, so I call it r which depends on x and s, and function within which has a imaginary part which also depends on x and s. So, this x is known as specific acoustic reactance and k is another function and it is specific acoustic reactance. So, that is my specific acoustic impedance. So this Z what does it depend on, it depend on the entire system, the boundary conditions, the position, the frequency, all of those things. But then there is another impedance which we had discussed earlier and it is known as Characteristic Impedance.

And this is equal to Z naught and this is defined as the ratio of P plus divided by the ratio of U plus their complex their magnitudes. So, the numerator is a number the denominator is a pure number and when we do the calculation it works out to be rho naught c. So, this does not depend on the system, it only depends on medium. And its value is equal to 415 pascals second per meter for air at 20 degree C and it is equal to 1.48 into 10 to the power of 5 - excuse me 6 pascals second per meter for water at 20 degree C.

So, it is important to understand these two impedances is specific acoustic impedance which changes from one place to other and it depends on the overall system. And characteristic impedance which is the ratio of complex pressure amplitude and complex velocity amplitude and that just rho naught c.

With this we conclude the discussion for today, and we will meet tomorrow. And in tomorrow’s lecture we will develop the relation for open tubes.

Thank you and we will meet tomorrow. Bye.