Fundamentals of Acoustics
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Lecture – 21
Transmission Line Equations – Part II

Hello, Welcome to Fundamentals of Acoustics. Today is the third day of forth week of this course, starting this week what we have been discussing is information about transmission line equations as they relate to one dimensional propagation of Acoustic waves and in last 2 lectures what we have covered is the transmission line equation related to pressure and then we have used that equations to solve for the pressure field in a infinitely long one dimensional tube, which is excited by piston moving at its initial point; now in waves you have not only pressure, but there is also a velocity wave equations one dimensional velocity wave equation.

The transmission line equation for velocity and the transmission line equation for pressure, they are somehow interconnected and what we will do today is to develop the relationship between the pressure wave function and the velocity wave function using some pressure velocity relationship. So, our aim is to develop the pressure velocity relations.

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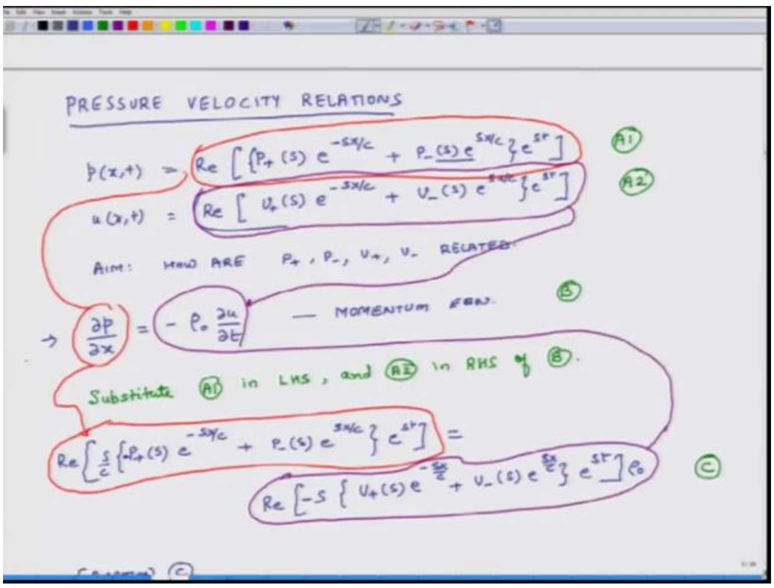

So, pressure velocity relations.

Now, in the last class or the class before that we had developed this relation p of x and t equals real of P plus of S, e minus s x over c plus p negative of s e s x over c and this entire thing is in parenthesis multiplied by e to the power of s t. So, that is my first equation and that corresponds to the transmission line equation for pressure.

Now, if I consider the velocity wave equation then I can develop a similar expression for velocity. So, u of x and t is equal to real of complex amplitude of the velocity wave moving in the positive x direction. So, I will call it capital U plus s e minus s x over c plus U minus s e s x over c multiplied by e to the power of s t. So, let us call this equation A 1 and call this A 2. So, I am getting equation A 1 from the pressure wave equation one dimensional equation for pressure waves and I am getting equation e 2 from 1 dimensional equation of velocity wave now our aim is to find out how are P plus P minus U plus U minus related this is what we want to figure out.

To figure that out, where we will go back to the momentum equation and the momentum equation says partial of pressure with respect to x equals negative of density at STP conditions times partial of u with respect to time. So, that is my momentum equation. So, I call this equation B. So, our aim is to figure out the inter relationship between U plus U minus P minus P plus and to figure that relationship we will use the momentum equation. So, what we do is we substitute A 1 in LHS and A 2 in RHS of B of equation B. So, when we do that we get the following relation real of so, I have to differentiate equation A 1 with respect to x in the partial way. So, I get P plus of s and then when I differentiate e to the power of minus s x over c, I get s over c and I will take this s over c outside the parenthesis. So, I get minus s x over c or actually let us keep this negative sign inside. So, this should have been minus P plus times s e to the power of minus s x over c plus p negative which depends on s times e to the power of s x over c. So, this entire thing is in parenthesis. So, I have differentiated the equation one with respect to x and this entire thing is multiplied by e to the power of s. So, that is the LHS of equation B and this equals real of s U plus which depends on s e to the power of minus s x over c plus U minus s e to the power of s x e to the power of s t because what I am doing here is I am differentiating equation a 2 with respect to time.

I am differentiating equation A 1 with respect to x which is this thing. So, I am connecting this with this and I am putting it here. So, these things are connected and similarly the left side which is in purple is connected to this negative and I have to multiply this by density rho naught and there is a negative sign here. So, I have to have a negative sign their and this purple sign this circle is coming from this equation.

Let us call this equation C, now equation C is basically are statement of momentum equation which is this equation and what is momentum equation it is essentially a modified form or a particular version of Newton's second law and Newton's second law is valid for at all points of time and at all positions. So, what; that means, is that equation C has to be valid at all values of x and it also has to be true for all values of time and that is going to be possible only if this component equals the component on the right side related to minus s x over c and this component times s over c is equal to the u negative term.

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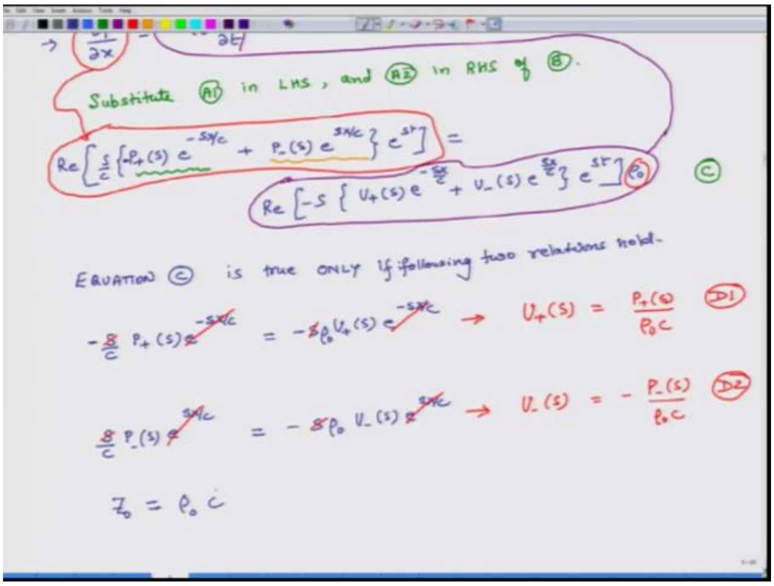

We write these equivalencies. So, equation C is true only if following 2 relations hold. So, what are those 2 relations? So, first relation relates to P plus and P plus is multiplied by s over c. So, s over c P plus s e to the power of minus s x over c equals minus s U plus of S e to the power minus s x over c. So, these 2 raise cancel out and of course, there has to be a density term rho naught it is the density term. So, what we get is eventually U plus of S is equal to. So, s also cancels out and what else and this is there was negative sign I missed here. So, these negative signs also go away. So, U plus of S is equal to P plus of s divided by rho naught c.

This is D 1. So, this is the first relation which has to hold to and the second relation which has to hold to is s over c times p negative, times e to the power of s x over c equals negative s rho naught U minus s e s x over c. So, once again s x over c gets canceled out. So, does S. So, what that gives me is U minus of s which is the complex amplitude of the wave traveling in the reflected direction is equal to negative of P minus of s by rho naught c, D 2. Now what we do at this stage is we say that let Z naught some constant Z naught or Z naught and we define it such that it is rho naught times C.

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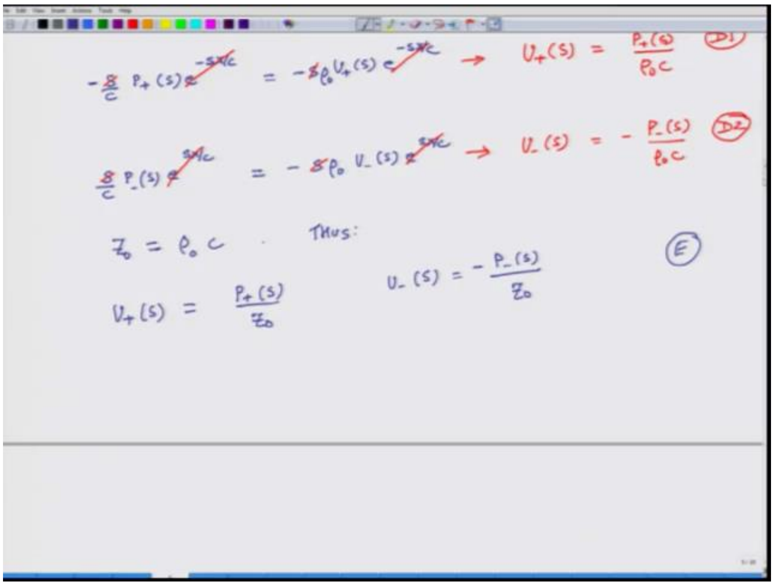

In that case, if we look at equations D 1 and D 2 from D 2 and D 2 what we get is that U plus of S equals P plus of s by Z naught and U minus of s equals P negative of s times Z naught and there is a negative sign here.

With this under this is equation E. So, now, what I do is I combine equation E and our first equation our equations for A 1 and A 2.

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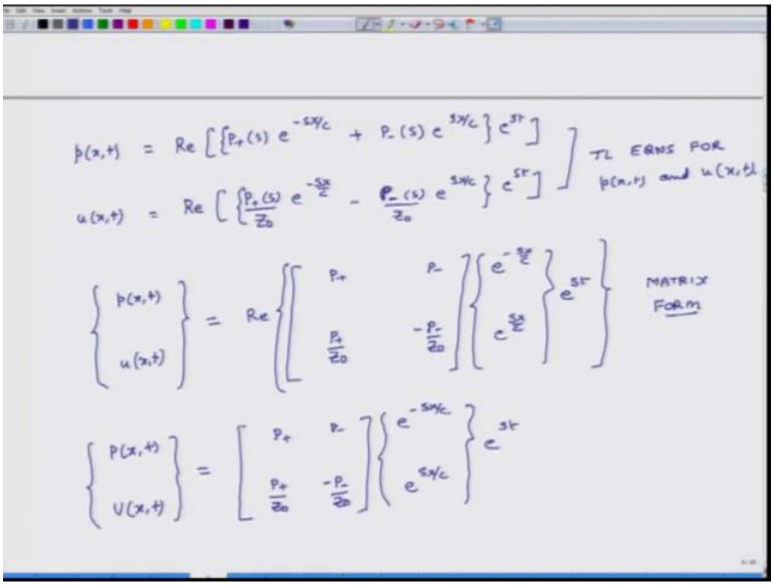

Once I do that I get the equation for pressure and velocity that p of x and t equals real of P plus of s e minus s x over c plus P minus of s e s x over c, e s t and u of x t I can express as real of U plus is nothing, but p positive divided by Z naught and then u negative is minus of p negative is depends on s divided by Z naught times e s x over c and this entire thing is multiplied by e to the power of s t.

These are transmission line equations for pressure and velocity. I can also write this in matrix form as p of x t and u of x t this is equal to real of P plus I am going to omit the s term in bracket just for purposes of gravity p negative P plus over Z naught and negative P minus over Z naught. So, this 2 by 2 matrix is multiplied by this vector e minus s x over c and e s x over c and this entire thing is multiplied by common term e t o the power of s t. So, and this entire thing is closed in early bases. So, this is the matrix form.

And if we do not want to use the term real then the left side can be expressed as complex pressure. So, complex pressure is capital P of x and t and complex velocity is this in. So, this is equal to P plus P minus P plus over Z naught minus P minus over Z naught e minus s x over c e s x over c times e to the power of s t. So, this is third form, but essentially all of these forms defect the same reality.

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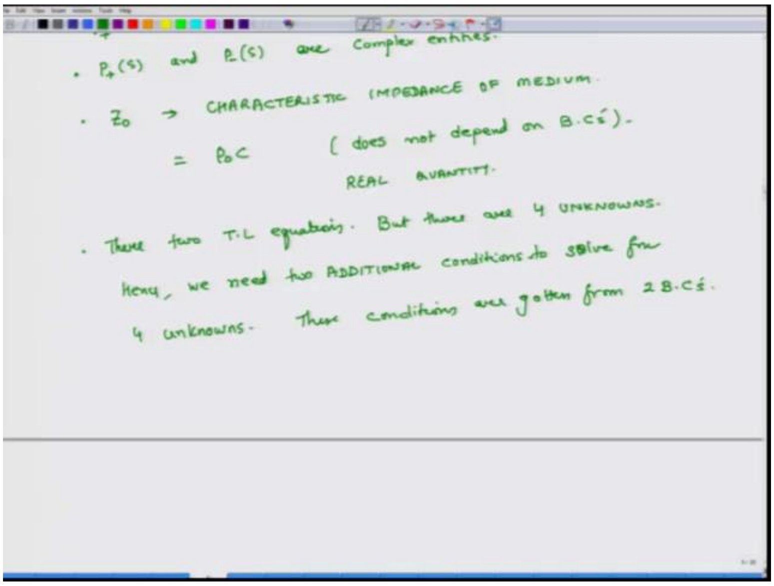

Now, some comments will look at some important comments, first thing P plus s and P minus s are dependent on complex frequency which is s. So, they can change with complex frequency, for each frequency they have to be evaluated.

Second thing is P plus of s and P minus of s are complex entities they complex entities in general they can be real, but there is no reason why they have to be necessarily real third thing Z naught is called characteristic impedance of medium of medium and it is equal to rho naught C. So, what is rho naught rho naught is the density of the medium and C is the velocity of sound and the medium. So, this Z naught or Z naught it does not depend on the boundary conditions it only depends on the medium if we change the medium then Z naught changes, but the boundary conditions do not influence the value of Z naught. So, it does not depend on boundary conditions or the length of the tube or whatever it only depends on the nature of the medium and the other thing is that it is a real quantity it is not a complex number because rho naught is real and c is also real. So, it is a real quantity, the forth thing is that this equation. So, let us call this equation F 1 F 2 F 3, so there are 2 transmission line equations, but there are 4 unknowns what are the 4 unknowns when we look at the transmission line equation what are unknowns p of x and t is unknown u of x and t is unknown P plus is unknown p negative is unknown.

Z naught is known because we know the density and the velocity of sound also x is known. So, everything is known, but other parameters are known, but these four are unknown parameters. So, what; that means, is hence we need to additional conditions hence we need to additional conditions to solve for four unknowns. So, these conditions are gotten from 2 boundary conditions. So, if we have a closed tube or an open tube or a tube of finite length it as 2 boundaries and you should know the conditions at both of these boundaries then you can solve for this four unknowns otherwise you cannot solve for this four unknowns. So, that concludes our discussion for transmission line equations what we have been able to do today is we have developed the relationship between pressure and velocity for these 2 transmission line equation and using that relationship we have expressed transmission line equation in 3 forms F 1 F 2 and F 3 and then what we also see is that the transmission line equation by themselves will not be able to solve help you solve for the actual value of pressure and velocity at all the points in a one dimensional space for that we need to know 2 extra conditions and those 2 extra conditions typically come from the boundary conditions has applicable on a case to case bases.

With that we conclude our discussion for today and we will meet once again tomorrow and we will continue this discussion on transmission line equations tomorrow as well.

Thank you, have a good night, bye.