Fundamentals of Acoustics
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Lecture – 15
Continuity Equation for 1-D Sound Propagation

Hello, welcome to Fundamentals of Acoustics. Today is the third day for this week and yesterday we had developed the momentum equation. What we planned to do in todays lecture is develop the second important equation which governs wave propagation and that is the equation for continuity. As we had discussed earlier the continuity equation essentially represents the conservation of mass. So, with that understanding, we will develop the continuity equation for one dimensional propagation of sound waves.

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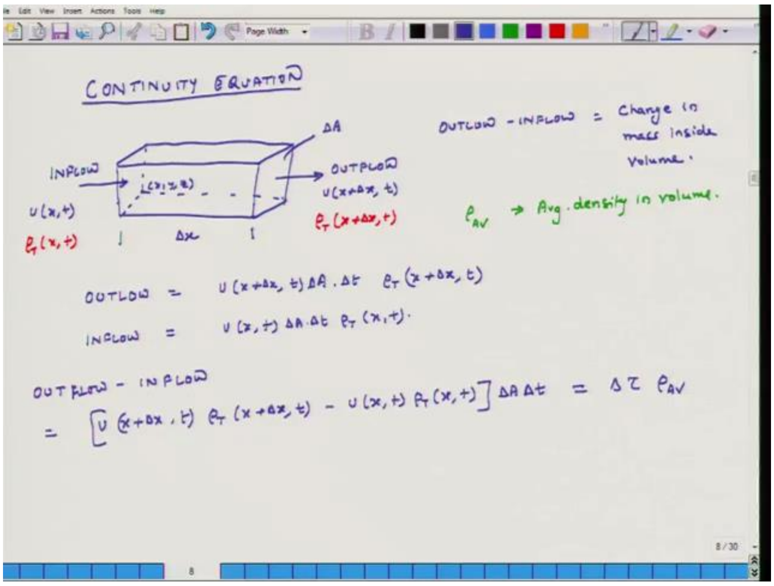

Our aim is to develop the continuity equation. So, we will once again draw a fluid element and actually this is the control volume. So, it is a volume into which flow is going in and flow is coming out. So, there is inflow an there is an efflux, the dimensions of this control volume are delta x and this cross sectional area is delta a now you should imagine this flow volume as some sort of a made up of some sort of a rubbery thing some sort of a rubbery thing. So, what happens if the amount of flow which is going in and amount of flow going out is same then the volume of this rubber volume, this will not it will remain same, but if there is less inflow and of there is more inflow and there is more outflow then mass will accumulate inside this rubbery volume and this rubber will expand because mass is going to accumulate in it. So that is going to happen. So, this is what we think this is how we are to develop the continuity equation.

The overall processes, the other thing I wanted to mention is that this expansion is going to happen only in the x direction and it is going to happen only in the x direction because we are assuming that the variation in Y and Z directions is 0. So, it will just become either longer or shorter, but it will not become fat or thin. So, outflow minus inflow.

So, this will equal to change in mass inside volume change in mass inside volume. So, outflow is what. So, outflow here the velocity is. So, once again the coordinates of this point are x Y and Z. So, outflow what is the velocity here inflow velocity is U x which is function of x and t at the outlet the velocity is U x plus delta x and time is same and that time itself. So, U x plus delta x comma T, we will also write down the density because the density of fluid going in and the fluid going out they may also be different. So, density is rho T x t and the density of the fluid coming out is rho T x plus delta x comma t and we will say that the density of fluid inside the volume see will change from point to point it will change from point to point because x is changing. So, it may have some average value rho AV is average density in volume.

So, what is outflow is going to be U x plus delta x time t comma t times area times area times delta t. So, in one second the amount of liquid which is going to flow out is going to be U x plus delta x the t. So, this is in cubic meters, but then we are interested in mass. So, we will multiply it by rho T x plus delta x t and then inflow is U x t delta a delta t rho T x comma t. So, we will develop an expression from of outflow minus inflow. So, that is equal to U x plus delta x t rho T x plus delta x t delta a times delta t. So, if outflow and inflow is same then the total sum of these 2 will be 0, but it may not be because it may be that flow inflow is higher outflow is less or whatever. So, that equals change in mass in the volume. So, what is change in mass it is the volume this element and the volume of this element is delta tau and times rho average. So, remember this delta tau is the change in the volume change in volume. So, volume may go up or go down and that change corresponds to the change in mass. So, change in volume times average density.

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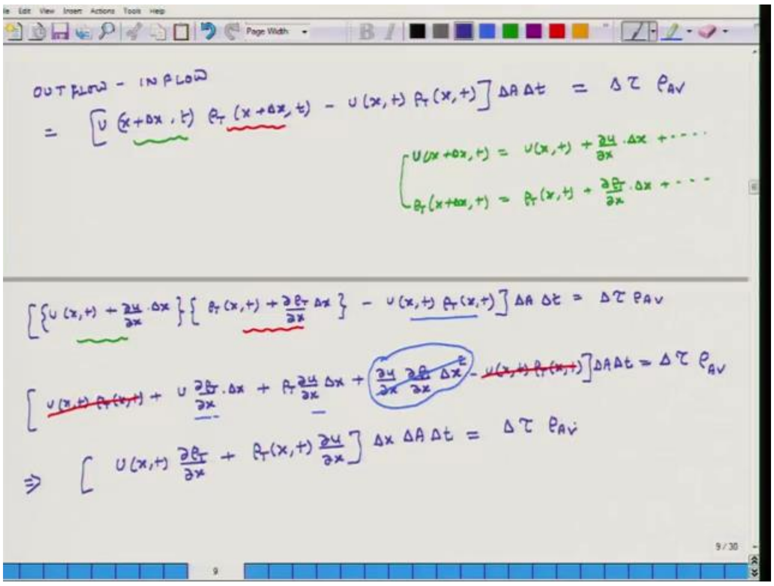

Now what we do is we expand these relations. So, from Taylor series we know that U x plus delta x comma t equals U of x t plus del U over del x del x plus higher order terms this from Tyler series mentioned. Similarly we know that rho T x plus delta x comma t is equal to rho T evaluated that x and t plus partial of rho T times delta x plus higher order terms. So, we plug these in the above equation and what we get is U x t plus del U over del x del. So, that is my first term then I multiply it by rho T minus that is my equation. So, corresponding to rho T this is the term I am getting. So, now, I multiply these 2 terms which are underlined in green and red and do all the math. So, what I get is, I made a small error this should have been delta t, I think I missed an important term. So, I have to write that again I missed this term. So, I have to write that minus this is.

First thing we notice that this term cancels out this term and I think I omitted one more term here in the parenthesis. So, I will write it again I missed the higher order term this is delta x square minus U of x t rho T x t change in volume times average density here now I think we are right. So, as I mentioned this these 2 terms cancel out the other thing we note is if delta x is very small then this term is negligible compared to these 2 guys. So, I can omit this term also I can omit this term also. So, essentially what I get is U and I am taking delta x out change in volume rho average.

Now, make 1 more assumption no we. So, we make one more simplification what we do is we shift this rho AV to the left hand side. So, I am going to erase average density from here and I am going to put it in the denominator of these terms.

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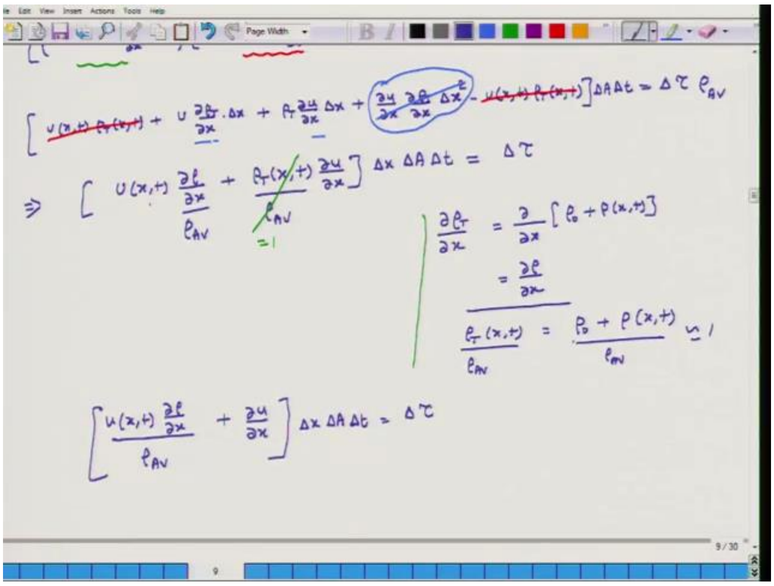

Now what do we observe we observe. So, we have already assumed that U is extremely small and. So, U is extremely small hat is one thing we have observed the other thing is what is partial of density with respect to x. So, this is equal to partial of initial density plus change in density right. So, this is equal to this density rho naught is constant. So, is this is essentially same as partial of rho with respect to partial of x. So, here I can omit this term t and I can make it partial of rho with respect to partial of x this is important.

Second thing rho T at position x t is equal to rho naught plus rho of x t and rho average. So, when I divide this by rho average what is rho average it is the value of rho T the average of different values of rho T along the whole length along this length at different cross sections I will have different values of rho T's, but at these cross section rho T will be rho naught plus rho x t rite and rho naught is extremely is large compare to rho x t. So, this ratio I can safely say is equal to 1. So, I can say this is equal to 1. So, I can rewrite my equation as U x t del rho over del x divided by rho average plus del U over del x times delta x delta A delta t equals change in volume this is my continuity equation.

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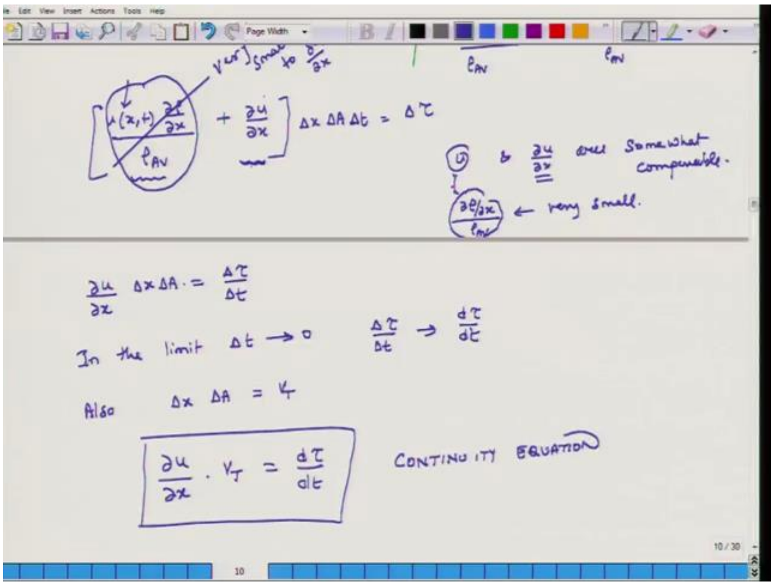

Now, we are going to simplify it further, we have to see compare these 2 terms we have to compare these 2 terms. So, u, this term is partial of U with respect to x and U is very small this is also U x t and this is also U is very small. So, U and partial of U with respect to x are somewhat comparable. Now let us look at del rho over del x and this is the change in density the change in density its partial derivative is shifted and I am dividing it by very large number rho average, this is very small. So, I am multiplying this number and these 2 numbers. So, this is small this is also very small I am multiplying these 2. So, when I multiply these 2 the product is significantly small compare to del U over del x. So, I can say that this is very small compared to del U over del x.

My continuity equation becomes del x del A times and I am going to change, move del t to the right side, this is what I get. Now in the limit del t going to 0 del tau over del t approaches d tau over d t also del x del a equals v t. So, my continuity equation becomes del U over del x times v t equals d tau over d t. So, that is my continuity equation.

So that concludes our discussion on the continuity equation, last day or in the previous lecture we had developed a momentum equation, today we developed the continuity equation, tomorrow we will develop the next equation which relates to the adiabatic process which is occurring in this gas. So, we will apply the gas law and we will develop that equation as well.

We will meet today or tomorrow again and have a great day, bye.