Fundamentals of Acoustics
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Lecture - 13
Bode Plots (Magnitude) for Complex Transfer Functions

Hello, welcome to Fundamentals of Acoustics. This is the third week of this present course, in the last week we were had concluded the last week by discussing Bode plots and specifically we had covered the phase and the magnitude plots for a simple zero and a simple pole. What we will do this week is we will continue that discussion on Bode plots by developing magnitude plots for more complex functions for illustration purposes and once we are done with that we will proceed to developing one dimensional wave equation for sound propagation. So, in this particular module which is today we will continue our discussion on Bode plots.

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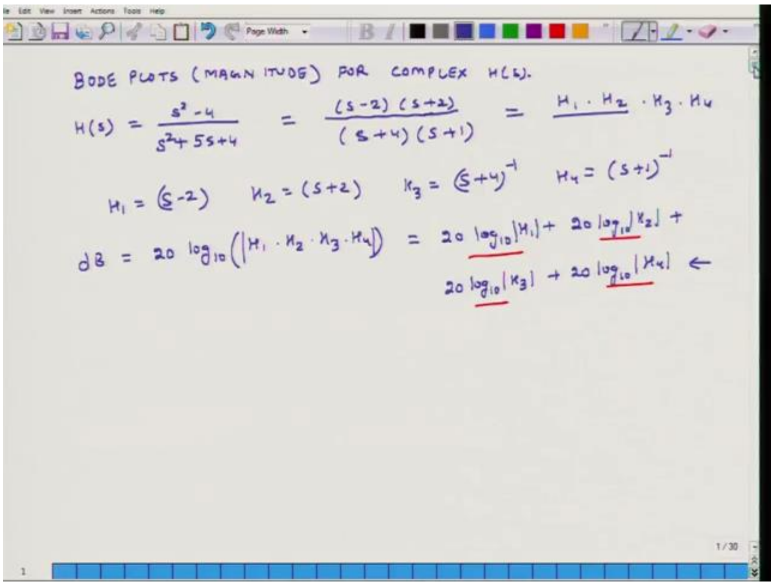

As I mentioned what we are going to discuss today is Bode plots and specifically we will do it for magnitude for complex transfer functions.

We will do it by an example. So, let us say that there are some system which has a transfer function defined as S square minus 4 divided by S square plus 4 S plus 5. So, the first step when I have a complex transfer function which is not just a simple pole or a simple zero is that I factorize the numerator. So, the numerator can be factored as S minus 2 and S plus 2 and the denominator can be factorized in terms of his linear factors as S plus 4. I am going to change this. So, this will be S plus S square plus 5 S plus 4. So, I can factorize it as S plus 4 and S plus 1.

This I can write it as a transfer function H 1 times of transfer function H 2 divided by 2 other transfer functions H 3 and H 4, where H 1 equals S minus 2, H 2 equals S plus 2, H 3 equals 1 over S plus 4 and H 4 equals 1 over S plus 1. So, the way we develop a Bode magnitude plot for this type of a transfer function is that developed Bode magnitude plots for individual linear factors and then we add them up. So, this is the H 3 is S plus 4 and this is S plus 1 or actually the better way would be because that will make mathematics slightly easier is I multiplied by H 3 times H 4. So, H 3 and H 4 are defined like inverse of H plus 4 and H plus 1. So, as I mentioned earlier we developed magnitude plots for each of this individual transfer functions and then we add them up. So, the overall transfer functions if I calculate it in decibels is equal to 20 log of 10 H 1 H 2 H 3 H 4. So, it is equal to 20 log of 10 H 1 plus 20 log of H 2 plus 20 log H 3 plus 20 log of H 4. So, if there are multiples or several factors several individual transfer functions and I have multiplied them up then the overall transfer functions Bode plot as seen in this relation because we are doing it on a log scale is just the some of the individual Bode plots. So, our aim will be to develop Bode plots for each of these individual factors and once we are able to do that then we just add them up and develop the final plot.

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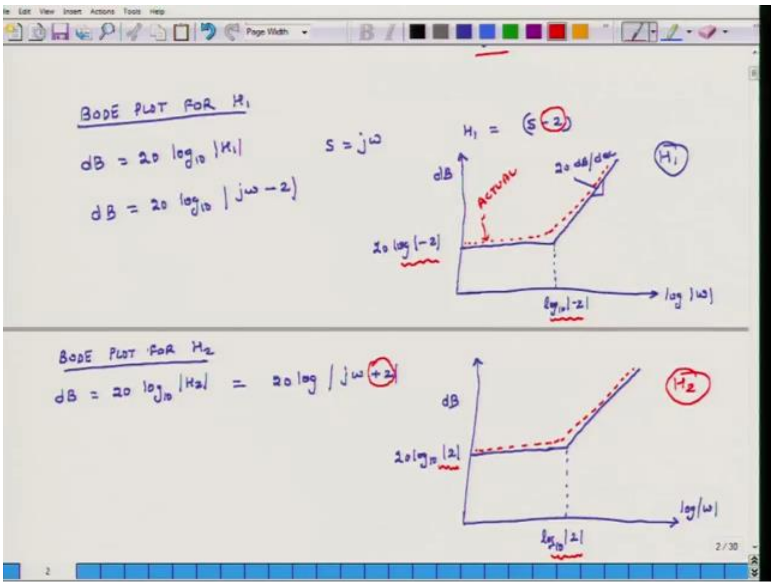

We will do these 1 by 1. Bode plot for H 1 at which d B equals 20 log of H 1 and when we are doing Bode plots S is equal to j omega. So, d B equals 20 and we also know that H 1 equals S minus 2. So, I what I get is j omega minus 2 now this is the simple zero type of a function. So, I know as we saw it in the earlier class that the Bode plots for magnitude for this function has 2 lines, the first line is the low frequency asymptote and the second line is the high frequency asymptote the low frequency asymptote corresponds to the condition when omega goes to 0. So, low frequency asymptote on the x axis, I am going to plot log of omega and on the y axis, it is going to be in decibels and this is what we are going for H 1. So, the low frequency asymptote is the straight line and this value when omega approaches 0 the straight horizontal line its value is going to be 20 log of minus 2 parenthesis.

It is not negative 20 log to 2 matrix. So, essentially it is 20 log 2 times 20 times log of 2 and the high frequency asymptote is 20 log of j omega. So, it is going to be a line with the positive slope and this slope is 20 d B per decade and the point of a inter section of this curve is going to be were log of omega is going to be same as log of 2. So, this is log of minus 2 log of absolute value of negative 2. So, this is my Bode plots for H 1 and the red curve is the actual curve actual disables where the dark blue line is for the asymptote is response. So, the next will be do is Bode plot for H 2. So, this is equal to d B equals 20 log of absolute value of H 2 and once I do the substation as equal's j omega and S 2 is S plus 2 what I get is 20 log of j omega plus 2.

Now, I can draw the Bode plot for this, but I did not do this in this particular case because I see that or actually I will just do that for sake of completeness. So, the Bode plot for this we once again. So, that is the Bode plot for H 2 the magnitude plot for H 2 now when you compare the magnitude plot for the H 1 and H 2 shape wise they are identical because both of them correspond to simple zero, but also in terms of values they are identical because this value log of absolute value of 2 and log of absolute value of negative 2 is same and also the cross over point in H 1 is log of absolute value of negative 2 and log of absolute value of 2.

These Bode plots for H 1 and H 2, they are just turned out to be same and the reason is that this term and this term, these 2 terms they have the same absolute magnitude. So, what does means is that if the absolute magnitude of the 0 is same then the Bode plot for simple zeros will be same the absolute the value of 0 could be positive or negative or even complex, but as long as its magnitude is same then the magnitude plot is going to be the same the phase plot may still not be the same because when we developed calculated the phase it will depend on how what is the real component and what is the imaginary component, but as long as the value of the 0 absolute value of the 0 is same the Bode plot for a simple zero is going to be the same thing.

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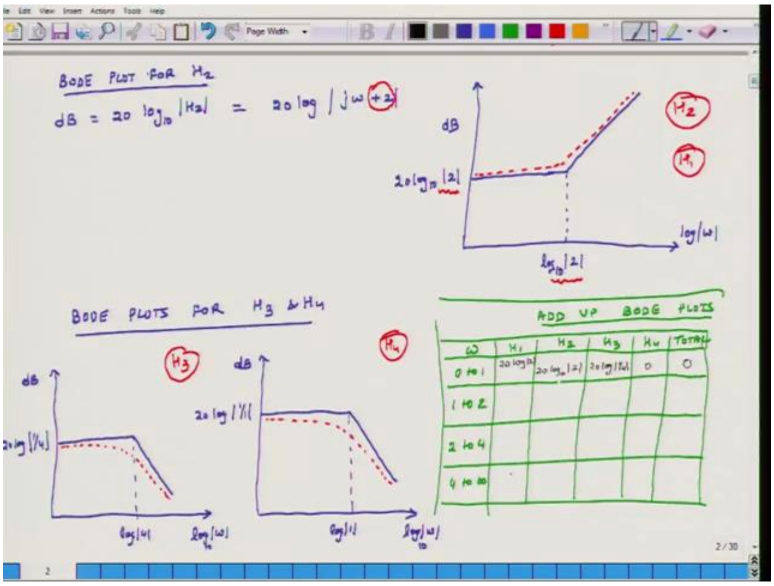

This is for H 1 and H 2 and next we will develop Bode plots for H 3 and H 4. So, we are going to directly develop the Bode plots because I thing we have done sufficient amount of practice. So, both of H 3 and H 4 are simple holes they are simple poles. So, what; that means, is that their Bode plots. So, my horizontal axis in both these cases log of omega and on the vertical axis or y axis I have decibels.

The Bode plots for these 2 guys because they are both of them are simple poles is going to be the low frequency asymptote is going to be a horizontal line and then the high frequency asymptote is negatively slope line something here and value here is going to be 20 log 1 by 4 for H 3 and in this case, it is going to be 20 log 1 by 4 for I am sorry this is going to be it is going to be 1 over 1. So, this is going to be for H 3 and this is going to be for H 4 and the cross over points are going to be. So, cross over point is going to occur when log of one over omega is going to be log of one over 4. So, which means this is going to be log of 4 and this is going to be log of 1.

What we have done is we have constructed all the 4 Bode plots for individual factors linear factors for H 1 H 2 H 3 and H 4 and we have seen that the curve for H 2 is same as curve for H one. So, I am going to also write it same as here also in H 3 and H 4 the actual response for simple poles is going to be like this is going to be below the asymptotic response in 0 simple zero its above in simple zero poles it below the asymptotic response. So, now, I make some observations. So, what is my next step my aim is to develop the Bode plot for the entire function.

What I do is I add up the Bode plots and the way I add up it is that on the frequency range I see what are the important what are the break points were the curve is expected to change. So, I construct a table. So, the first range is 0 to 1 because the Bode plot for H 4 changes it slope after one the next break point is 2 because H 2 and H one the change at 2. So, basically 1 to 2 that is the next range for omega then we see that S 3 changes at slope at 4. So, my next range is going to be 2 to 4 in this range in this individual range is shape of the curve is not going to change and then the next and final range is 4 to infinity.

I will fill up this table. So, what we are going to do is we are going to list down contributions of individual transfer functions H 1 H 2 H 3 H 4 and then we are going to total it up. So, the contribution from H one in the range 0 to 1 is. So, we look at it contribution from H 1 in the range 0 to 1. So, this is 20 log 2 20 log 2 contribution from H 2 is also 20 log 2 contribution from H 3 is in this range 20 log 1 by 4 and contribution from H 4 is 20 log 1 over 1 which is 0 log of 10 or log of 1 is 0.

When I add these up the total decibels log 2 plus log 2 is 2 log 2 and log of 1 over 4 is log of 1 over 4 is minus of log 4. So, that. So, the H 1 H 2 and H 3 will add up to cancel each other and H 4 is any ways 0. So, in the range 0 to 1 the contribution from over all contribution is going to be 0 decibels and we can fill up other all other parts of the table, but it just turns on that at list in this case we do not necessarily have to fill up all the other parts of the table.

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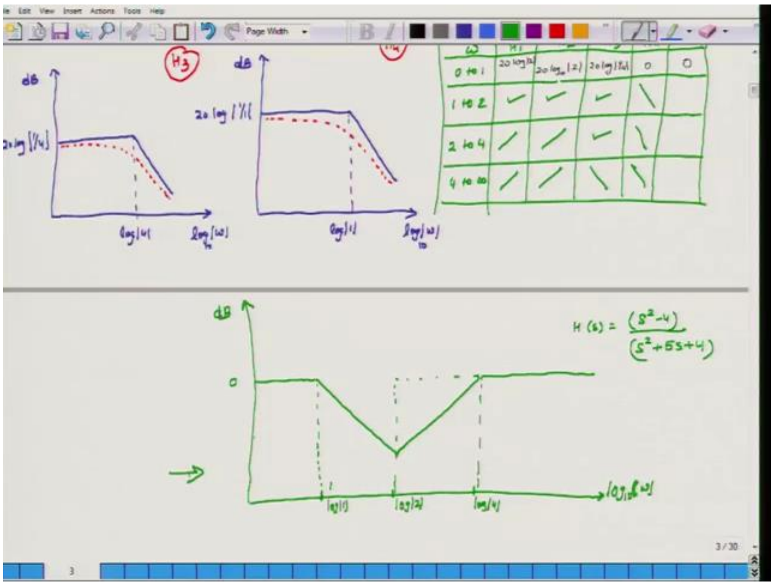

What we will do is we will now construct the overall Bode plot. So, overall Bode plot vertical x is in decibels. So, first we will do is we will marked down different points. So, first is 0 to 1. So, this is log of 1 then 1 to 2. So, this is log of 2 then 1 to 4 2 to 4. So, this is log of 4 and then 4 to infinity.

In the range 0 to 1 the contribution is 0 decibels, from 1 to 2 what we see is that contribution from H 1 is still the same, it does not change from H 2 it is still the same because H 1 and H 2 are same up to 2 and for H 3 it is still the same, but from H 4, I have a negatively slope line when we look at H 4 after 1 the high frequency asymptote takes over. So, what; that means, is that the contribution from H 1 H 2 3 H 2 is still remain 0, but because of H 4 the overall curve is starts sloping down. So, actually I am going to modify this curve a little bit. So, that I can draw easily, I am going to draw this is 0 decibels. So, after this it is going to be minus 20 decibels per decade this is going to be slope.

Now, let see what happens to the range 2 to 4, when we look at H 2 and H 1 after 2, the slope is positive after 2 slope is positive and it is 20 decibels per decade. So, the overall slope of this H 1 and H 2 will become 40 decibels per decade. So, this is 20 decibels per decade 20 decibels per decade this is still 0 20 log 1 over I am sorry 20 log 1 over 4.

And this is negative. So, what; that means that the contribution from H 1 and H 2 will make sure that this should. So, I am getting 20 decibels positive decibels from H 1 positive 20 decibels from H 2 and negative 20 decibels per decade from H 4. So, the overall slope becomes like this and then from 4 to infinity the slopes are positive 20 decibels for H 1 positive 20 decibels per decade for H 2 and negative 20 decibels for per decades for H 3 and H 4. So, the overall slope becomes 0. So, that is my overall transfer function. So, this is Bode plot for H is that is the overall Bode magnitude plot.

Now, if we are interested in developing the actual plot then what we have to do is we have to actually plot the thing and at some points the actual plot may curve the, cut the green line at some and at other points it may be somewhat parallel to it. So, this is the overall Bode plot. So, in this way we can construct the overall Bode plots for transfer functions which are not simple zeros are simple poles.

Similarly, if we were interested in developing Bode plots for phase, we know that for a complex function which is the multiple of different simple functions phase of the overall function will be phase of first part plus phase of second part plus phase of third part and so on so forth.

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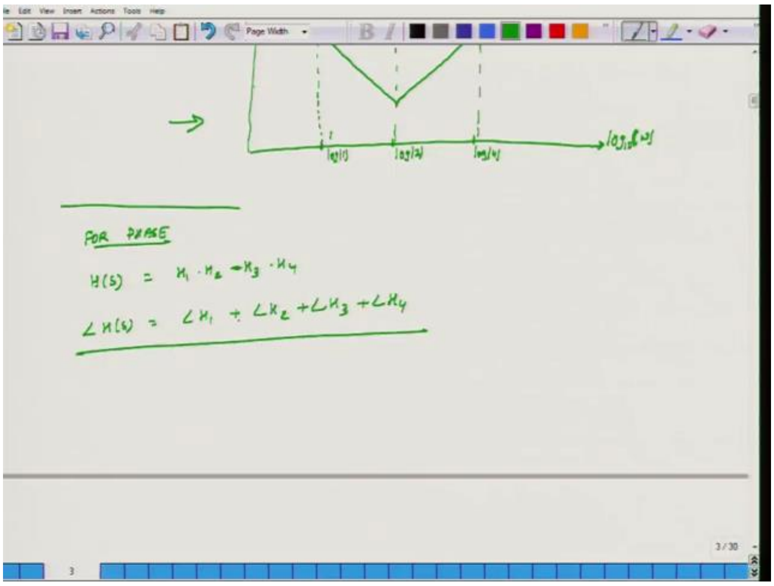

What do I mean by that for phase we are shown that we can break this H in this case, as H 1 times H 2 times H 3 times H 4.

Then phase of H S is equal to phase of H 1 plus phase of H 2 pus phase of H 3 plus phase of H 4. So, if we can develop individual Bode plots for phase plots for H 1 H 2 H 3 H 4 we can add them up and again develop the overall Bode plot. So, the method for developing magnitude plot and Bode phase plot, it means essentially the same.

This concludes our discussion on Bode plots and we will close for today and starting tomorrow, what we will do is we will start developing the one dimensional wave equation for sound propagation.

Thank you very much and I look forward to seeing you tomorrow, bye.