Fundamentals of Acoustics

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Lecture – 14
Momentum Equation for 1-D Sound Propagation

Hello, welcome to Fundamentals of Acoustics. Today is the second day of this week which is the third week of this course and starting today and in next 4 5 modules, what we plan to do is develop the 1 dimension wave equation for sound propagation. For this we will make a number of assumptions, but before we start doing and going over this process, we should have a board understanding at what we are trying to accomplish.

Now in one of our earlier lectures I had mentioned that sound, the way we perceives, it is essentially a pressure fluctuation in air or liquid or some fluid media. So, when sound propagates from point A to point B, essentially physically what is happening is that this pressure fluctuation which is originating at point A is somehow getting transmitted to the second point and what we are interested in knowing is how is the sound or this pressure fluctuating pressure fluctuation getting transmitted between 2 different points.

In that context what that is, what the context is and with that understanding we will try to develop the wave propagation equation or 1 dimensional wave equation.

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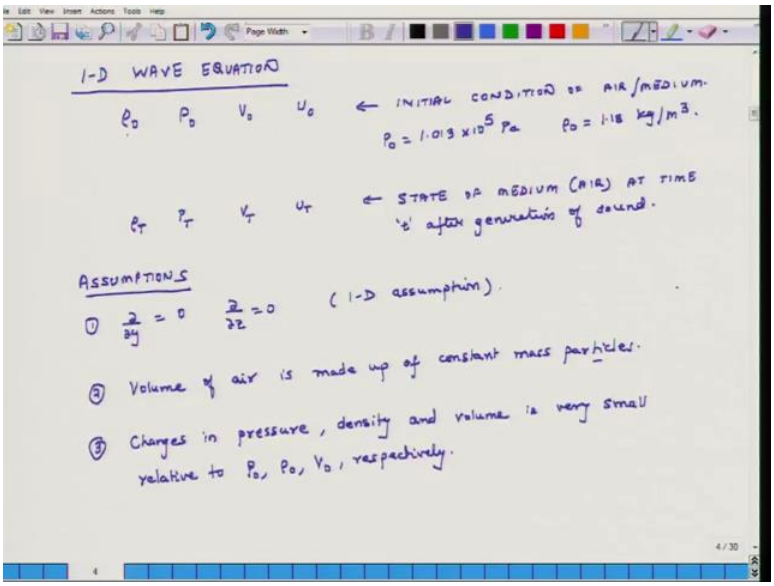

The overall theme is 1-D wave equation. Let us consider a situation that there is a room and in this room, there is initially absolutely no sound and there is no and there is no other disturbance in the air in the room. So, the air in the room is at a steady state and it is not changing with time at all. So, its density is rho naught, the pressure in the room P naught, the volume of the air is V naught and the velocity of the air is U naught. So, U naught could be 0 or it could be just steady flow of air which is just not changing with time. So, this is the initial condition of air or in general I can say this is the steady state initial condition for the medium at standard temperature pressure conditions and if the medium is air then P naught equals 10.13 times 10 to the power of 5 pascals and the density of air is 10.18 kilo grams per cubic meters.

Now, in this air let us think that some sound is produced. So, when the sound gets produced then there is a pressure fluctuation in the air and this pressure fluctuation will initially be at the source and it will over a period of time propagate to all other points in the medium. So, let us call the pressure. So, this is the initial condition and then at time T the density may be rho T, the pressure in the room may shift from P naught to P T and this P T will change from point one point to other point then the volume of air may also the air may expand or contract. So, it may be V T and the velocity may also change. So, initially it was U 0. So, it may change.

These are parameters which define the state of medium and in and especially it could be air at time T after generation of sound. So, now, what we will do is we will make some assumptions. So, when we are trying to figure out how is sound propagating in one specific direction. So, it is not spreading in other directions then it is one dimensional propagation. So, in that case, any variation of density pressure volume or velocity is only going to happen in one specific direction and let us say that direction is X. So, in that case, the partial derivative in all other directions is 0. So, the partial of any entity and this entity could be density pressure velocity or volume in X in Y and Z directions is 0.

This is the 1-D assumption and here we are assuming that it is this 1-D propagation is in Cartesian frame. So, that is why I am using X Y Z coordinates, otherwise I could have chosen R P T or theta coordinates or some other coordinate system, but here we are going to develop wave propagation in one dimension corresponding to Cartesian frame of reference.

The second assumption we make is that volume of air is made up of constant mass particles. So, the overall, so air; the overall air is composed of small particles of air and each particle as it moves in the overall space, its mass does not change. So, what does that mean? What it means is that there is no flow separation happening in the room or in the media. So, this is an important assumptions and then the third thing is that changes in pressure density and volume is very small relative to P naught rho naught V naught respectively.

These are the 3 important assumptions. So, what the third assumptions means is that compare to initial pressure which was atmospheric pressure changes in fluctuations and pressure in the medium at every point were extremely small and the same is true for density as well as volume. So, with this understanding what we can do is I can express pressure.

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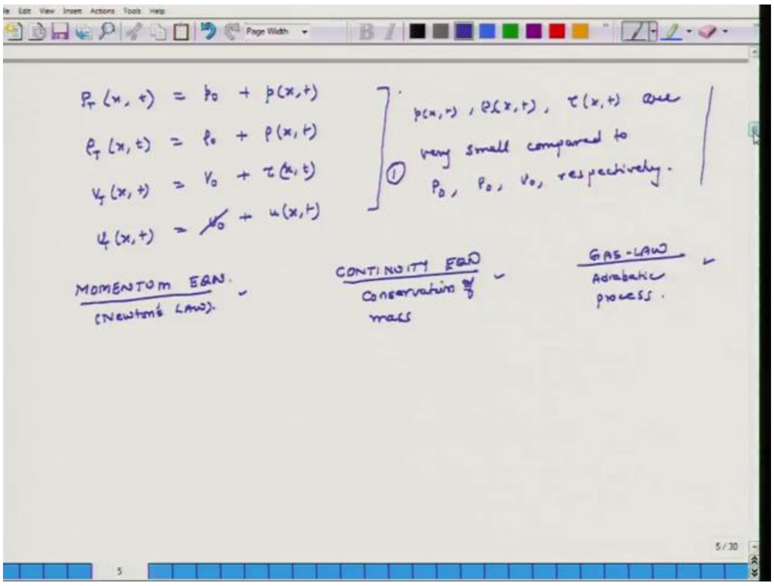

And now we know that pressure is going to change only in one direction which is X and it can also change with respect to time. So, it is not a function of Y and Z because partial of P with respect to Y and Z is 0. So, P T which is the pressure at given time and at a position is equal to P naught which was constant in space as well as time plus some fluctuation in pressure which is very small. So, p small p is the pressure fluctuation and when this occurs then the sum of this and P naught gives us P T similarly rho T equals rho naught plus fluctuation in density which is the function of X and time then volume;

total volume is the sum of original volume plus. So, here change in volume is designated as tau and not as V because later in some context we may use V as voltage.

That is why we are using tau and then U T is equal to U naught plus U of X T. So, in all this 4 cases P X T, rho X T, tau of X T, are very small compared to P naught rho naught and V naught respectively. So, this is by equation 1 and then we can say that if initially air was at rest then U naught was 0 then U T is same as U X T if U naught was 0 to begin with. So, this is my first set of equations this is what is happening and our goal is to figure out how is pressure moving from point a how is pressure fluctuation which is initially happening at some location, how is it getting transmitted to another location that is what we are interested in. So, now, we are set up the overall frame work and now we have to start developing mathematical relations which connect pressure at point a to pressure at point B in that context we have to think about 3 important equations.

The first equation is called momentum equation and essentially what it means is that if you have a piece of air or piece of matter and if it is being acted upon by external forces then the acceleration of this piece of mass which in this case could be a small volume of air will be equal to its mass will be equal to its rate of change of momentum. So, it is essentially coming from Newton's second law and that is why the first equation which we are going to develop will be known as momentum equation.

This will come from Newton's law the second equation will relate to the fact that mass is conserved. So, if you have a volume into which mass is entering and at the other end of the volume mass is exiting and over of period of time and so whatever is the influx and whatever is the out flux or it flux the difference of these two will cause accumulation of mass in that volume and this is related to the fact that mass is conserved for any system. So, that equation is we will call it as continuity equation and that comes from conservation of mass.

The momentum equation comes from Newton's second law continuity equation comes from conservation of mass and the third one is called gas law and that essentially comes from the fact that if you have a pieces of gas and if you compress it depending on how you going are you going to compress it you may compress it either adiabatically or isothermally or isobarically or whatever so or polytropically. So, if a gas is undergoing some change that change is governed by some law. So, if it is going to be an adiabatic compression then the behavior of the gas is going to obey the law P 1 V 1 gamma equals P 2 V 2 gamma, if it is going to observe isothermal expansion then P times V is equal to constant and so on and so forth.

That is the third law and that law we will express in terms of a gas law and here we are going to use adiabatic expansion and compression of the gas, why we will chose that we will be explain later, but at this time we will just write gas law as based on adiabatic process. So, with this framework we are going to develop the momentum equation the continuity equation and the gas law. So, we will start working on the momentum equation.

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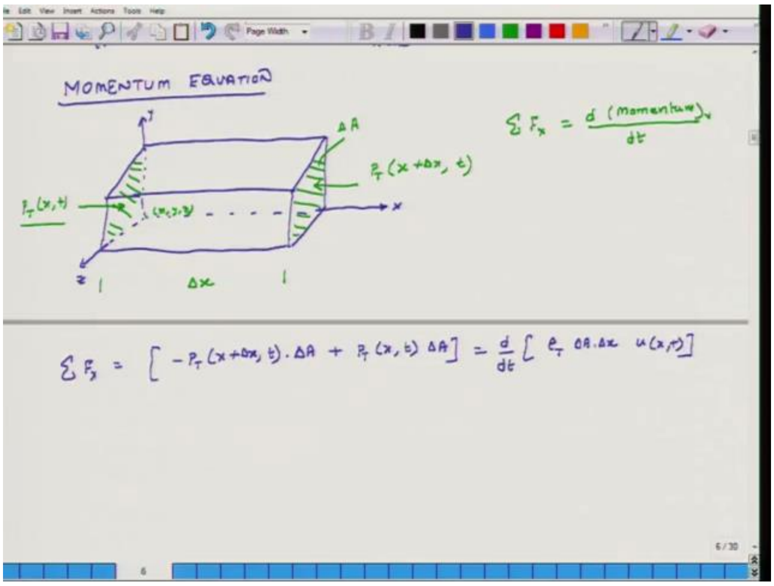

Consider a volume of finite volume of gas and let us say this is my X axis, this is my Y axis, and this is the Z axis. Let us say the pressure on this face P T x t. So, let us say the coordinate of this point is X Y Z. So, at on this plain the pressure P T x t, P T which is the function of X and time and then we assume that these dimension is delta x in the length direction and this area is area is A or delta A.

Now, P T does not change with Y in the Y and X direction because we have initially assumed that partial of P T with respect to Y and Z direction is 0 on this face P T is constant on this phase, but as I change the X coordinate P T may vary similarly on the other face opposite face they will be the fluid will be experiencing some pressure and that pressure is going to be again P T, but because the coordinate has changed from X to

X plus delta x it will be P T X plus delta x and time is same. So, essentially what we are looking at is we are taking a picture of the fluid element at time T freezing it and then at that particular time we are seeing or trying to measure what is the force on the positive face and what is the force on the negative face of the fluid element all pressures in the Y direction and Z direction they will be the same.

So, because of that the fluid will not experience any acceleration in Y or Z direction because all other pressures P T on other phases they will be equal and opposite on opposite phases because of the assumption because of the assumption partial of pressure with respect to Y and partial of pressure with respect to Z is 0 what we are bothered about is only variation of pressure in the X direction.

In momentum equation essentially what we are trying to do is we are trying to put all the forces on the body, when we will some up all those forces and then we will equate it to rate of change of momentum for the system. So, my Newton's law says that sum in X direction sum of all the forces in X direction is equal to D of momentum in X direction with respect to time rate of change of momentum in X direction with respect to time is equal to the external force on the body. So, I am going to develop this expression further. So, sum of forces equals.

So, on the positive phase the force experienced by the body is minus P T X plus delta x times T that is the pressure times area delta A and on the negative face it is. So, on the negative face the direction of force is positive and on the positive face the direction force is negative. So, it is P T x t times delta A and this sum of forces equals rate of change of momentum in the X direction. So, I say D over D T and momentum is mass times velocity. So, the mass of the system is density of the body rho T times volume of the body which is delta a times delta x and times its velocity and velocity is U of x t.

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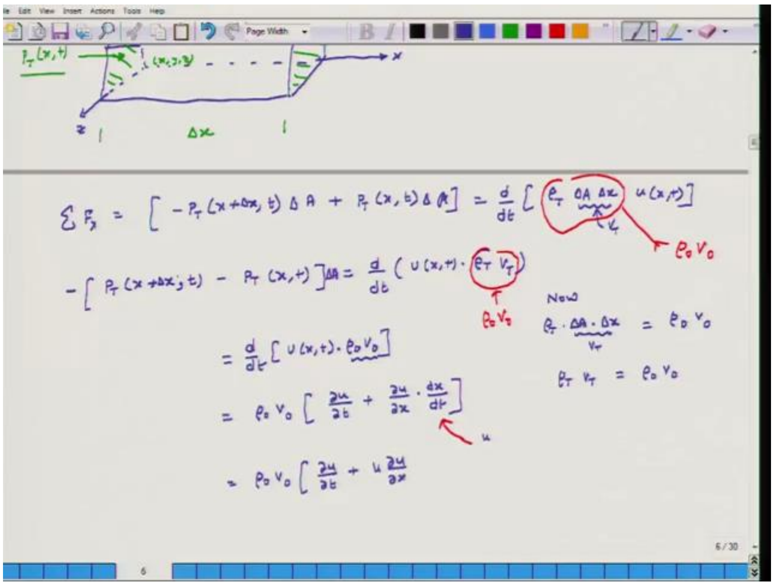

This delta A, it cancels out. So, what I am left with this. So, I will take negative sign outside and I will write P T X plus delta x T minus P T x t equals D over D T U x t times rho T delta x now we know that we had made a constant mass assumption which means constant mass particle assumption which means that rho T times volume was delta a and times the length delta x. So, this is V T delta x a times delta x is V T was same as rho naught V naught is same as rho naught V naught. So, I can write rho T V T equals rho naught V naught. So, I had erase this delta A from here it may be little convenient to not erase at this time.

So, I have introduced it back in the system. So, this is V T this is V t. So, this entire thing can be replaced by rho naught V naught or this thing can be replaced as rho naught V naught. So, I rewrite this equation has D over D T U of x t times rho naught V naught and there is also a delta a here yeah. So, I now rho naught V naught is a constant. So, I can take it out of my differential operator. So, this is equal to rho naught V naught and D over D T can be written as del U over del T plus partial of U with respect to X times D X over D T.

Now, D X over D T what is X, X when we look in this picture is not just some coordinate it is the coordinate of this fluid element the beginning of the fluid elements coordinate is x. So, the change in X represents the change in position of fluid elements change in position of fluid elements which means D X over D T is velocity. So, this can be further written as rho naught V naught partial of U with respect to T plus U del U over del X we had assumed that all changes in U essentially P rho tau and U all these are very small quantities and if they are also very small.

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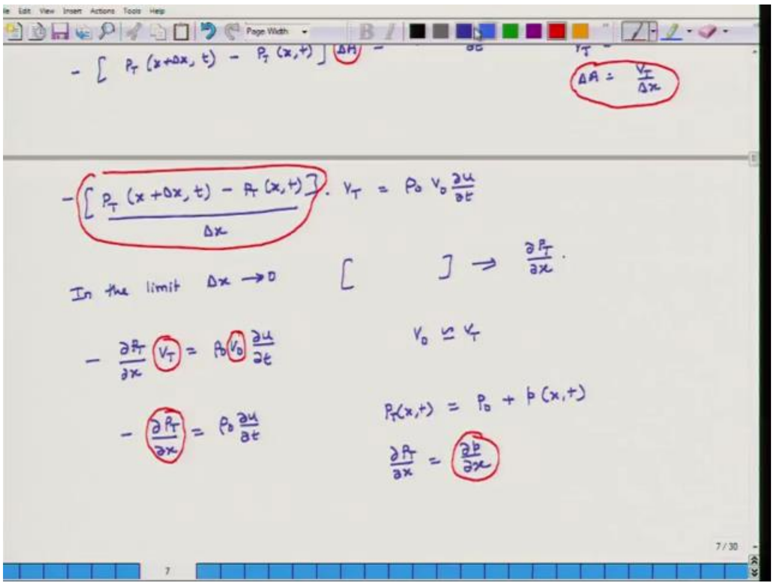

Then I am multiplying partial of U with respect to X with a very small quantity. So, compare to this entity del U over del T, this entity is going to be very small mathematically explain I can express it as partial of U with respect to T is fairly large in turn compare to U times del U over del X.

In that case, I can drop this term and this becomes rho naught V naught del U over del T. So, I can write this overall equation as negative of P T X plus delta x and at T minus P T x t delta a is equal to rho naught V naught del U over del T now we know that V T volume of this fluid elements volume of this fluid elements is delta a times delta x. So, this is equal to delta a times delta x or delta A is equal to V T over delta x. So, I put this thing here and what I get is minus P T X plus delta x times T minus P T x t divided by delta x times V T equals rho naught V naught del U over del T.

The next thing we do is we shrink the size of this delta x. So, we say in the limit as delta x is approaching 0 this term becomes. So, this entire term it approaches the value partial of P T with respect to partial of X why is it because that is the definition of partial derivatives that if times is held constant variable is depended on 2 or 3 more entities and if all other entities are constant which in this cases times. So, T is not changing then difference in that entity which is P T when I change that by a small amount and divide it by delta x is the partial derivative.

So, I can write it as minus P T partial of P T with respect to partial of X and finally, we note that V naught is approximately equal to V T because the changes in volume are very small. So, when I take the ratio of V T and V naught, it is very close to 1. So, if that is the case then these terms cancel out and what; that means, is minus del P T over del X is equal to rho naught del U over del T and we also know that P T is equal to P naught plus P of X T and P naught is constant. So, partial of P T with respect to X is same as partial of P with respect to X.

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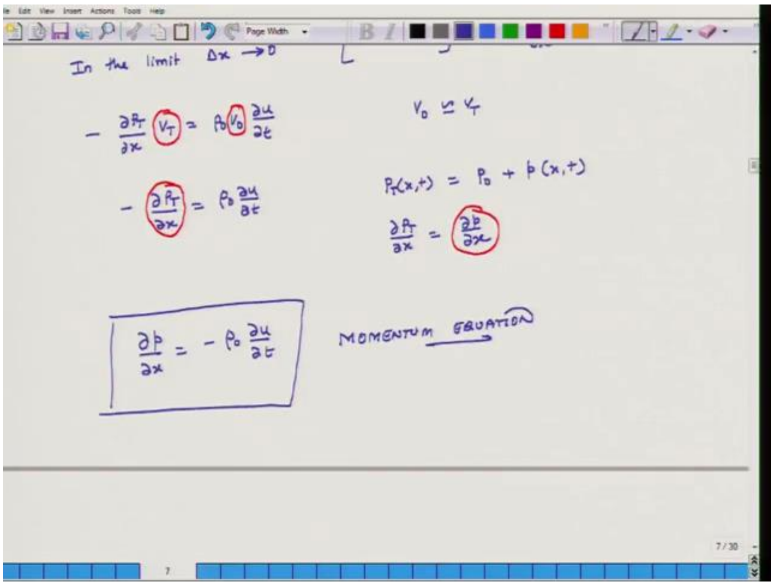

We plug this relation, this equivalence and finally, we get the momentum equation which is partial of P with respect to X is equal to minus rho naught partial of U with respect to T. So, that is my momentum equation. So, that is the momentum equation and that concludes the discussion for today, what we have done today is develop the momentum equation from fundamentals principles. In the next lecture we will develop the continuity equation which is based on conservation of Maths.

Thank you and have a great day, bye.