Fundamentals of Acoustics
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Lecture – 17
1-D Wave Equation

Hello, welcome again to Fundamentals of Acoustics. Today is the fifth day of this course of the third week of this course and what we planned to do today is develop the 1 dimensional wave propagation equation and then the way we are going to do it is we will write down all the 3 equation which we have derived and developed till so far which are the momentum equation, the continuity equation and the gas law and we are going to synthesized all this 3 equations into 1 final wave equation. So, that is goal for today.

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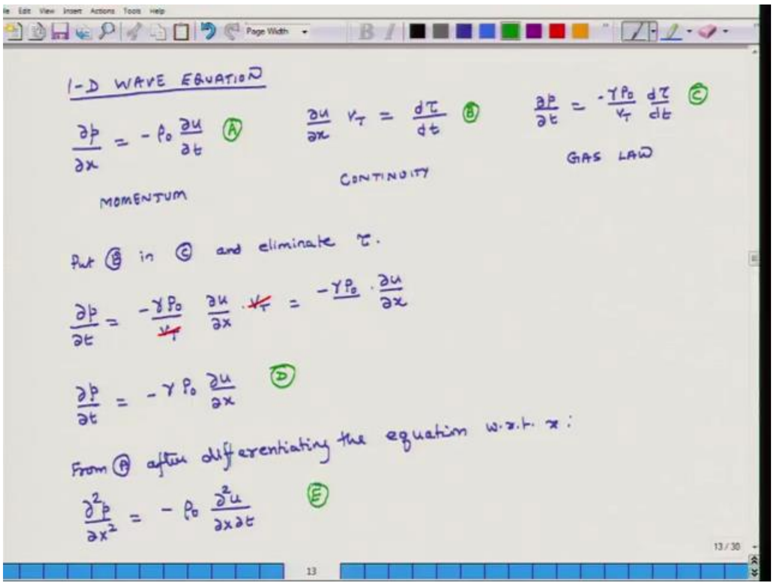

What we will do is we will develop the 1 D wave equation, 1 D wave equation. So, let us first write down all the 3 equations which we have developed. So, this is the first equation partial of p with respect to x equals minus rho naught partial u respect to time.

The second equation is, this is the momentum equation, equation for momentum the second equation is partial of u with respect to x times V T equals total derivative of change in volume over time. So, that is my continuity equation and the third equation which we have developed was from the adiabatic process equation and that is del p over del t to equals minus gamma p naught over V T d tau over d t. So, this is gas law.

Let us number gives as equation A equation B or actually I am going to equation C. So, we see that in these 3 equations there are 3 unknowns, the first unknown is pressure, the second rho naught is known which is the density of air at standard temperature and pressure condition, the second unknown is u the third unknown is volume and then there is also fourth unknown tau. So, therefore, unknowns, but we know that V T equals tau plus v naught v naught is known. So, effectively there are 3 unknowns, p u and tau because V T is nothing, but v naught which is known plus tau. So, 3 unknowns and then we have 3 equations all other parameters in this equations rho naught in equation A and then in equation B gamma p naught all these parameters are known. So, we have 3 equations 3 unknowns. So, we should be able to solve for these unknowns. So, what we will do is we will del over one single equation for pressure and one single equation for velocity and those equations are known as wave equation.

So, let us get started. So, first what we do is we compare equation B and C. So, in B, we have d tau over d t equals del u over del x times V T. So, we put B in C and eliminate tau. So, what we get is del p over del t equals minus gamma p naught over V T and here I have d tau over d t which equals del u over d x times V T. So, I put in there del u over del x times V T, this V T and V T cancel out.

I get minus gamma p naught times del u over del x. So, I can I just rewrite this del p over del t equal minus gamma p naught del u over del x. So, I will number this equation as equation D. So, now, we compare equation A and equation D and what we see is that on the left side, on the right side I have a del u over del t and on the left side right side of D I have del u over del x. So, I can eliminate u from this if I differentiate a with respect to x and I differentiate b with respect to time and I equate the cross directive derivative. So we will do that.

From A after differentiating the equation with respect to x, what we get? Second derivative pressure with respect to x equals minus rho naught second derivative of u del x del T. So, I will call this equation E.

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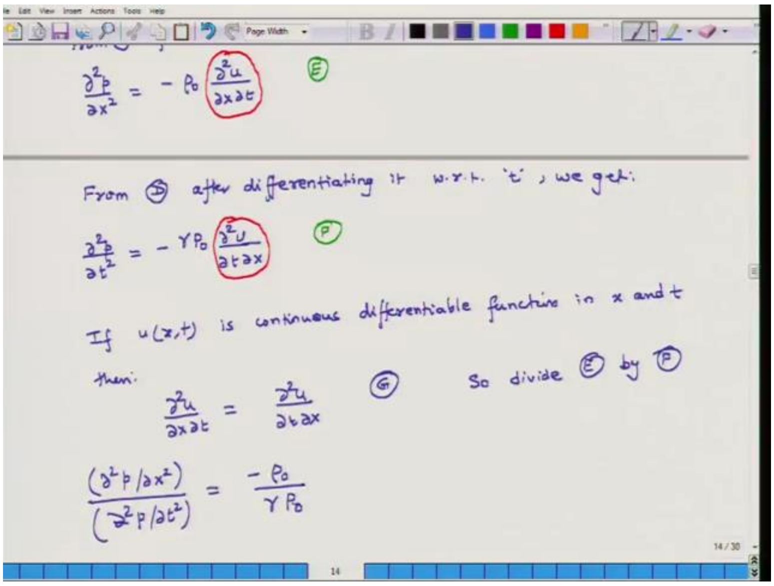

And then from equation D after differentiating it with respect to time we get. So, from D we are getting second derivative of pressure with respect to time equals minus gamma p naught del 2 u over del t del x this is equation F.

We compare this term and this term. So, these are cross derivatives of u and in the first case for equation u we are first differentiating u with respect to time and then with respect to x and in the second case, we have first differentiating u with respect to x and then with time and if u was a continues and differentiable function. So, if u which depends on x and t is continuous differentiable function in x and t then if that condition is true. So, if it continuous in x and t at least for the first order then del 2 u over del x del t equals del 2 u over del t del x I can actually equate those G.

From E, F and G, what we can write it as, if I divide, divide E by F, if I divide E and F, E by F then I can eliminate u. So, what do I get del 2 p over del x square divided by del 2 p over del t square is equal to minus rho naught by gamma p naught or I can write it has del 2 p over del t square and there is a negative sign here.

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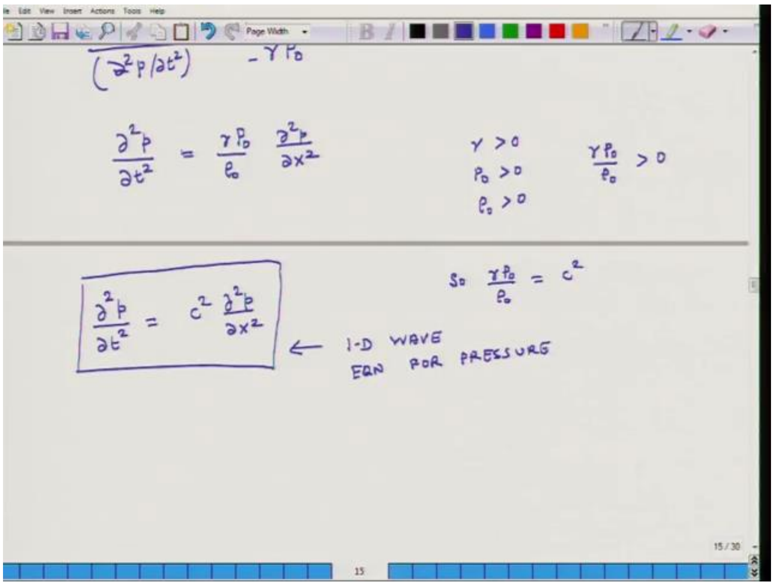

Del 2 p over del t square equals rho naught over gamma p naught del 2 p over del x square, now I think I need a error. So, this is equal to gamma p naught over rho naught and I have to write this clearly.

Now, gamma is a positive number p naught is the pressure addressed in the fluid, it is a positive number, rho naught is a positive number. So, gamma p naught over rho naught is positive, it is a positive number and we defined. So, we can also write gamma p naught over rho naught as some constant and I can square it. So, C is always a positive terms C square. So, if that is the case then second derivative of pressure with respect to time equals gamma p naught over. So, I can replace this c square. So, that is my 1 D wave equation for pressure 1 D wave equation for pressure, I can get a similar wave equation for velocity and the way I do it is we should see equation D and A and when we were del a eliminating u we had differentiated first equation with respect to x and second equation D with respect to time and because of that I got cross derivatives for u, I can eliminate pressure from these two equations from A and B by differentiating the first equation A with respect to time and equation D with respect to x and then I get cross derivatives in pressure and I get an identical equation in u and the form of the equation is similar and that is second derivative of u with respect to time is equal to C square, second derivative of u over x. So, that is my 1 D wave equation.

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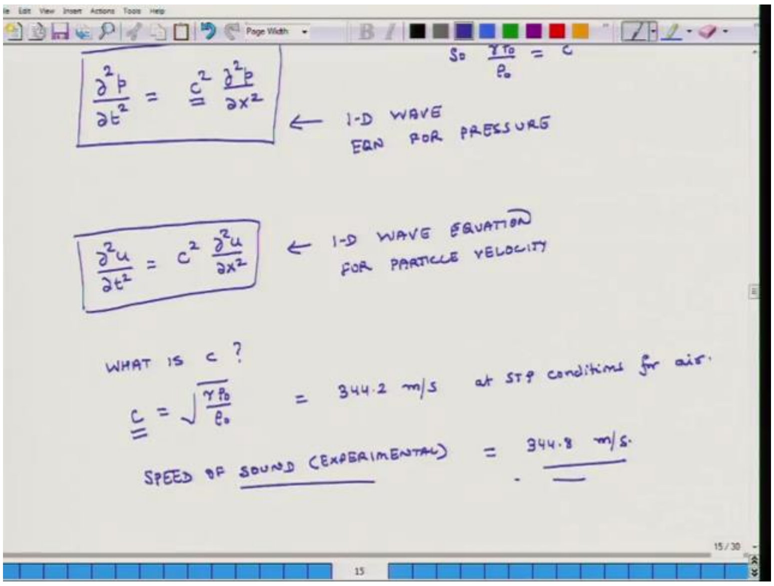

This is 1 D. So, this is 1 D wave equation for particular velocity because this deals with u and u represents the velocity of the fluid particle which is transmitting tau.

Now, you may wonder what is C. So, we have define C as C is equal to square root of gamma p naught over rho naught and if we considered p naught to be 1.01, 3 to 5 times 10 to the power of 5 pascals which is 1 atmospheric pressure and rho naught to be 1.18 meters kilograms per cubic meter which is the density of air at standard temperature pressure conditions then add the value of c is equal to 344.2 meter per second at STP conditions for air, if you measure experimentally the speed of sound experimental value speed of sound that comes to be 344.8 meters per second. So, it has looks like that C is very close to the experimentally measured value of speed of sound. So, maybe it is speed of sound, but at this point of time I have explained you whether C is indeed speed of sound or it is something different.

But what I have explained till. So, far is that if I calculate the value of C at standard temperature and pressure conditions for air, its value comes out to be 344.2 and if I measure the speed of sound using some experiments I find that at STP conditions, its value is 344.8 meter per second. So, C is awfully close to the speed of sound.

In the next class, which is the last class for this week, we will discuss this issue further and we will actually show that see mathematically indeed represents the speed of sound. So, that concludes the discussion for today. what we have done today is we have developed 1 D wave equation for pressure and particle velocity and we have also defined C as a square root of gamma p naught over rho naught and for air at standard temperature and pressure conditions, its value is 344.2 meters per second which it just happens to be very close speed of sound and in the next class we will explain the relationship between speed of sound and the parameter C.

That is pretty much for today; Thank you and we will meet once again tomorrow, bye.