Fundamentals of Acoustics
Prof. Nachiketa Tiwari
Department of Mechanical Engineering
Indian Institute of Technology, Kanpur

Lecture - 16
Gas Law for 1-D Sound Propagation

Hello, welcome to Fundamentals of Acoustics. Today is the 4th day of this week and what we will be discussing today is the gas law equation as it concerns the development of one dimensional wave equation.

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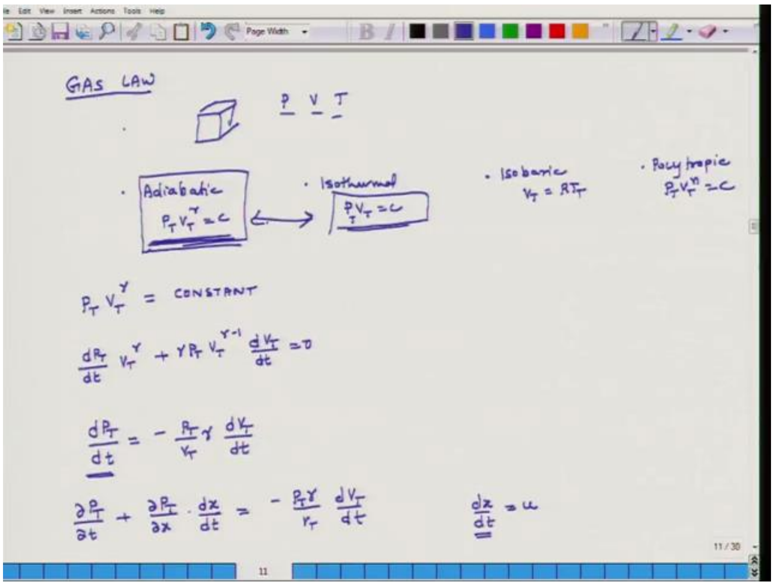

The law which we will develop or the equation which we will develop it will relate pressure and volume for the gas which is undergoing change as sound is getting propagated in the medium. Now in reality there are several processes which can potentially occur. So, as a piece of gas which has pressure volume temperature, it undergoes change there could be it can undergo change following the verity of processes, some of these processes are adiabatic process. So, here the governing relation is P T V T to the power of gamma is constant or it could be isothermal. So, here the governing relation is P T V T equals constant or it could be isobaric. So, here the governing relation will be V T pressure is constant is equal to RT T or R temperature or it could be poly tropic explanation. So, here it could be P T V T to the power of some constant n is constant and so on and so forth.

This gas can undergo you can follow anyone of these processes. So, we have to make a choice as to which process does it follow and in that context the only way we can make that choices by having a some sense of reality as to how gases or the fluid media is changing its pressure and temperature and volume as sound gets propagated through it. So, based on experience based on lot of experimental data, we can say with certainty that when the gas experience such changes in pressure volume and temperature, it is very close to the adiabatic process and the only reason we are saying that is that this is based on lot of experimental data and understanding of the physical phenomena with theoretically I can take either one of those and I can develop the wave equation, but then that final wave equation may or may not be correct, it will be incorrect if one of these assumptions, if the assumptions we make here are incorrect.

So, we are selecting the adiabatic process because we know for sure based on a lot of experimental data that when sound propagates in air the gas or the air experiences adiabatic expansion or compression because based on experimental data and a lot of observation. So, if that is the case then we say that P T V T to the power of gamma is constant or if I differentiate it I can write it as partial this total derivative of pressure with respect to time V T to the power of gamma plus P T V T to the power of gamma minus 1 times gamma times partial or this total derivative volume with respect to time is equal to 0 or total derivative of pressure with respect to time is equal to minus P T over V T times gamma d V T over dt.

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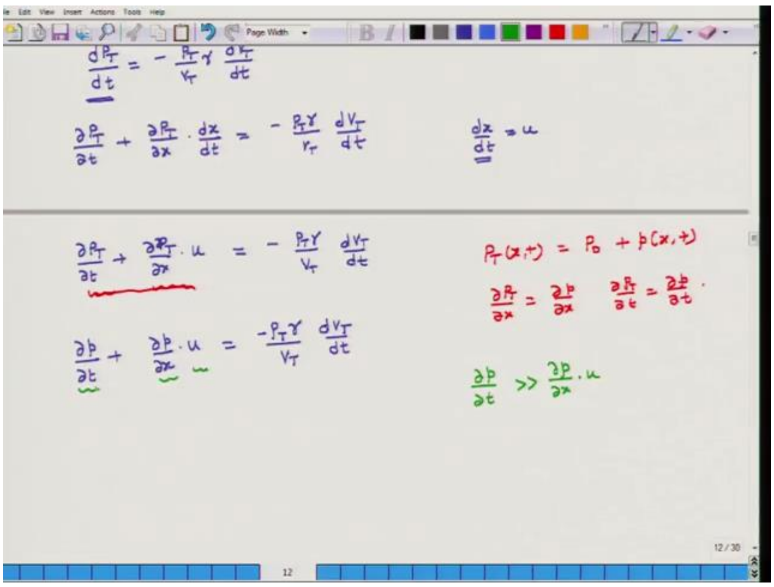

Now, what I do is I expand the LHS and once I do that I can write is as total derivative of pressure with respect to time is partial of pressure with respect to time plus partial of pressure with respect to x times dx over dt and that equals minus P T gamma over V T d V T over dt.

Now, dx over dt is equal to u because x in this system represents the position of the fluid element which we saw in while we were develop the momentum equation. So, x represents the position of the fluid element. So, change in x represents change in position of the fluid element. So, dx over dt is velocity of the fluid element. So, I can write it as partial of pressure with respect to time plus partial of pressure with respect to x times u equals minus P T gamma over V T, d V T over dt.

Now, we look at these terms and we realize that pressures velocities all of these entities are very small, but before we do that we will simplify this right side once more time. So, we realize that P T which can change with x and time is equal P naught plus P x and time. So, del P T over del x is equal to del P over del x and del P T over del T is equal to del P over del t. So, I can write this relation as del P over del T plus del P over del x times u is equal to minus P T gamma over V T d V T over dt. Now we realize that partial of P with respect to time is very large compare to partial of x times u for the simple reason the this is the product of 2 small entities partial of P with respect to x and u and this is just one small entity.

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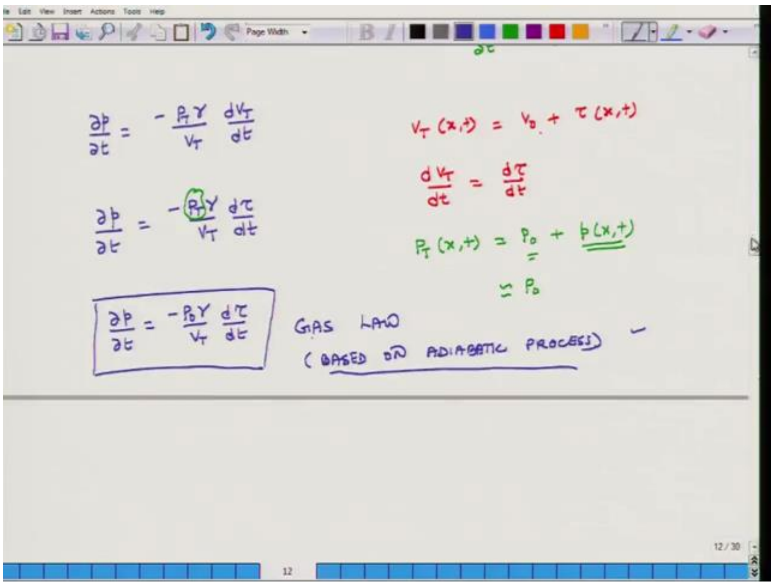

That is the case then I can simplify this relation as del P over dt equals minus P T gamma over V T d V T over dt.

Finally we say that volume which is the total volume which can change with respect to x and T is equal to initial value of the fluid element plus change in volume which is tau since we naught is constant. So, d V T over dt is equal to d tau over dt. So, this relation further gets modified as partial of P with respect to time equals minus P T gamma over V T d tau over dt then we say that P T x T is equal to P naught plus P x T and P is very small compare to P naught. So, in this term P T, I can approximate P T as P naught. So, I write this relation finally, as partial of incremental pressure with respect to time is equal to minus P naught gamma over V T d tau over dt. So, that is the gas law and it is based on adiabatic process.

If we had assumed that this process was isothermal in nature then the overall governing equation would have been P T V T to is equal to constant and when we compare this relation with this relation the only difference is factor of gamma. So, if I remove gamma from here and make it one that will be the gas law based on isothermal process. Historically this relation was first developed by Isaac Newton and he assumed because that time probably he dint have a lot of he did not have a lot of experimental data on how sound propagates. So, he assumed that the air when it expends in contrast due to transmission of sound it follows the isothermal process. So, the law which he developed gas law which he developed was partial of pressure with respect to time equals negative P naught, but it not take gamma as gamma, but it was one divided by V T times d tau over dt.

But once the wave equation got developed using that assumption it was found that the velocity of sound which it predicted was significantly less than the velocity of sound which was actually measured. So, later Sir Raleigh made that correction and he used the assumption of adiabatic process and once that adiabatic process was used, then the predictions were much better.

That concludes are discussion for today and tomorrow, which is the fifth day of this week, we will continue this discussion and what we will do in that discussion will be that we will then merge this gas law, momentum law and the continuity law into 1 single equation and develop the final 1-D wave equation.

Thank you and we will meet once again tomorrow, bye.