Fundamentals of Acoustics
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Lecture - 20
Transmission Line Equations - Part I

Hello. Welcome to Fundamentals of Acoustics. Today is the second day of the fourth week of this course, and what we will do today is a continuation of our discussion which we had yesterday. So, yesterday what we had been able to accomplish is that we were able to develop a transmission line equation for pressure wave. So, what this essentially equation tells us is how pressure wave travels in a long one dimensional pipe or a field in one dimensional field. So, now what we will do is we will actually use this equation to solve one particular problem.

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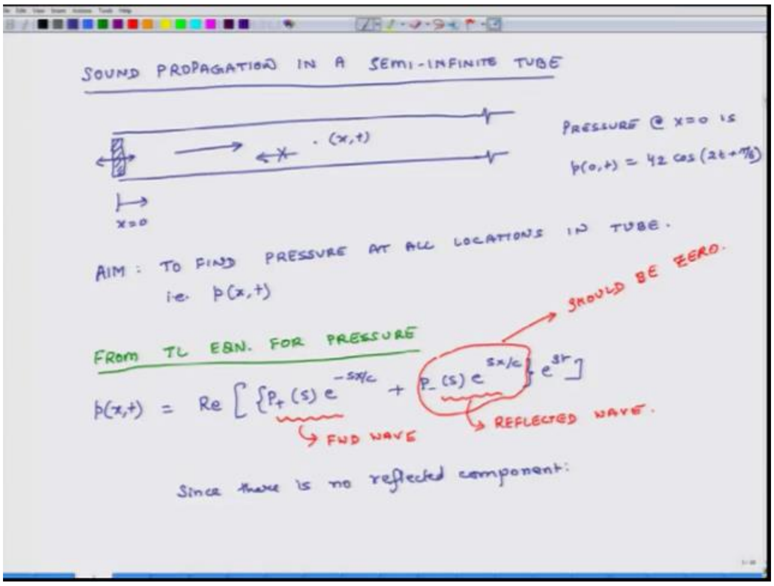

First we will write down the; so what we are going to solve is Sound Propagation in a semi-infinite tube. So, how does it look like? So, consider a long tube. So, this tube goes on forever it has no end point, but it has a start point and at its beginning we have a piston, and this piston is such that it seals the tube. So air cannot flow from this gap, this gap I just shown for illustration purpose. So, if the piston seals the tube and this piston is moving back and forth and are coordinate system is such that x is equal to 0 at the initial point of the tube. Now what we are told is that pressure at x is equal to 0 is p 0 t and we are told that it is a given number and it is something like 42 cosine 2 t plus pi by 6.

The pressure at the beginning of the tube is 42 cosine 2 t plus pi by 6 and our aim is to find pressure at all locations in tube. So, we have to find the value of pressure at any general point which is x distance away from the initial position, from the beginning of the tube and at any time. So, we are interested in finding at this find and at any general time the pressure. So, that is we have to find what is the function for p x and t. Now so what we do? We start with the transmission line equation. So, I will abbreviate transmission line STL. So, we say that from transmission line equation for pressure, so we will write down that relation p of x and t is equal to real of P plus s e minus sx over c plus P minus s e sx over c e to the power of st. So, this is my transmission line equation.

Now, we have explained earlier that this term corresponds to a forward travelling wave and this term corresponds to reflected wave. So, P plus s times e minus sx over c corresponds to forward travelling wave and then P minus s e to the power of sx over c corresponds to a wave moving in the negative x direction. Now, when we look at the physics of the problem, we see that there is no reason why; so there is reason why wave could travel in the positive x direction, but there is no reason why wave should get reflected because the tube is infinitely long.

So, if the wave has no reflected component then this term should be 0. And it should be 0 at all times and at all locations of x which means.

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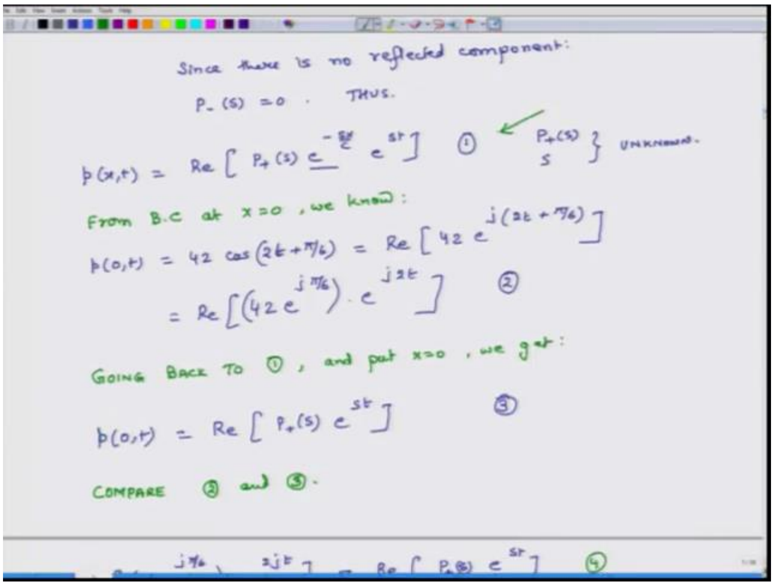

Since there is no reflected component; so this entire thing which I have encircled in red it should be 0 for all values of x and all values of t; it implies P negative of s is equal to 0. So, this is the first thing we get from the physics of this situation the P negative of s is 0.

So, now we move forward. We say thus p of x and t equals real of P plus e minus sx over c e to the power of st. Now, here so in this equation P plus is unknown and s both are unknown. And if we are able to know these terms then we have solved problem and then we know how pressure is travelling in this infinitely long tube. So, our aim is to find P plus which is the function of s and s which is the complex frequency.

So, these things we find out from the fact that at x is equal to 0 we know that pressure is 42 cosine 2 t plus pi over c. Let us call this equation number 1, and then we will say from boundary condition which I call B.C at x is equal to 0 we know. What we know that p at location 0 is equal to 42 cosine 2 t plus pi over 6 or I can call it as real of 42 e to the power of j times 2 t plus pi over 6 or I can further change it or transform it in this form 42 e to the power of j pi over 6 times e j times 2 t. So, that is my equation 2.

Now going back to 1; so we go back to equation 1 and we find out the value of pressure at x is equal to 0. So we put and we put x is equal to 0 we get p o and 0 at the origin is equal to real of P plus s and when we put x is equal to 0 in this situation then this term goes away e to the power of minus s (Refer Time: 10:44) to the only thing we are left with this e to the power of st. So, that is our equation 3.

Once again our aim is to find P plus of s and the parameter s the values of these 2 things. So now, what we do is compare 2 and 3, we see that an equations 2 and 3 the left hand side is the same. If the left hand side is same then I can equate the right hand side of both the terms also. So, what I can write it as real of 42 e to the power of j times pi over 6 times e to the power of 2 t; excuse me 2 j times t equals real of P plus s e to the power of st. So, that is my equivalence. And I am getting this equivalence from the boundary condition (Refer Time: 12:14) x is equal to 0.

Now, this equation, so this is let us call this equation 4.

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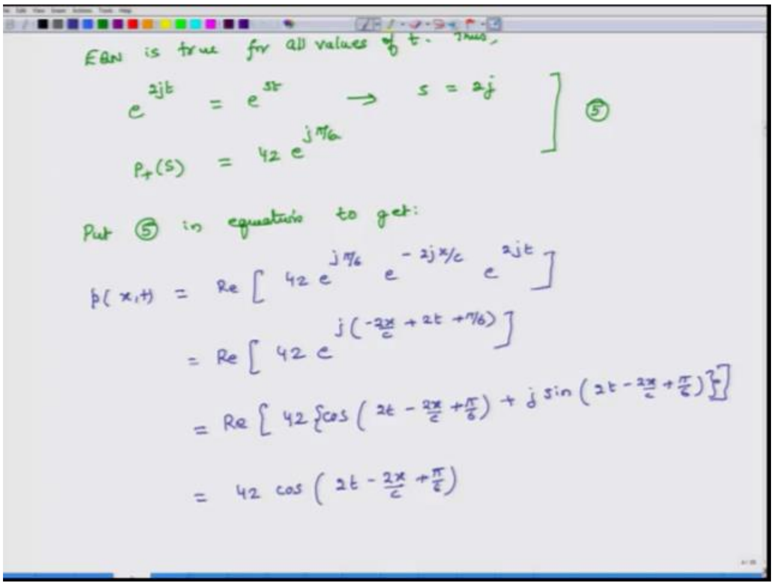

So, equation four is true at; so it I true for all values of t. Only it is going to be true if e to the power of 2 j t equals e to the power of st which implies that s is equal to 2 j and the other equivalence is that P plus s equals 42 e to the power of j pi over 6. So, let us call these equations 5. Now, we have calculated the value of s and we have also figured out what is the value of P plus which may depend on s, but in this case it just happens that it is not depended; does not change f I change s.

But it is a complex number. So P plus may or may not be a real number it can be a complex number. So, what we do is put 5 in equations. So, which equation do we put in? So we put it in this equation one, so we get. So, actual pressure in the tube at any location x and at anytime t is equal to real of P plus of s which is 42 e to the power of j pi over 6 times e to the power of minus sx over c and s is 2 j. So, 2 j x over c and e to the power of st and s is again 2 j, so 2 j t. Or I can express it as 42 e to the power of j minus 2x over c plus 2t plus pi over 6. So that equals real of 42, and this is e to the power of j something I can resolve it into a cosine component and sin component. So, it is cosine of 2t minus 2x over c plus pi over 6 plus j times sin of 2t minus 2x over c plus pi over 6.

And if I take real of everything in the parentheses what I get is 42 cosine 2t minus 2x over c plus pi over 6.

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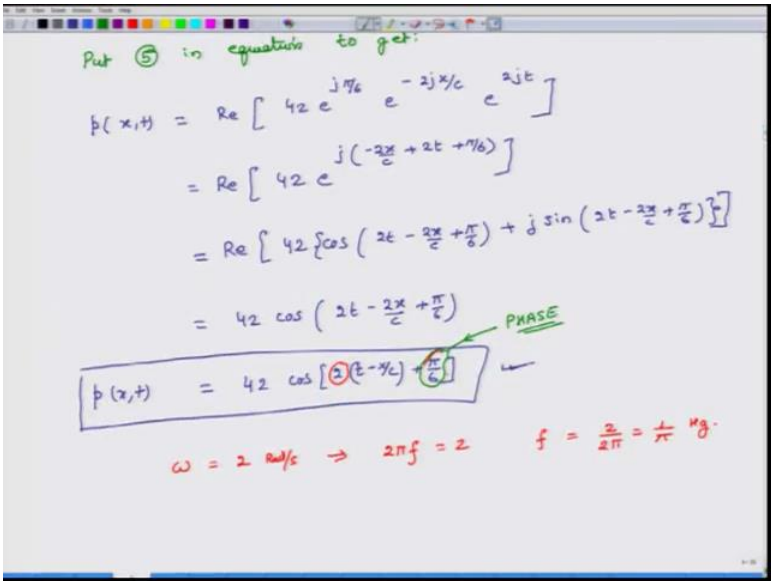

So, I can also rewrite pressure in the tube and I can write it as 42 cosine 2 t minus x over c plus pi over 6. So, that is the pressure in the system, in the tube which is infinitely long and at one end it is excited by a piston where an, at x is equal to 0 it is excited by piston in such a way that the pressure at x is equal to 0 is defined by this relation. I like you to check one thing.

So, we had said that in this tube there is only a wave which is travelling in the positive x direction and there is no wave which is reflected or which is travelling in the negative x direction, which means that any solution of this system has to be such that its of the nature f 1 of t minus x over c.

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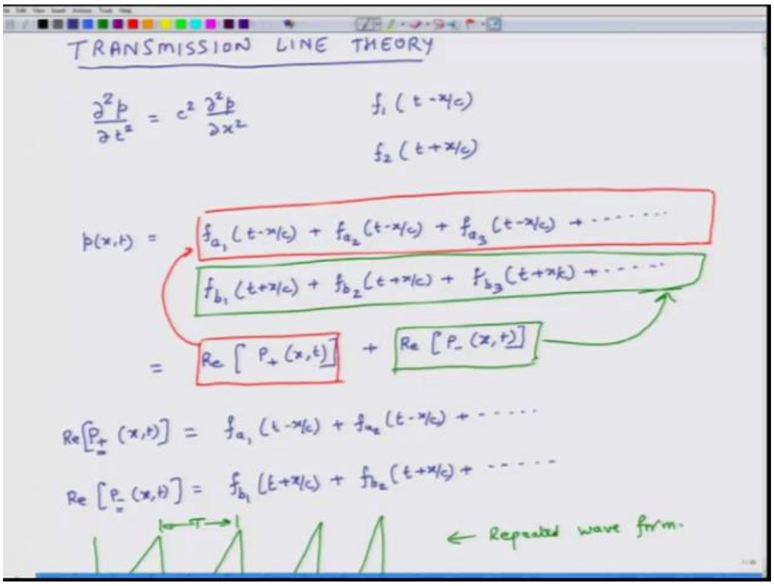

Because, I mean this is what we had developed in the last class that these terms were represents forward travelling waves. So, it is f 1 of t minus x over c these are all forward travelling waves. So, we have to make sure that our final solution is consistent with this mathematics. So, we go back and we see that in this relation t does not appear by itself it comes as t minus x over c. And because its t minus x over c it means that the wave is for travelling in the forward direction.

The other thing is that what does two represent is two. So, here the wave is travelling in such a wave that its angular frequency omega is 2 or we can say that 2 pi f is equal to 2. Or its frequency is equal to 2 over 2 pi or that equals 1 over pi hertz, and here the units are radians per second. So, the angular frequency is two radians per second, the frequency is 1 over pi hertz, and then what does pi over 6 represent. So, pi over 6 represents phase of the wave. Where does this come from? It comes from our initial condition. So, both these parameters 2 and pi over 6 they come from the boundary condition curve related to x is equal to 0 and so does this term 42.

So, that I think covers pretty much the behaviour of a wave travelling in an infinitely long tube in such a way that at its one end which corresponds to position x equals 0 it is excited by some piston, and we happen to know if we know the pressure at x is equal to 0. Then using that initial condition and the fact that there is not going to be any reflected wave we can solve for pressure at any point in this infinitely long tube. And that is the relation for the problem which we have solved.

So, with that I conclude the discussion for today, and tomorrow we will continue our discussion on transmission line equations.

Thank you and have a great day. Bye.