Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture – 18
Solution for 1-D Wave Equation

Hello. Welcome to Fundamentals of Acoustics. Today is the last day of the third week of this course, and what we will do today is continue our discussion on the 1-D Wave Equation, and also its solution and we will also develop a physical interpretation for the term c.

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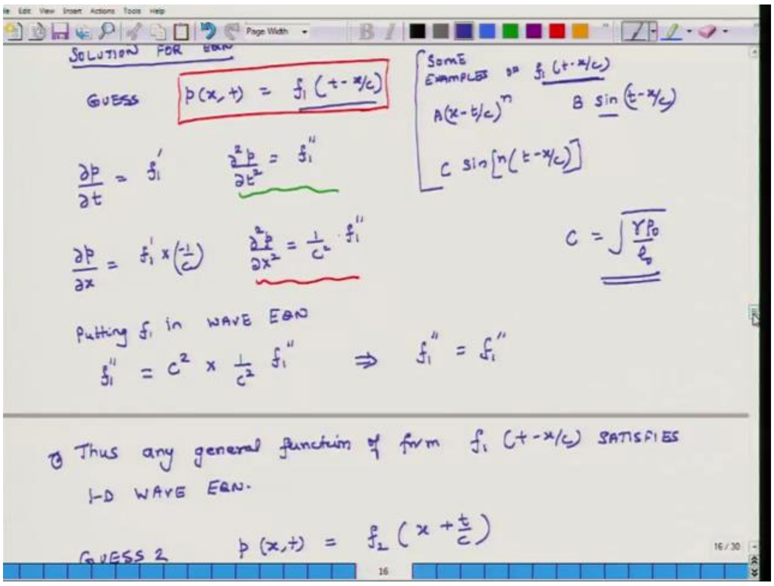

So, the agenda is some more discussion on 1-D Wave Equation. So, what we have done in the last class was we have developed this relation. So, this is what we had said was 1-D wave equation for pressure; what does that mean? What this means is that this equation tells us how a pressure fluctuation in air: if it is at point BA it travels to another point in the medium. So, the left side tells how pressure will change in time at a given point and this right side tells us how pressure is going to change in space. So, it tells us how pressure is traveling in air or in water or in a fluid media.

Now, this is partial differential equation because and it is a second order partial differential equation and on the left side the partial differentiation is with respect to time on the right side it is with respect to x. So, what we will do as a first step is we will develop a solution for this. And we will develop a general solution for it. A one way to develop a general solution is to guesses. So, we will make a guess and then we will plug that guess into this equation, if that guess satisfies the equation then it is a valid guess otherwise our guess is wrong. So, we will say that we guess that p of x t is equal to sum function f 1 and that f 1 does not depend on time alone, it does not depend on x alone but it depends on t minus x over c. So, this is a guess we are making.

And if this guess is correct then when I plug f 1 this a definition of p a back into the wave equation it will be satisfying this wave equation which means the left and right sides will be equal, if the guess is incorrect then the equation will not be satisfied. But before we do that let us see what kind of functions these could be. So, examples of f 1; so 1 could be x minus t over c to the power of n. This is one possible value of pressure and I can multiply it by some constant.

Another possible function could be B sin t minus x over c. So, a and b these could be constants they need not be real but they have to be constants. So, this is also of the form f 1 t minus x over c. Another possible function could be C into sin n where n could be a parameter 1, 2, 3, 4 whatever times t minus x over c. So, these are some examples So, all these possible functions are consistent with a definition for f 1, because t does not combine itself x does not combine itself rather t and x in this function f 1 they come as a combination and they come as in the form t minus x over c. So, these are all possible function.

Now let us see whether this function satisfies the wave equation or not. So, if this is the case then with respect to t, partial of p with respect to t; let us define it as f 1 prime. And partial of p with respect to t second derivative is f 1 prime time. Then partial of p with respect to x is equal to what, is equal to f 1 prime into minus 1 by c. And partial of p with respect to second derivative, with respect to x is 1 by c square times f 1 prime time. So, I plug this here and the second derivative of pressure in time here and then what I get is finally. So putting f 1 in wave equation we get f 1 double prime equals c square times 1 over c square f 1 double prime or f 1 prime prime equals f 1 prime prime.

So, what we see is that any general function of form f 1 t minus x over c; so we can say thus any general function of form f 1 t minus x over c satisfies 1-D wave equation, any general function. So, which means that our guess that p of x and t is equal to f 1 t minus x is a correct guess; it may not be the only guess possible, but it is at least one guess which is correct and it satisfy the equation. So, this is 1.

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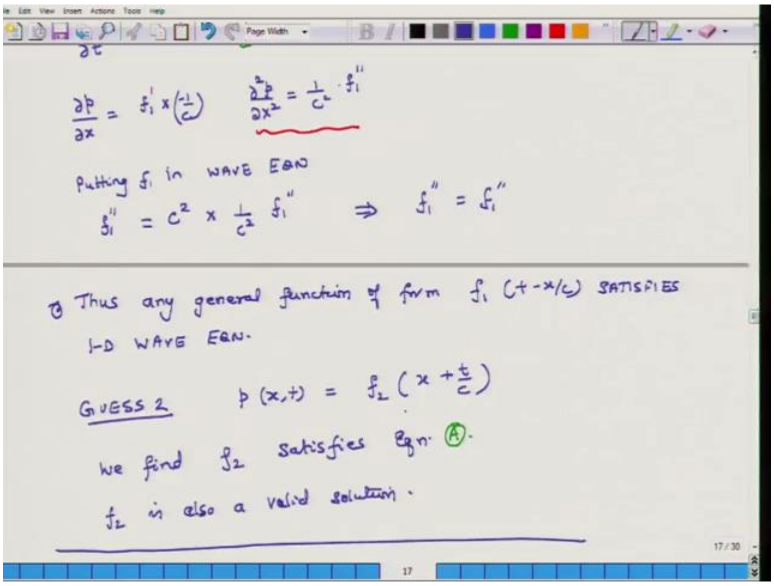

We make another guess; guess two and here we assume that p of x t is equal to another function at f 2 x plus 2 over c; so this is second wave. And if we do the same mathematics for this also we find we find f 2 satisfies equation and I will number this equation, I will call this equation A; so equation A.

So f 2 is also valid solution; it is also valid solution. And then we can try guessing others, but we will find that we would not be able to figure out any general function beyond these two to be satisfying the pressure wave equation.

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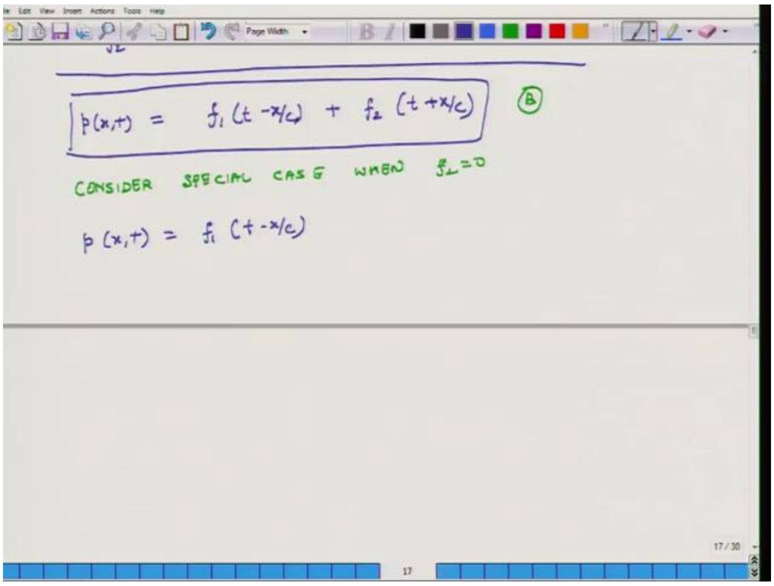

So, in general pressure equals f 1 t minus x over c plus f 2 t minus x over c. So, f 1 could be a series of function by itself right f 1 could be all (Refer Time: 09:57) of sins, cosines, square, etcetera; as long as t minus x over c comes as a unit and f 2 could be a by itself a series function, but this is a general solution. So, this is equation B.

Now what we do is, consider special case when f 2 equals 0. So, in that case p of x t equals f 1 t minus x over c. Now, what we are going to do is we are going to interpret the meaning of this term c and what it implies. So, we are going to consider this (Refer Time: 10:56) when f 2 is equal to 0 then f p of x and t is f 1 t minus x over c. We have seen that f 1 satisfies the wave equation, so I can write p equal to f 1 t minus x over c.

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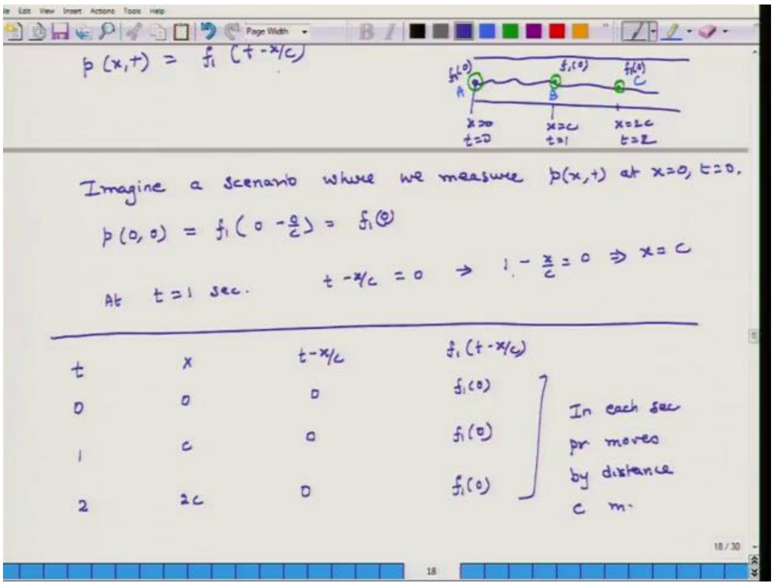

And, let us imagine a scenario. So, imagine a scenario we measure p x t at x is equal to 0 t is equal to 0. What does that mean? Suppose there is sound traveling in a tube it is a 1-D system. So, at the beginning of time when we start making recordings that is t is equal to 0 we record the pressure at the beginning of the tube which where x is 0 and initially we measure it so time is also 0.

So, p of 0, 0 is equal to f 1 and what is a value of time 0 minus 0 by c, so it is equal to f 1 of 0. So, consider a tube and sound is traveling like this, so here x is equal to 0. So, we are measuring time at time t is equal to 0 pressure is f 1 of 0. Now we will see what happens after time t. So, at p is equal to 1 second t minus x by c. And suppose t minus x by c is 0 then t is equal to 1 second then x is equal to c; so this is x is equal to c. So, what I am saying is that at x is equal to c and at time t is equal to 1 second. So, here t is equal to 0 what is a pressure here, f 1 0 at time t is equal to 1 second and x is equal to c what is the pressure here, I have to put t is equal to 1 in this x is equal to c here. So, it is f 1 0.

So, we make a table; time x. In the beginning time is 0, x is 0 t minus x over c is 0 this is at point at this point. Then after 1 second time is 1 and let us see what is the value of at x is equal to c. T minus x over c is 0 and this is f 1 over 0. Then after 2 seconds and distance 2 c; t minus x over c is still 0 f 1 0. Now, let us interpret this data. So, let us call this point A let us call this point B and let us call this point C. So, what does this mean? Initially at time t is equal to 0 let us look at point A in this picture, at point A in this picture the pressure was f 1 0 at time t is equal to 0. At time 1 second the pressure was f 1 0 when at location x is equal to c. And at time 2 seconds the pressure was f 1 0 at location x is equal to 2 c.

So what that means is, that this pressure which was initially at the origin became the pressure at point B after 1 second and this B was located C distance away and then this pressure travel to point C which was at time 2 seconds which is located 2 c distance away. So, in each second this pressure is moving by distance c. So, in each second pressure moves by distance c meters.

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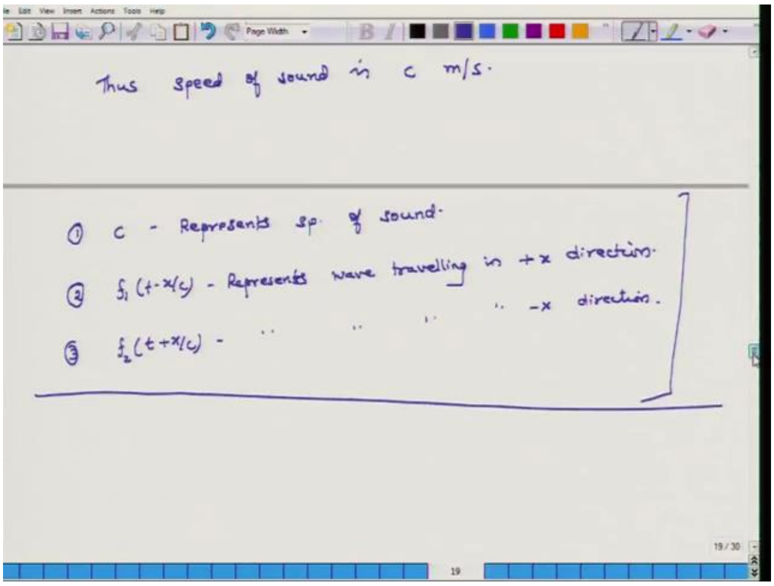

Which means thus speed of sound is c meters per second; so this is the first thing. So, one is c represents speed of sound. Second, f 1 t minus x by c this function it represents that sound is moving in the positive x direction, because sound is moving in this direction physically. So, f 1 t minus x over c represents wave traveling in positive x direction. And third is; similarly we had another solution f 2 which is t plus x over c it represents wave traveling in negative x direction. How do I say that? I say that based in this table here if I construct the same t able, but instead of t plus minus x over c I replace it t plus x over c. We will see that this function represents the wave traveling in the reverse direction.

Physically what it means is that when I am talking in a room or in a pipe, when I speak there is wave which is traveling in the positive direction, but it could also happen that if the tube is closed at the end and then when the wave hits it some of the wave can get reflected. So, some wave is traveling in the positive direction and some wave is moving in the negative x direction.

So, these are the three important conclusions from today's discussion first is c represents the speed of sound and it is value which is calculated theoretically which is c equals gamma P naught over rho; this theoretical value is pretty close to the experimentally observed value which we had discussed in the last class. And the second thing is that the solution of any wave propagation equation in one dimension it can be represented either as f 1 or f 2. F 1 t minus x over c it represents a wave traveling in the positive x direction, f 2 which is a function of t plus x over c represents the wave traveling in the negative x direction.

So, with these three important conclusions we conclude our discussion for this week. I look forward to seeing you next week as well, and next week we will start developing some more details solutions for these equations using the concept of transmission line equations. So, till then have a great day and I look forward to seeing you next week, bye.

Thank you.