Fundamentals of Acoustics
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Lecture – 19
Wave Guide

Hello. Welcome to Fundamentals of Acoustics. This is the fourth week of this particular course, and in this week what we will be covering is descriptions of a wave guides and transmission lines. And essentially what this will help us understand is how do waves travel in a 1-D space. So, in the last week we had developed the governing equation for 1-D pressure wave and one dimensional velocity wave. Now, we will learn how to solve these equations using concepts of transmission lines.

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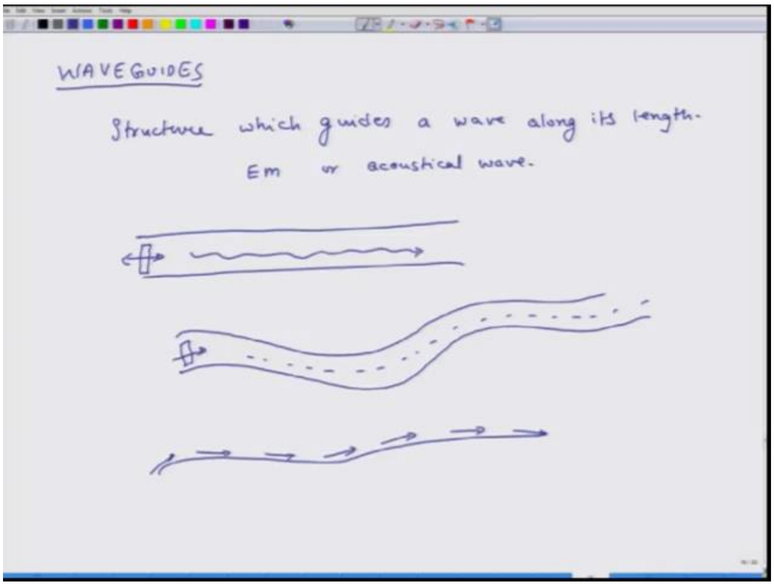

So, first we will discuss in brief about Wave Guides. So what is a wave guide? It is a structure which guides a wave along its length. Now this wave could be any wave; it could be an EM wave electromagnetic wave or it could be an acoustical wave.

Now, these wave guides could be one dimensional or two dimensional in nature, but what we will be covering in this course is essentially one dimensional wave guides. So, some examples of such wave guides. So, you consider a long tube and I am; so this tube is filled with air and I am exciting a piston in the tube such that the piston is moving back and forth. So, the sound wave travels in along the length of the pipe. So, this pipe acts as a wave guide. Now this pipe need not be straight, but as long as its curvatures are gentle then the sound will travel along the length of the pipe. So, both of these are examples of one dimensional wave transmission.

Another example of wave guides in context of electromagnetic wave is a fiber optic cable. So, this cable is very thin and the light travels along the length of this cable. Actually travels inside the cable along the length of the cable. And the cable need not be a straight it can be bend and it can be hundreds of kilo meters long in the light travels along its length. Another example of a wave guide are; transmission lines for electricity. So, you have a source where electricity is generated and may be thousand kilo meters away electricity is being consumed. So, there are wires which connect the source and the place of consumption. And in this case electricity which is again an examples of electromagnetic wave guides they travel along the length of these cables.

So, this is what wave guides are and these wave guides, how sound or an electromagnetic wave travels along these wave guides is based on a particular theory and that theory is known as Transmission Line Theory.

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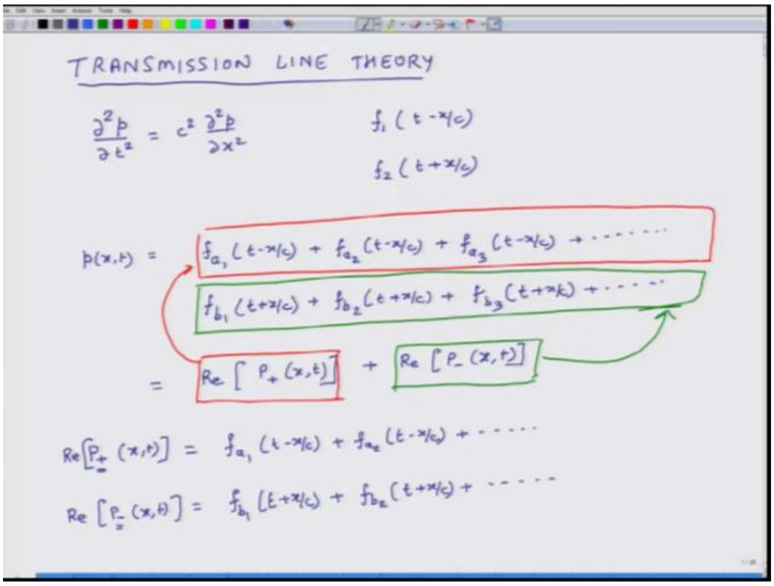

Transmission line theory; so the theory which helps us understands how waves travel along wave guides is known as theory of transmission lines. So, we are going to discuss the basics of this transmission line theory in context of acoustical wave. So, what we had expressed earlier is that; so let us consider the pressure wave equation. So, that is my pressure wave equation.

So, for this particular governing differential equation we had shown earlier that there could be two possible general solutions. So solutions one is, f 1 t minus x over c. And then another possible solution is f 2 of t plus x over c. Now f 1 itself could be a series of functions and f 2 itself could be a series of functions. So in general the most general solution for this would be a series of f 1s and a series of f 2s. So, what would that look like? So, first we will write a series of f 1s; f a 1 t minus x over c plus f a 2 t minus x over c plus f a 3 t minus x over c plus, and if this could go up to n terms. So, these are all f a 1, f a 2, f a 3, means all depict components of the wave which is traveling in the positive x direction. And then there could be several components of the wave traveling in the reverse direction and those we can designate as f b 1 t minus x over c plus f b 2; so excuse me t plus t plus x over c plus f b 3 t plus x over c and so on and so forth. So, that our most general solution.

Now, I can write this entire term as. So, this has two components: a forward traveling wave and a wave traveling in the reverse direction. So, I can also write this as this is equal to real of P plus, so P plus is a function of x and t. And this P plus function is could be a complex function and its real component. And then there is another one, real of P negative which is a function of x and time. So, what does real of P plus x over t means? It essentially means the sum of this f a 1, f a 2, f a 3. And real of P negative could be equated to f b 1, f b 2, f b 3, and so on and so forth.

So, using such an equivalence I can write P plus of x and t, if I take its real portion. So, this is equal to f a 1 t minus x over c plus f a 2 t minus x over c plus and so on and so forth. Similarly, real of P negative which is a function of x and time could be f b 1 t plus x over c plus f b 2 t plus x over c and so on and so forth.

So, here the P plus implies that it corresponds to a pressure wave moving in a positive x direction and P negative corresponds to a pressure wave moving in the reflected direction of the negative x direction. There is a reason we have expressed our solution in this form and the reason relates to some practical considerations.

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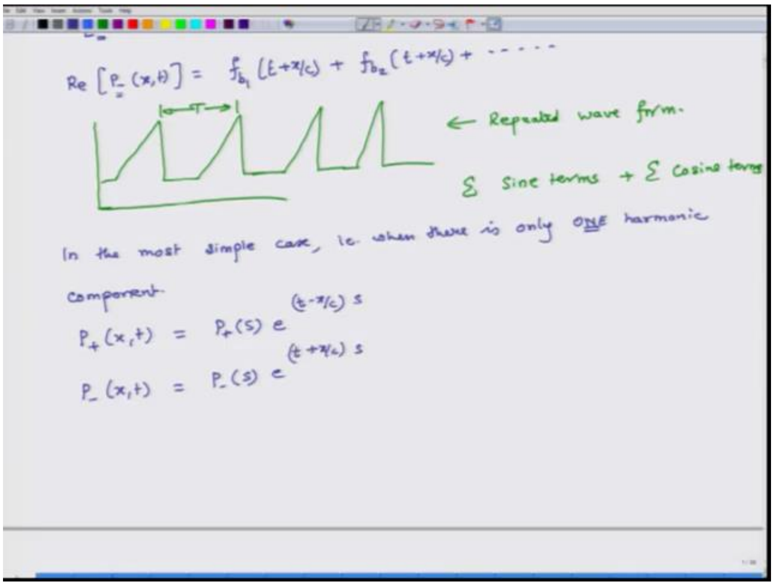

So, consider a wave form; some repeated wave form. Let us say this is some repeated wave form and it has a time period of t. So, it repeats itself after every t seconds. So, while I was doing this may be I was little less careful, so this is like this like that; so this is my time period.

Then I know from principles of Fourier series or even Fourier transform that I can express this has a sum of sine terms plus a sum of cosine terms. If I can resolve this wave form which is repeating a itself, this pattern is repeating itself after every t seconds in terms of a series or sine terms and number of cosine terms using the mathematics associated with Fourier series. So, one way to look at f a 1, f a 2, f a 3 could be that they individually; so if the wave form is repeating itself then f a 1 could be the first sine term, f a 2 could be the second sine term, f a 3 could be the third sine term and so on and so forth. So, that is why we have expressed it in this way.

And if that is the case then in the most simple case that is when there is only one harmonic component then I can write P plus which is a function of x and t as p constant which depends on complex frequency times e to the power of t minus x over c times s. If there were multiple harmonic components if there was long sine series then it would be P plus 1 s e to the power of t minus x over c s plus P plus 2 s times e to the power of 2 t minus x over c and so on so forth. But for learning purposes we are saying that we have only one harmonic component, so only one my sine wave or cosine wave is progressing along the x direction.

And similarly P negative of x and t equals some constant p minus s e to the power of t plus x over c to the power times s.

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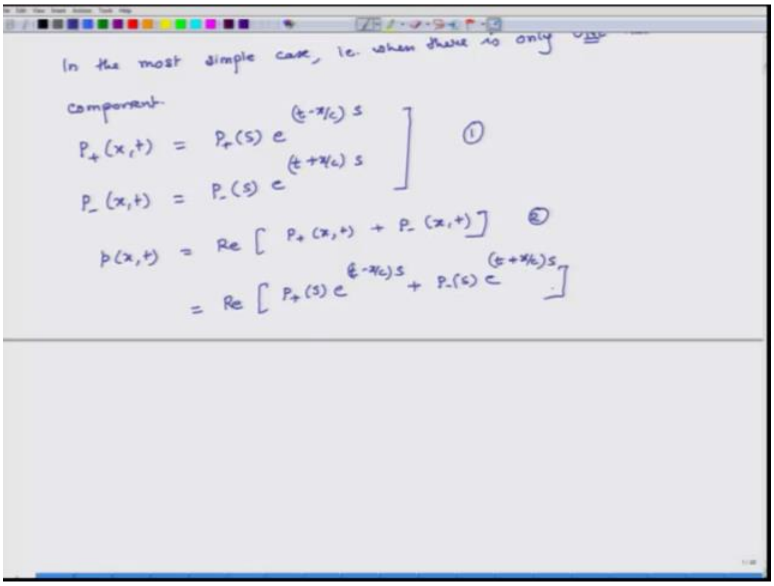

So, if that is the case then pressure is what, is real of P plus x over t; x and which is the function of x and t plus P negative x and t and this is equal to real of; so let us number this equation as 1 and 2. So, we put in equation 2 the definitions of P plus x t and p minus x t. So, I get P plus some constant P plus s e t minus x over c s plus P negative s e t plus x over c s.

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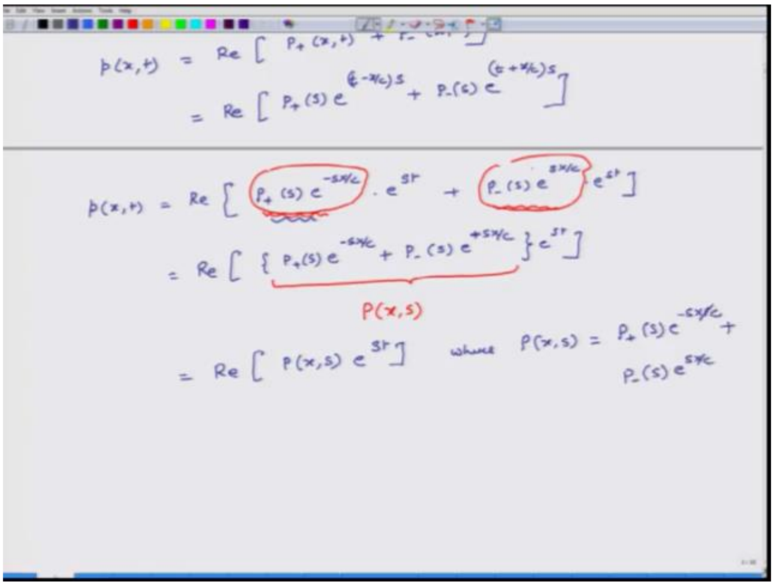

Or I can write it as that the real pressure or actual pressure physical pressure is real of and I am going to expand this, so it is P plus s e minus sx over c times e st plus P negative of which is dependent on s e sx over c times e st.

So, I have this term and I have this term and I can take call this entire term as something, so first what we will do is we will take st out. So, this is real of P plus s e minus sx over c plus p minus s e minus sx over c; excuse me e to the power of st. And I call this entire thing in the bracket. So, the entire thing in the bracket is dependent on two entities which is complex frequency and the position. So, I can call it the complex amplitude of the wave which is complex spatial portion of the wave. So, I can write it as real of P x s e st; where, P x s is equal to P plus s e minus sx over c plus p minus s e sx over c.

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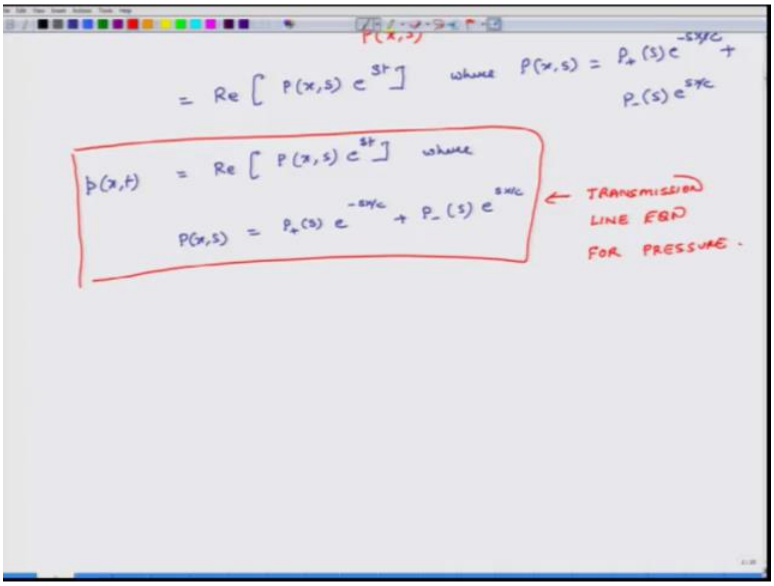

So, to recap the actual pressure is equal to real of some function which depends on x and s and it is a complex function times e to the power of st. Where, this function which is complex function of x and s; P x and s equals a constant number which can change on with respect to frequency times e to the power of minus sx over c plus P negative which again depends on s times e to the power of sx over c. So, this equation is called the Transmission Line Equation for Pressure.

So likewise, we also have a transmission line equation for velocity, but right now we are just starting so this is what this equation is. So, transmission line equation for pressure and what it says is that the actual pressure which can vary with respect to x and t is equal to real of a complex function which depends only on x and s times exponent of s times t where s is the complex frequency. And this complex function p x comma s is equal to P plus s e minus sx over c plus p minus s e to the power of sx over c.

So, I think for today's purposes we have done pretty much what I planned to cover. And what we have been able to accomplishes that we have develop the transmission line equation for pressure, and in the next class we will see how we can actually use this equation to solve some practical problem. So that is pretty much for today and look forward to seeing you tomorrow.

Thank you very much and have a great day. Bye.