Fundamentals of Acoustics
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Lecture - 22
Transmission Line Equations - Part III

Hello, welcome again to Fundamentals of Acoustics course. Today is the fourth day of the current week and over this week we have been discussing transmission line equations and what we have done till so far is develop transmission line equations for pressure velocity, we have also develop the relationship which connects P plus with u plus and P minus with u minus and using these relationships we have developed the overall transmission line equations and we have also solved an example for a pressure wave travelling in an infinitely long tube. So, what we will do today is continue this discussion and we will actually use these equations to solve 2 different examples the first one corresponds to a close tube. So, for that transmitted in a close tube we would like to explore how pressure wave and velocity wave transvel get transmits in a close tube. So, waves in closed tubes. So, how does the tube look?

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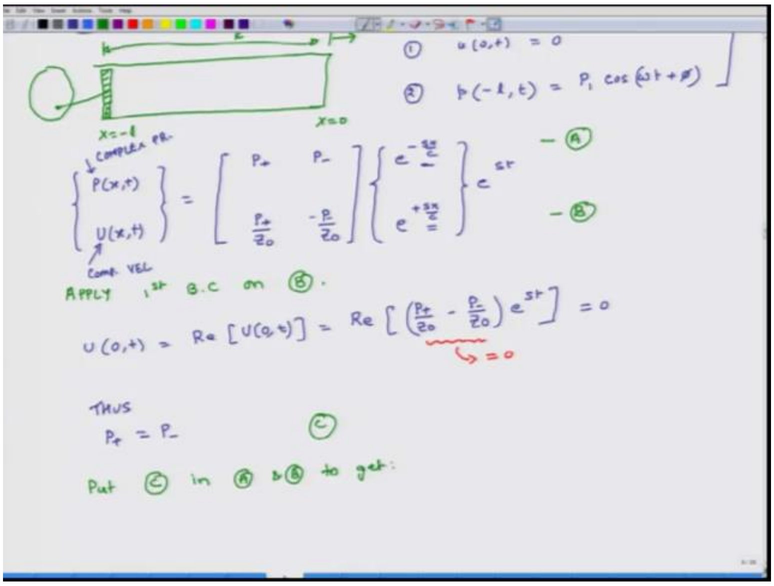

It is closed at one end and at the other end I have a piston which moves back and forth because of some rotary mechanism and the coordinates are x is equal to 0 at the closed end and x is equal to minus l at the l wave I have a piston at motion. So, my coordinate system is positive x in the in this particular direction.

Now, we had said that the transmission line equations there are 2 such equations in number, but there are four different unknowns. So, we have to 2 different 2 additional conditions and once we identify those 2 additional conditions we can use them and the 2 transmission line equations to develop expressions for pressure field in the tube and velocity field in the tube. So, what are these boundary conditions? So, boundary conditions, the first boundary condition is that u is 0. So, at x is equal to 0 at all times velocity is 0 why is it because it is a close boundary no air can go out or no air can come in. So, air is stationary at the boundary. So, velocity is 0 at this end.

The second condition is and this is based on our measurement or we are told this boundary condition is given that at x is equal to minus l the value of pressure. So, at x is equal to minus l and at time t the value of pressure is P 1 where P 1 is a real number cosine omega t plus phi, these are the 2 boundary conditions. So, with these conditions and also the 2 transmission line equations we can solve for the pressure and velocity field in the tube. So, once again the length of the tube is l. So, how do we go around it we start with transmission line equations and we know that complex pressure and complex velocity is equal to P plus P minus P plus over Z naught minus P minus over Z naught e to the power of minus s x over c e to the power of positive s x over c times e to the power of s t. So, let us call this equation A and let us call this equation B.

Now we have to solve for P plus if we know P plus P minus and s then we can calculate the value of complex pressures p and u and if we take the real component we will get the actual pressures. So, our aim is to find P plus P minus and S. So, now, we apply the first boundary condition. So, the first boundary condition says that u at x is equal to 0 is 0. So, which means we apply it on equation B. So, what we get. So, u at 0 t is equal to real of u at 0 t is equal to real of P plus over Z naught minus P minus over Z naught. So, what I am doing is putting x in this equation as 0. So, these terms become 1 and then time's e to the power of s t. So, this is complex pressure and this capital U is complex velocity and if I have to find actual pressure or velocity I take its real portion. So, this is what I had developed in the last class also. So, what we are using is form F 3 capital P capital U is equal to P plus P minus P plus over Z naught and so on and so forth, but if I am interested in finding the actual pressure then I take the real of this entire thing.

This entire thing from equation from the first boundary condition at x is equal to 0, we know that it is equal to 0 and this is going to happen if and only if this term equals 0. So, which means thus P plus equals P minus, let us call this equation C. So, what we do is put C in A and B to get we put this condition and see in the first 2 transmission line equations.

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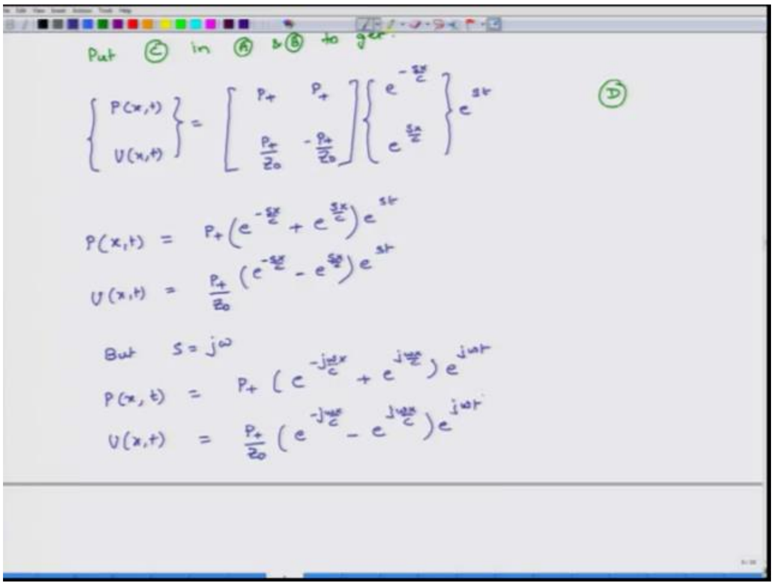

And we get complex pressure, it depends on x and t, complex velocity is depends on x and t and this equals. So, P minus is same as P plus. So, let us call this equation D. So, now, we have eliminated 1 variable which is P plus or which is p negative, but we still have to figure out P plus and S and that we will get once we implement the second boundary condition.

But before we do that we will simplify equations D. So, we expand from here, we get p of x and t equals P plus e minus s x over c, plus e s x over c, e to the power of s t and U of x t which is complex pressure is equal to P plus e minus s x over c and here minus e to the power of s x over c, e to the power of s t and here we have a Z naught in the denominator. Now we make an observation that at the source in the excitation has a angular frequency of omega and because this is a linear system, it implies that at all positions in the system by steady state solution will be such that the angular frequency of the wave at all positions will be same as omega. So, with that understanding what we can say is that s is equal to j omega. So, once we do that substitution we get. So, we know that s is equal to j omega. So, thus what we get is P of x and t and U of x and t equals P plus e minus j omega over c x plus, e j omega x over c e j omega t and velocity is P plus over Z naught e minus j omega x over c minus e j omega x over c, e j omega t.

Now, this term in the parentheses if I expand it in terms of sins and cosines, the sin term is going to cancel away and what we will get is 2 cosine omega x over c and from here what we will get is if I add these 2 contributions from each of these terms what we will see is that the cosine term will go away and what we will be left with this. So, what we will be left will be 2 j sin omega x over c and there will be negative sign out here.

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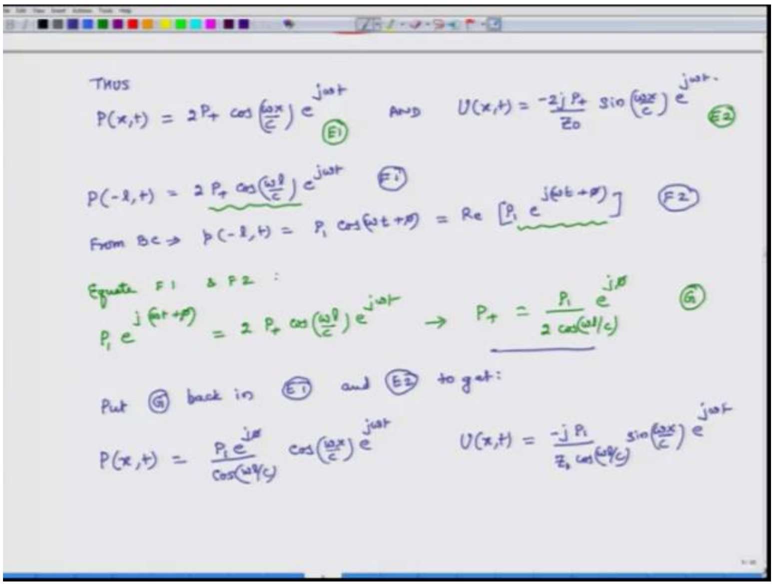

We make these simplifications. So, we can state thus P of x and t which is the complex pressure is equal to P plus and this is 2 cosine omega x over c, e j omega t and complex velocity is equal to P plus over Z naught and there is a. So, I will need some space, minus 2 j P plus over Z naught, sin omega x over, c e j omega t. So, these are 2 relations for one is complex pressure, another one is for complex velocity and if I take the real components I will get the actual pressures and velocities, but in both these expressions we do not know P plus. So, we have to still figure P plus and I had said that P plus is going to come, if we use this condition that pressure at x is equal to minus l is P 1 cos omega t. So, we use that condition. So, before we do that we will give these numbers. So, let us call this E 1 and let us call this equation E 2.

The value of pressure complex pressure at minus l t is equal to 2 P plus and I substitute x equals minus l. So, I get cosine omega l over c times e j omega t. So, I call this F 1 and fro our boundary condition from B C, we know that the actual pressure P minus l t equals a real number P 1 cosine omega t plus phi and this I can re express it as real of P 1 e j omega t plus phi. So, this is F 2. So, what we do is we equate F 1 and F 2. So, how do we equate I mean this is a complex entity and this is a complex entity. So, we equate these 2 terms. So, we get P 1 e j omega t plus phi equals 2 P plus cosine omega l over c e j omega t. So, e j omega t goes away from here and what we are left with is P plus equals P 1 divided by 2 cosine omega l over c times e j phi, so this is equation G.

Now we have calculated the value of P plus which is a complex entity. So, this is P 1 is real, but e to the power of j phi is complex and the denominator is 2 cosine omega l over c. So, now, what I do is I put G back in E 1 and E 2 to get. So, I can get my complex pressures. So, complex pressure at location x and time t is equal to P 1 e j phi over cosine omega l over c, cosine omega x over c e j omega t and complex velocity of the particle is minus j P 1 by Z naught cosine omega l over c sin omega x over c, e j omega t.

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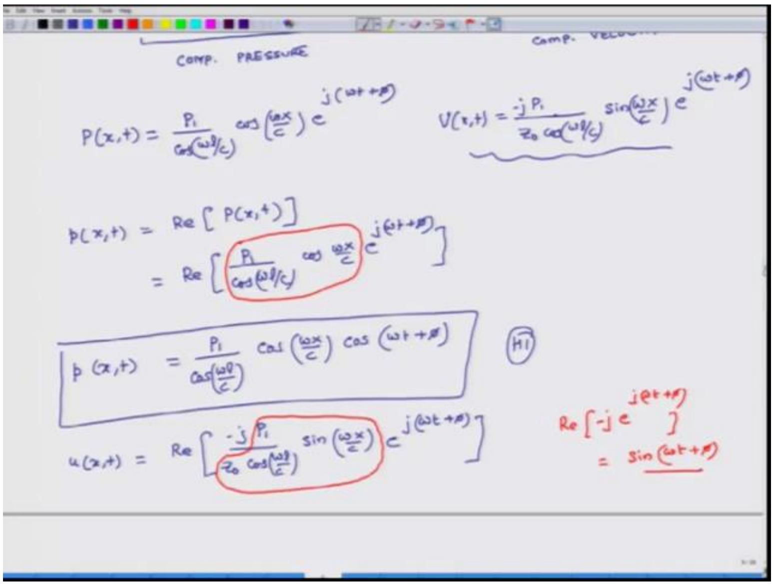

So, this is complex pressure and this is complex velocity and I can further simplify this somewhat by writing complex pressure equals P 1 divided by cosine omega l over c cosine omega x over c, e j times omega t plus phi and my complex velocity is U of x t is equal to P 1 minus j over also I missed e j phi here, Z naught cosine omega l over c sin omega x over c e j omega t plus phi.

Then my pressure is real of p x t and this is equal to real of P 1 over cosine omega l over c cosine omega x over c e j omega t plus phi now in this entire expression whatever I am encircling in red everything is real. So, this can just come out of the bracket right away P 1 is real because we have defined it as such, we have said that what is P 1 P 1 is this entity and we are saying that we know it and its are real number. So, P 1 is real omega is real l is real c is real x is real. So, everything here in the red is real only thing complex is e j to the power of omega t plus phi. So, this red part can come out and the real component of e to the power of j omega t plus phi is cosine of omega t plus phi. So I can finally, write as p of x and t is equal to P 1 divided by cosine omega l over c cosine omega x over c, cosine omega t plus phi; so this is 1 equation and what this equation tells is that how is pressure changing in this close tube and I call it equation H 1.

The next thing is that we figure out what is velocity. So, u of x and t is equal to real of minus j P 1 divided by Z naught cosine omega l over c sin omega x over c. So, where are we getting from? We are getting it from this relation e to the power of j. So, here what is real this term in the red is real and real of minus j e to the power of j omega t plus phi is equal to sin of omega t plus phi because e to the power of j omega t plus phi is cosine omega t plus phi plus j times sin of omega t plus phi and when I multiply all that by minus j, this is the term which becomes real and the other term is becomes imaginary.

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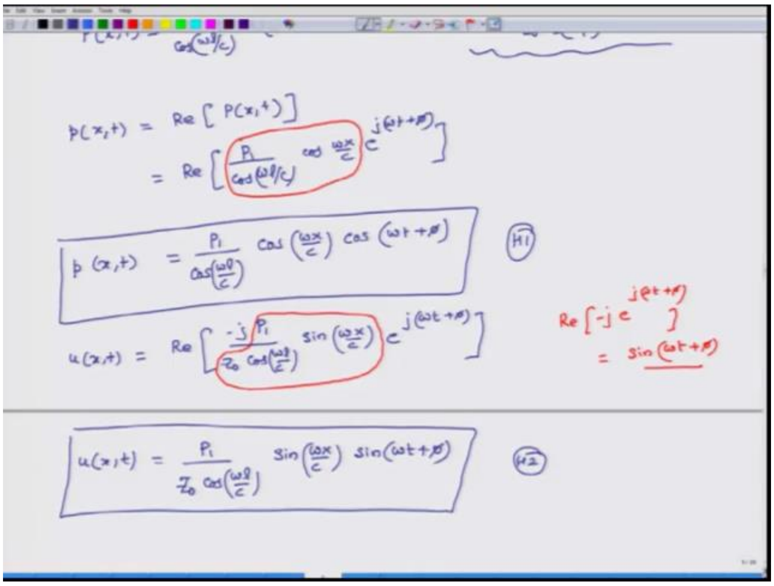

Actual velocity is equal to P 1 divided by Z naught cosine omega l over c, sin omega x over c, sin omega t plus phi. So, that is my second relation and this relation tells us how velocity is changing in a closed tube and that tube which is excited at one end by a piston where the pressure at the beginning is defined by this function. So, H 1 and H 2 give us expressions for pressure in the tube and velocity in the tube and in these expressions everything is known, P 1 is known, omega is known, l is known, c is known and if we know the value of x and t we can calculate at that location, pressure and velocity at all times and all for all positions.

This concludes our discussion for today. We will continue the discussion on closed tubes tomorrow as well and specifically we will discuss standing waves in context of closed tubes. So, that is all for today and we will meet once again tomorrow.

Thank you, bye.